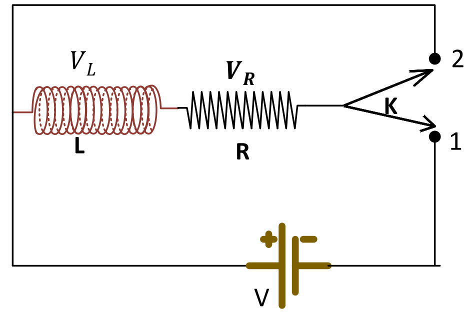
# TRANSIENT PHENOMENA IN ELECTRIC CIRCUITS

During the **time interval** between **switching operation** and the **final steady state** of current or charge the current and voltage in the circuit change with time **in a non-periodic manner**. Such variations using exist for a short time and this is called the transient phenomena or simply transients in electric circuits.




- **Growth and decay of currents in series LR circuit.**

❖ **Growth of current**

If the switch K suddenly thrown to the position 1 at time $t = 0$.
As the current in the circuit begins to grow the magnetic flux linked with the inductor also increases and consequently according to Faraday's law of induction a back emf $-\boldsymbol{L\frac{di}{dt}}$ is generated in the circuit which oppose the driving emf V and thus, delays the process of the growth of current.

So the net emf in the circuit is $(V - \boldsymbol{L}\frac{di}{dt})$ and hence, from Ohm's law, the instantaneous current $i = \frac{(V-\boldsymbol{L}\frac{di}{dt})}{R}$

Or, $\boldsymbol{L\frac{di}{dt}} + Ri = V$ or, $\boldsymbol{L\frac{di}{dt} = V - Ri}$ **or,** $\frac{di}{V-Ri} = \frac{1}{L}dt$ or, $\frac{di}{i-\frac{V}{R}} = -\frac{R}{L}dt$

**Assuming that $\boldsymbol{i = 0}$ at $\boldsymbol{t = 0}$** We get on integration $\int_0^i \frac{di}{i-\frac{V}{R}} = -\frac{R}{L}\int_0^t dt$

Or, $\ln\frac{i-\frac{V}{R}}{-V/R} = -\frac{R}{L}t$ or, $\frac{i-\frac{V}{R}}{-V/R} = e^{-\frac{R}{L}t}$ or, $i - \frac{V}{R} = -\frac{V}{R}e^{-\frac{R}{L}t}$ or, $i = \frac{V}{R}(1 - e^{-\frac{R}{L}t})$ or, $\boxed{\boldsymbol{i = I_0(1 - e^{-\frac{R}{L}t})}}$ **[ where $\boldsymbol{I_0 = \frac{V}{R}}$ ]**

The **rate of growth** of the current is $\boldsymbol{\frac{di}{dt} = \frac{I_0R}{L}e^{-\frac{R}{L}t} = \frac{R}{L}I_0e^{-\frac{R}{L}t} = \frac{R}{L}(I_0 - i)}$ **,**

**Hence rate of growth depend upon $\boldsymbol{\frac{R}{L}}$**. The quantity $\boldsymbol{\frac{L}{R} = \tau}$ has the dimension of time and is called **time constant** of this circuit. The **time constant** may be defined **as the time** in which the **current in the circuit grows from 0 to 63%** of its final steady state.

NOTE THAT $\boldsymbol{V_L = L\frac{di}{dt} = L\left(\frac{R}{L}I_0e^{-\frac{R}{L}t}\right) = RI_0e^{-\frac{R}{L}t}}$ **and** $\boldsymbol{V_R = Ri = RI_0(1 - e^{-\frac{R}{L}t})}$.

**Hence instantaneous $\boldsymbol{V = V_L + V_R = RI_0e^{-\frac{R}{L}t} + RI_0(1 - e^{-\frac{R}{L}t}) = RI_0}$=constant**

When the current reaches the maximum steady value the variation of current with time **stops so back emf is zero.**
At this stage the circuit behaves as if there were **no inductor in the circuit** the instantaneous voltage drops across L and R and given by figure -2

