

# *Lecture notes on Nuclear and Particle Physics (Theory)*

By Suman Sadhukhan, assistant Professor P R T G College.

Paper: PHS-A-DSE-A2-TH Credits: 5 (+1 for Tutorial)

**1 Introduction Recapitulation of general properties of nuclei, nuclear models and radioactivity.**

**2 Nuclear Reactions (10 Lectures)**

Types of Reactions, Conservation Laws, kinematics of reactions, Q value, reaction rate, reaction cross section, Concept of compound and direct Reaction, resonance reaction, Coulomb scattering (Rutherford scattering).

**3 Interaction of Nuclear Radiation with matter (15 Lectures)**

Energy loss due to ionization (Bethe-Block formula), energy loss of electrons, Cerenkov radiation. Gamma ray interaction through matter, photoelectric effect, Compton scattering, pair production, neutron's interaction with matter.

**4 Detector for Nuclear Radiations (15 Lectures)**

Gas detectors: estimation of electric field, mobility of particle, for ionization chamber and GM Counter. Basic principle of Scintillation Detectors and construction of photo-multiplier tube (PMT). Semiconductor Detectors (Si and Ge) for charge particle and photon detection (concept of charge carrier and mobility), neutron detector.

**5 Particle Accelerators (15 Lectures)**

Accelerator facility available in India, Different type of accelerators

- Van-de Graaf generator (Tandem accelerator)
- Linear accelerator
- Cyclotron
- Betatron
- Synchrotrons

**6 Particle Physics (15 Lectures)**

Fundamental particles and their families. Fundamental particle-interactions and their basic features. Gellmann Nishijima formula. Quark structure of hadrons. Symmetries and Conservation Laws: energy and momentum angular momentum, parity, baryon number, Lepton number, Isospin, Strangeness and charm. concept of quark model, color quantum number and gluons.

# Lecture notes on Nuclear and Particle Physics (Theory)

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- 1 Introduction Recapitulation of general properties of nuclei, nuclear models and radioactivity. (SEE PREVIOUS NOTES)

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## 2 Nuclear Reactions (10 Lectures)

Types of Reactions, Conservation Laws, kinematics of reactions,  $Q$  value, reaction rate, reaction cross section, Concept of compound and direct Reaction, resonance reaction, Coulomb scattering (Rutherford scattering).

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### Nuclear Reaction

This process of bombarding a **target nucleus** by fast moving **projectiles** and the subsequent interaction between the two, which alters the composition, energy, etc. of the target nucleus is known as nuclear reaction. This is also known as transmutation of one element into another.

Suppose a target nucleus  $X$  is bombarded by particle  $a$ . During this process a new nucleus  $Y$  is formed and a particle (or  $\gamma$  ray)  $b$  is emitted, then this nuclear reaction is written as:  $a + X \rightarrow Y + b$

This reaction is also denoted as  $X(a, b)Y$ .

For example, consider the following reaction:  ${}^4_2\text{He} + {}^9_4\text{Be} \rightarrow {}^{12}_6\text{C} + {}^1_0n$

It can be written as:  ${}^9_4\text{Be}({}^4_2\text{He}, {}^1_0n){}^{12}_6\text{C}$  or  ${}^9_4\text{Be}(\alpha, n){}^{12}_6\text{C}$

If more than one particle say  $b_1, b_2$  and  $b_3$  are emitted, then the reaction is written as:

$$a + X \rightarrow Y + b_1 + b_2 + b_3$$

and it is denoted as  $X(a, b_1 b_2 b_3)Y$ .

Consider the following reaction:  ${}^{238}_{92}\text{U} + {}^{14}_7\text{N} \rightarrow {}^{243}_{97}\text{Bk} + {}^4_2\text{He} + {}^1_0n + {}^1_0n + {}^1_0n + {}^1_0n + {}^1_0n$

Alternatively, we can write it as:  ${}^{238}_{92}\text{U} + {}^{14}_7\text{N} \rightarrow {}^{243}_{97}\text{Bk} + {}^4_2\text{He} + 5 {}^1_0n$

The concise version is:  ${}^{238}_{92}\text{U}({}^{14}_7\text{N}, \alpha, 5n){}^{243}_{97}\text{Bk}$

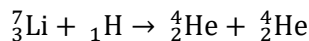
[ In most of the nuclear reactions, mass and atomic number of both target and resulting nuclei are also written. Further, if the incident particle, emitted particles are composite particles (like heavy ions), their mass and charge numbers are also displayed. If there is no confusion, then in some cases, charge number is dropped.]

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The first nuclear reaction was performed by Rutherford in 1919. He bombarded nitrogen gas with  $\alpha$ -particles obtained from natural radioactive source. The reaction is:  ${}^4_2\text{He} + {}^{14}_7\text{N} \rightarrow {}^{17}_8\text{O} + {}^1_1\text{H}$

The first nuclear reaction by accelerating protons was performed by Cockcroft and Walton in 1932.



The study of nuclear reactions is extremely important. Almost all the properties of the nucleus such as mass, size, charge distribution, etc. have been obtained from such a study. Further, this study provides information about the nature of nuclear forces and reaction mechanisms.

## Types of Nuclear Reactions