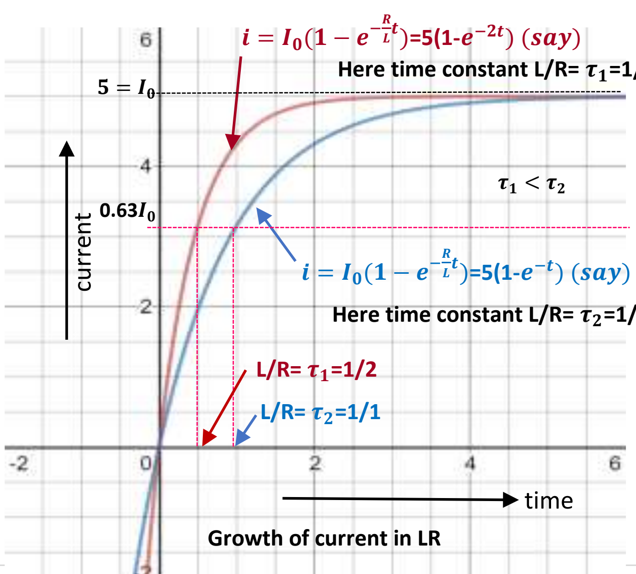



FIGURE 1

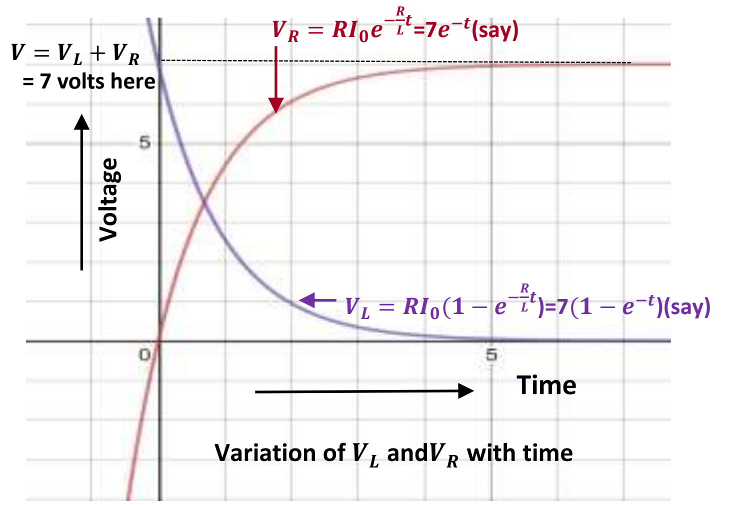



FIGURE 2

## ❖ Decay of current

suppose a steady current has been set up in the LR circuit by connecting the switch K **to** the position 1 for sufficient time. Now the key is suddenly thrown from position 1 to position 2. As a result, the current in the circuit is cut off and current in the circuit begins to fall hence the flux linked with inductor decreases with time and consequently an emf $-\boldsymbol{L}\frac{\boldsymbol{dI}}{\boldsymbol{dt}}$ is generated to oppose the decay of current.

In this case $V = 0$ and the instantaneous current $i = \frac{(0-L\frac{dI}{dt})}{R}$

**Assuming that $i = I_0$ at $t = 0$** We get on integration $\int_{I_0}^{i}\frac{di}{i} = -\frac{R}{L}\int_0^t dt$

Or, $\ln\frac{i}{I_0} = -\frac{R}{L}t$ or, $\frac{i}{I_0} = e^{-\frac{R}{L}t}$ or, $\boxed{i = I_0 e^{-\frac{R}{L}t}}$ [ where $I_0 = \frac{V}{R}$ ]

The **rate of decay** of the current is $\frac{di}{dt} = -\frac{I_0R}{L}e^{-\frac{R}{L}t} = -\frac{I_0}{\tau}e^{-\frac{R}{L}t}$ , or, $\left.\frac{di}{dt}\right|_{t=0} = -\frac{I_0}{\tau}$

**Hence rate of decay current depends upon time constant** $\tau = \frac{L}{R}$. At $t = \tau$, $i = \frac{I_0}{e} \approx 0,37I_0$

The time constant $\tau = \frac{L}{R}$ maybe also defined **as the time** in which the **current decays to 37%** of its initial value.

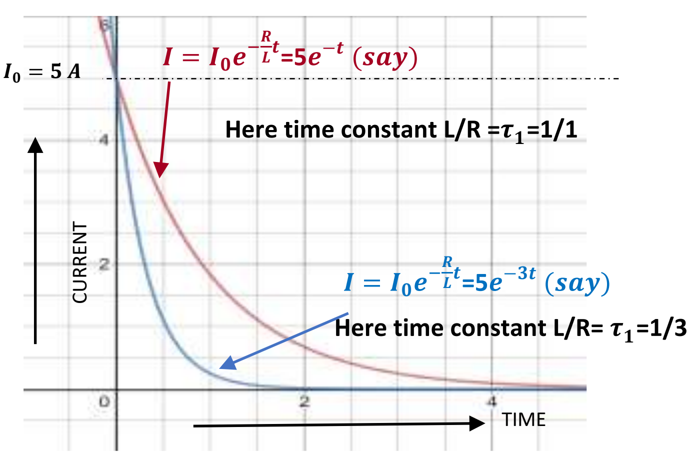
The instantaneous voltage drop across L and $R$ are given by $V_L = L\frac{dI}{dt} = L\left(-\frac{I_0R}{L}e^{-\frac{R}{L}t}\right) = -I_0Re^{-\frac{R}{L}t} = -Ve^{-\frac{R}{L}t}$

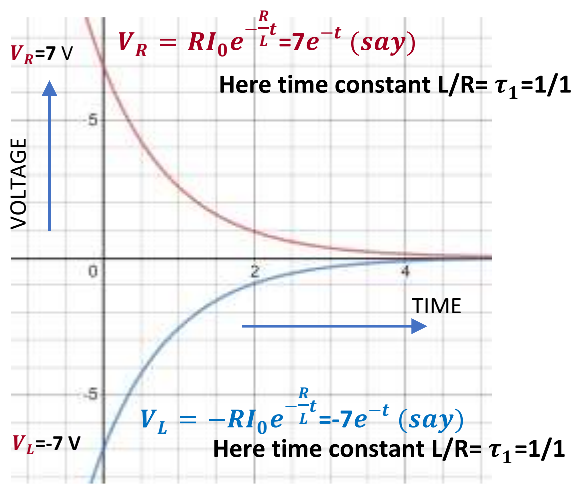
**And** $V_R = Ri = RI_0e^{-\frac{R}{L}t} = Ve^{-\frac{R}{L}t}$

Thus $V_L$ and $V_R$ Are always equal and opposite due to the fact now $V_C + V_R = 0$.

NOTE THAT

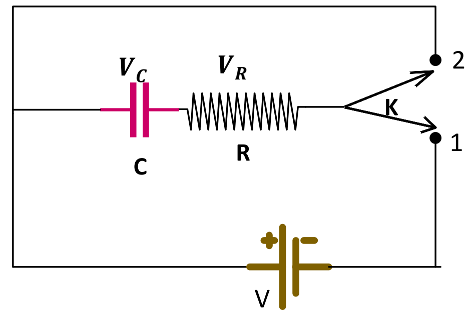
1) **when an inductive circuit is suddenly broken** the air gap at that point of breaking makes the effective resistance of the circuit very large hence time constant very small. The current decreases very rapidly which in turn **produces a large back emf. Hence the insulation of air gap may break down and electric sparks may appear.**
2) During growth of current the source V supplies energy a part of which lost due to joule heating and another part goes to build up magnetic field around L. During decay of current **the stored magnetic field energy is used up in driving current in the circuit**



## ▪ Charging and discharging of a capacitor in series CR circuit

### ❖ Charging of capacitor

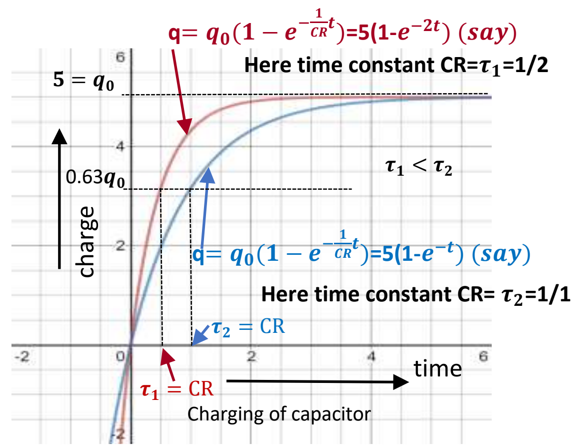



If the switch K suddenly thrown to the position 1 at time $t = 0$.
The capacitor is now charged by the source. Some electron from the negative terminal of the source flow to one plate of the capacitor and equal number of electron flow out of the other plate into the positive terminal of the source. This is known as charging of a capacitor.
But no steady current can flow through the dielectric of the capacitor.
When potential difference across C is equal to the supply emf V then charging becomes complete and current in the circuit becomes zero.

**Note that the source tries to introduce more charge on the capacitor but the charges already acquired by the plate oppose the introduction of further charges on the plates. So the source has to do some work in charging the capacitor. A part of the energy supplied by the source in lost as Joule heat in R and rest goes to build up electrostatic field around C.**

Let $q$ be the charge on the positive plate of the capacitor and $i$ be the current at any time $t$

so the instantaneous emf equation of the circuit is




$V_C + V_R = V$ or, $\frac{q}{C} + Ri = V$ or, $\frac{q}{C} + R\frac{dq}{dt} = V$ or, $\frac{dq}{q-CV} = -\frac{1}{CR}dt$

**Assuming that $q = 0$ at $t = 0$** we get on integration

$\int_0^q \frac{dq}{q-CV} = -\frac{1}{CR}\int_0^t dt$ or, $\ln\frac{q-CV}{-CV} = -\frac{1}{CR}t$

or, $\frac{q-CV}{-CV} = e^{-\frac{1}{CR}t}$ or, $q - CV = -CVe^{-\frac{1}{CR}t}$ or,

$q = CV(1-e^{-\frac{1}{CR}t})$ or, $\boxed{q = q_0(1-e^{-\frac{1}{CR}t})}$ **[ where $q_0 = CV$ ]**

The rate of charging or instantaneous current in the circuit is given by

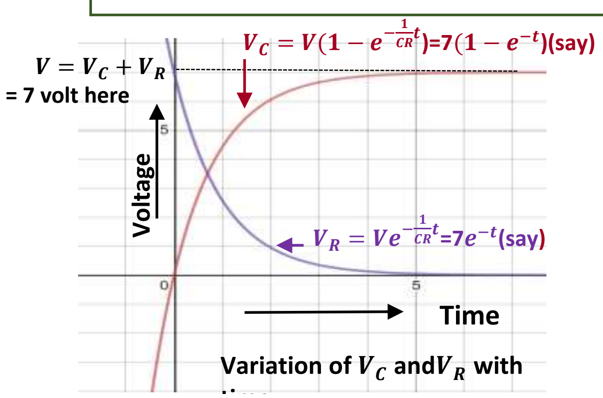
$i = \frac{dq}{dt} = \frac{q_0 1}{CR}e^{-\frac{1}{CR}t} = \frac{CV}{CR}e^{-\frac{1}{CR}t} = \frac{V}{R}e^{-\frac{1}{CR}t} = I_0 e^{-\frac{1}{CR}t}$ **[ where $I_0 = \frac{V}{R}$ ]**

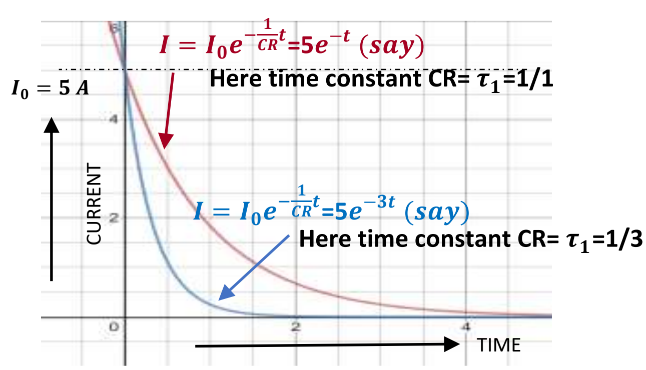
Note that initially there is no potential difference across C and the total applied emf is across R.

**Hence rate of charging depends upon CR**. The quantity $CR = \tau$ has the dimension of time and is called **time constant** of this circuit. As for $t = \tau = CR$, $q = q_0\left(1-e^{-\frac{1}{CR}CR}\right) = q_0(1-e^{-1}) = q_0\left(1-\frac{1}{e}\right)$=0.63$q_0$

The **time constant** may be defined **as the time** in which the **charge in the circuit grows from 0 to 63%** of its final steady state.

| |
|---|
| **NOTE THAT** $V_C = \frac{q}{C} = \frac{q_0}{C}\left(1-e^{-\frac{1}{CR}t}\right) = V\left(1-e^{-\frac{1}{CR}t}\right)$ **and** $V_R = Ri = R\frac{V}{R}e^{-\frac{1}{CR}t} = Ve^{-\frac{1}{CR}t}$ |
| **Hence** $V = V_C + V_R = V\left(1-e^{-\frac{1}{CR}t}\right) + Ve^{-\frac{1}{CR}t} = V$ |



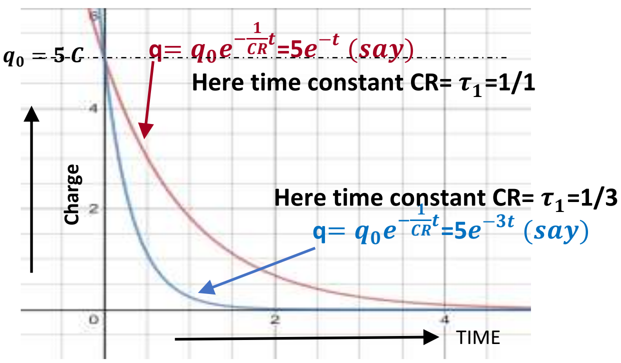



## ❖ Discharging of capacitor

Suppose the capacitor has been fully charged by connecting the switch K to the position 1 for sufficient time. Now the key is suddenly thrown from position 1 two position 2 as a result the source is cutoff. The charge stored in the capacitor begins to discharge through R. Physically excess electrons from the negative plate of the capacitor flows through R to its positive plate where there is a deficiency of electrons. The process continues till the potential difference across capacitor becomes zero. the phenomenon is known as **discharging of capacitor** in this process the **electrostatic energy stored in the capacitor is dissipated as a Joule heat in the resistor R.**

Let $q$ be the charge on the positive plate of the capacitor and $i$ be the current at any time $t$

so the instantaneous emf equation of the circuit is




$V_C + V_R = 0$ or, $\frac{q}{C} + Ri = 0$ or, $\frac{q}{C} + R\frac{dq}{dt} = 0$ or, $\frac{dq}{q} = -\frac{1}{CR}dt$

Assuming that $q = \boldsymbol{q_0}$ at $t = 0$ we get on integration

$\int_{q_0}^{q} \frac{dq}{q} = -\frac{1}{CR}\int_0^t dt$ or, $\ln\frac{q}{-q_0} = -\frac{1}{CR}t$

or, $\frac{q}{-q_0} = e^{-\frac{1}{CR}t}$ or, $\boxed{q = \boldsymbol{q_0} e^{-\frac{1}{CR}t}}$ or, **[ where $\boldsymbol{q_0 = CV}$ ]**

The rate of decay of charge or instantaneous current in the circuit is given by

$\boldsymbol{\frac{dq}{dt} = -\frac{q_0 1}{CR} e^{-\frac{1}{CR}t} = -\frac{CV}{CR} e^{-\frac{1}{CR}t} = -\frac{V}{R} e^{-\frac{1}{CR}t} = -I_0 e^{-\frac{1}{CR}t}}$ **[ where $\boldsymbol{I_0 = \frac{V}{R}}$ ]**

**[negative sign indicates discharging current flow in a direction opposite of charging current.]**

**Hence rate of decay of charge depends upon time constant $\boldsymbol{\tau = CR}$**. At $\boldsymbol{t = \tau = CR}$, $\boldsymbol{q = \frac{q_0}{e} \approx 0,37q_0}$

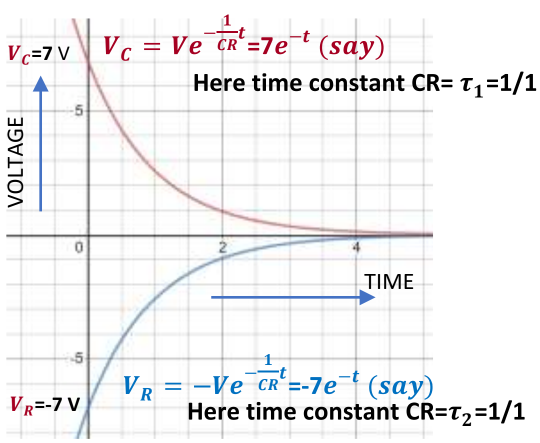
The time constant $\boldsymbol{\tau = CR}$ maybe also defined **as the time** in which the **charge decays to 37%** of its initial value.

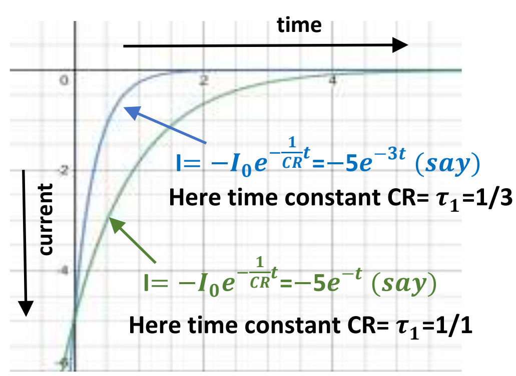
The instantaneous voltage drop across $C$ and $R$ are given by

$$V_C = \frac{q}{C} = \frac{\boldsymbol{q_0}}{\boldsymbol{C}} e^{-\frac{1}{CR}t} = \boldsymbol{V e^{-\frac{1}{CR}t}}$$

**And** $V_R = Ri = -R\boldsymbol{I_0 e^{-\frac{1}{CR}t}} = \boldsymbol{-V e^{-\frac{1}{CR}t}}$

Thus $V_C$ and $V_R$ Are always equal and opposite due to the fact $V_C + V_R = 0$.



# Series LCR circuit

<u>Charging of the capacitor:</u>

Let consider circuit if the switch K suddenly thrown to the position 1 at time $t = 0$. The capacitor is now charged by the source of voltage V. Let $q$ be the charge on the positive plate of the capacitor and $i$ be the current at any time $t$ so the instantaneous emf equation of the circuit is

$V_L + V_C + V_R = V$ or, $\boldsymbol{L}\frac{\boldsymbol{dI}}{\boldsymbol{dt}} + \frac{q}{C} + Ri = V$ or, $\boldsymbol{L}\frac{\boldsymbol{d^2q}}{\boldsymbol{dt^2}} + \frac{q}{C} + R\frac{dq}{dt} = V$ or, $\boldsymbol{L}\frac{\boldsymbol{d^2q}}{\boldsymbol{dt^2}} + R\frac{dq}{dt} + \frac{q}{C} = V$

Or, $\frac{\boldsymbol{d^2q}}{\boldsymbol{dt^2}} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{CL} = \frac{V}{L}$ or, $\frac{\boldsymbol{d^2q}}{\boldsymbol{dt^2}} + 2b\frac{dq}{dt} + \omega_0^2 q = \frac{V}{L}$ Let $\frac{R}{L} = 2b$, $\frac{1}{CL} = \omega_0^2$

Introducing parameter $Q$ such that $\omega_0^2 Q = \omega_0^2 q - \frac{V}{L}$ or, $Q = q - \frac{V}{L\omega_0^2}$ we get , $\frac{\boldsymbol{d^2Q}}{\boldsymbol{dt^2}} + 2b\frac{dQ}{dt} + \omega_0^2 Q = 0$…..(i)

Let $Q = Ae^{\alpha t}$ then we have from equation (i) $\alpha^2 Ae^{pt} + 2b\alpha Ae^{pt} + \omega_0^2 Ae^{\alpha t} = 0$

Or, $\alpha^2 + 2b\alpha + \omega_0^2 = 0$ as $Ae^{\alpha t} \neq 0$

Then $\alpha = -b \pm \sqrt{b^2 - \omega_0^2} = -b \pm p$ (say) where $p = \sqrt{b^2 - \omega_0^2}$ Then three cases may arise

I) <u>when $b > \omega_0$</u> (overdamped condition) then $Q = A'^{e^{(-b+p)t}} + B'^{e^{(-b-p)t}} = e^{-bt}(A'e^{pt} + B'e^{-pt})$

then $q = \frac{V}{L\omega_0^2} + e^{-bt}(A'e^{pt} + B'e^{-pt}) = q_0 + e^{-bt}(A'e^{pt} + B'e^{-pt})$ where $q_0 = \frac{V}{L\omega_0^2}$

now assuming that at $\boldsymbol{t = o\ q = 0\ and\ i = 0}$

**we get 0** $= q_0 + A' + B'$ and $0 = (A' - B')p - (A' + B')b$

**then** $A' + B' = -q_0$ & $(A' - B') = -q_0\frac{b}{p}$ then $A' = -\frac{q_0}{2}\left(1 + \frac{b}{p}\right)$ & $B' = -\frac{q_0}{2}\left(1 - \frac{b}{p}\right)$

**Hence finally** $\boldsymbol{q} = q_0 + e^{-bt}\left[-\frac{q_0}{2}\left(1 + \frac{b}{p}\right)e^{pt} - \frac{q_0}{2}\left(1 - \frac{b}{p}\right)e^{-pt}\right]$

$$\boxed{\boldsymbol{q} = q_0 - \frac{q_0}{2}e^{-bt}\left[\left(1 + \frac{b}{p}\right)e^{pt} + \left(1 - \frac{b}{p}\right)e^{-pt}\right]}$$

Hence charge on capacitor grows gradually and ultimately acquires a steady state value $q_0$.

II) <u>when $b = \omega_0$ (critical condition)</u> then $p = \sqrt{b^2 - \omega_0^2} \to 0$ and $e^{pt} \to 1 + pt$ & $e^{-pt} \to 1 - pt$

then $q = q_0 - \frac{q_0}{2}e^{-bt}\left[\left(1 + \frac{b}{p}\right)(1 + pt) + \left(1 - \frac{b}{p}\right)(1 - pt)\right] = q_0 - \frac{q_0}{2}e^{-bt}\left[1 + pt + \frac{b}{p} + bt + 1 - pt - \frac{b}{p} + bt\right]$

$$\boxed{q = q_0 - q_0 e^{-bt}(1 + bt)}$$

hence again charge on capacitor grows gradually and ultimately acquires a steady state value $q_0$.But compared to overdamped value a smaller time is required to reach the steady state.

NOTE THAT As $b = \omega_0$ then $\frac{R}{2L} = \sqrt{\frac{1}{CL}}$ or, $R = 2\sqrt{\frac{L}{C}}$ is called **critical damping resistance**.

III) <u>when $b < \omega_0$</u> then $p = \sqrt{b^2 - \omega_0^2} = \sqrt{-(\omega_0^2 - b^2)} = j\omega$ (say) where $\omega^2 = (\omega_0^2 - b^2)$

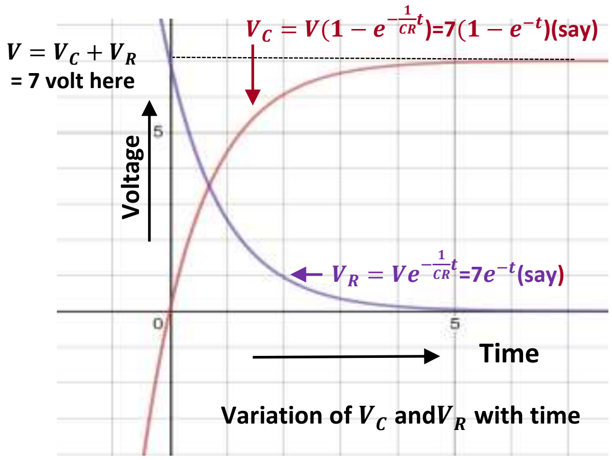
**then** $\boldsymbol{q} = q_0 - \frac{q_0}{2}e^{-bt}\left[\left(1 + \frac{b}{p}\right)e^{j\omega t} + \left(1 - \frac{b}{p}\right)e^{-j\omega t}\right]$

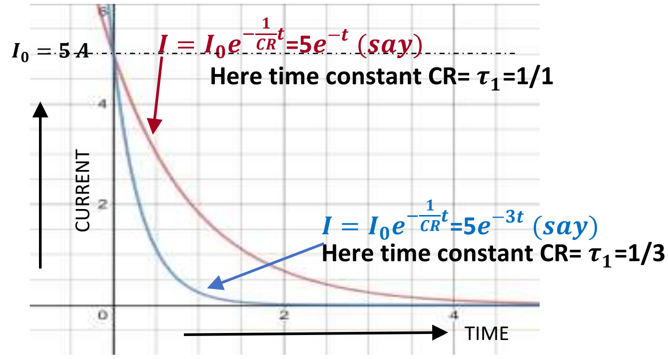
or, $q = q_0 - \frac{q_0}{2}e^{-bt}\left[(1 + \frac{b}{p})(\cos\omega t + j\ \sin\omega t) + (1 - \frac{b}{p})(\cos\omega t - j\ \sin\omega t)\right]$

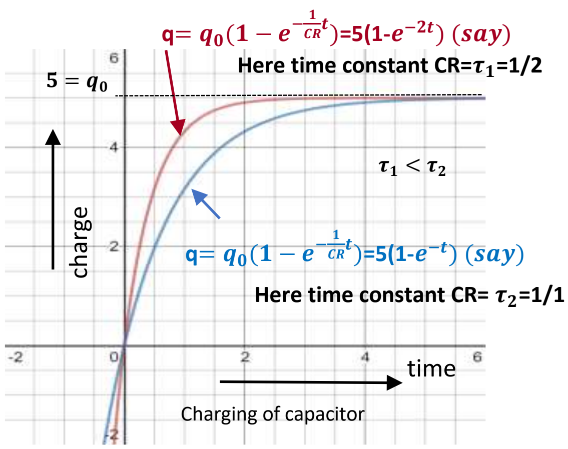
$= q_0 - \frac{q_0}{2}e^{-bt}\left[\cos\omega t + j\ \sin\omega t + \frac{b}{p}\cos\omega t + \frac{b}{p}j\ \sin\omega t + \cos\omega t - j\ \sin\omega t - \frac{b}{p}\cos\omega t + \frac{b}{p}j\ \sin\omega t\right]$

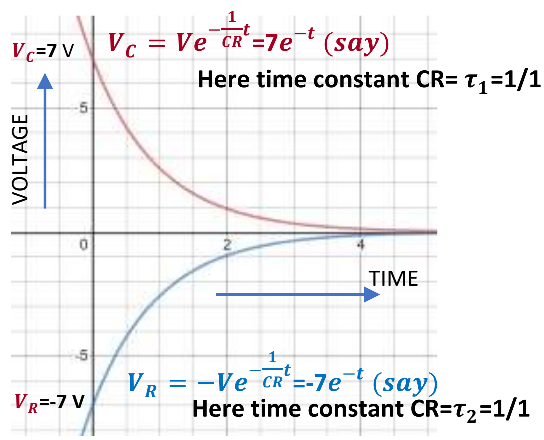
$= q_0 - \frac{q_0}{2}e^{-bt}\left[2\cos\omega t + 2\frac{b}{p}j\ \sin\omega t\right] = q_0 - \frac{q_0}{2}e^{-bt}\left[2\cos\omega t + 2\frac{b}{j\omega}j\ \sin\omega t\right] =$

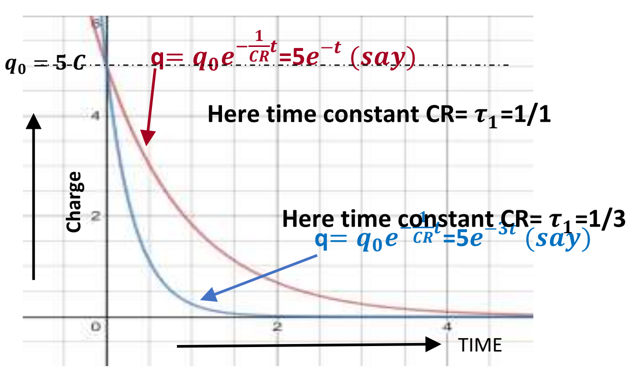
$= q_0 - q_0 e^{-bt}\left[\cos\omega t + \frac{b}{\omega}\sin\omega t\right] = q_0 - \frac{1}{\omega}q_0 e^{-bt}[\omega\cos\omega t + b\ \sin\omega t]$

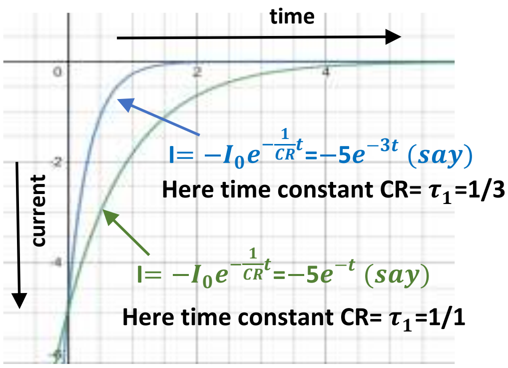

$V = V_C + V_R$
= 7 volt here
$V_C = V(1-e^{-\frac{1}{CR}t})$=7$(1-e^{-t})$(say)
Voltage
$V_R = Ve^{-\frac{1}{CR}t}$=$7e^{-t}$(say)
Time
Variation of $V_C$ and$V_R$ with time

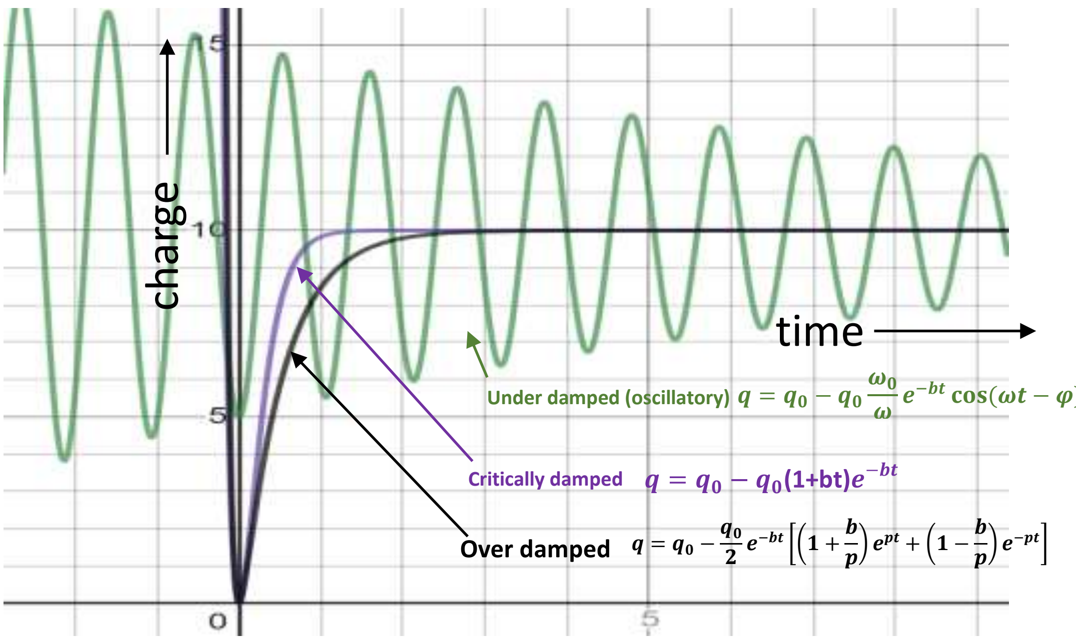

$I_0 = 5\,A$
$I = I_0e^{-\frac{1}{CR}t}$=$5e^{-t}$ $(say)$
Here time constant CR= $\tau_1$=1/1
CURRENT
$I = I_0e^{-\frac{1}{CR}t}$=$5e^{-3t}$ $(say)$
Here time constant CR= $\tau_1$=1/3
TIME

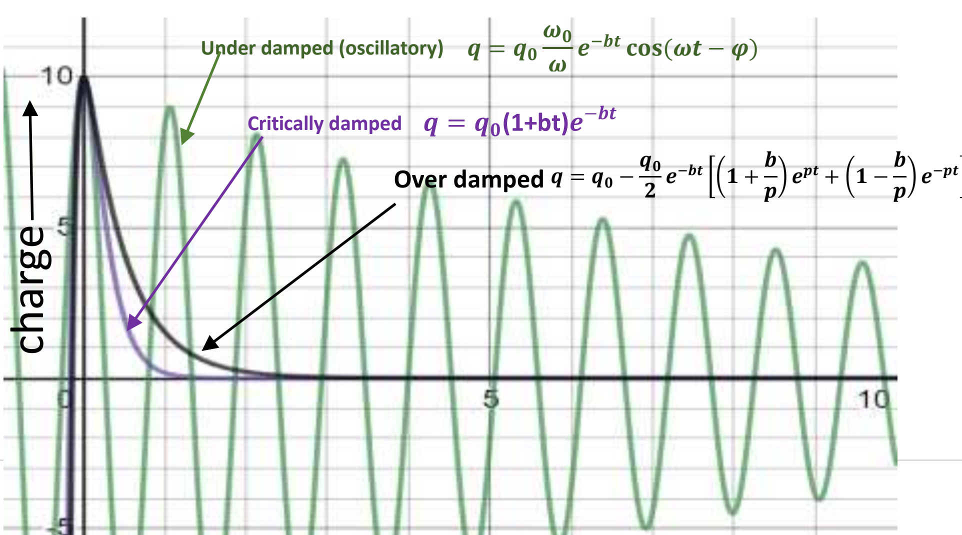

q= $q_0(1-e^{-\frac{1}{CR}t})$=5(1-$e^{-2t}$) $(say)$
Here time constant CR=$\tau_1$=1/2
$5 = q_0$
$\tau_1 < \tau_2$
charge
q= $q_0(1-e^{-\frac{1}{CR}t})$=5(1-$e^{-t}$) $(say)$
Here time constant CR= $\tau_2$=1/1
time
Charging of capacitor


$V_C$=7 V
$V_C = Ve^{-\frac{1}{CR}t}$=$7e^{-t}$ $(say)$
Here time constant CR= $\tau_1$=1/1
VOLTAGE
TIME
$V_R = -Ve^{-\frac{1}{CR}t}$=-$7e^{-t}$ $(say)$
$V_R$=-7 V
Here time constant CR=$\tau_2$=1/1


$q_0 = 5\,C$
q= $q_0e^{-\frac{1}{CR}t}$=$5e^{-t}$ $(say)$
Here time constant CR= $\tau_1$=1/1
Charge
Here time constant CR= $\tau_1$=1/3
q= $q_0e^{-\frac{1}{CR}t}$=$5e^{-3t}$ $(say)$
TIME


time
current
I= $-I_0 e^{-\frac{1}{CR}t}$=$-5e^{-3t}$ $(say)$
Here time constant CR= $\tau_1$=1/3
I= $-I_0 e^{-\frac{1}{CR}t}$=$-5e^{-t}$ $(say)$
Here time constant CR= $\tau_1$=1/1


charge
time
Under damped (oscillatory) $q = q_0 - q_0 \frac{\omega_0}{\omega} e^{-bt} \cos(\omega t - \varphi)$
Critically damped $q = q_0 - q_0$(1+bt)$e^{-bt}$
Over damped $q = q_0 - \frac{q_0}{2} e^{-bt}\left[\left(1+\frac{b}{p}\right)e^{pt} + \left(1-\frac{b}{p}\right)e^{-pt}\right]$


Under damped (oscillatory) $q = q_0 \frac{\omega_0}{\omega} e^{-bt} \cos(\omega t - \varphi)$
Critically damped $q = q_0$(1+bt)$e^{-bt}$
Over damped $q = q_0 - \frac{q_0}{2} e^{-bt}\left[\left(1+\frac{b}{p}\right)e^{pt} + \left(1-\frac{b}{p}\right)e^{-pt}\right]$
charge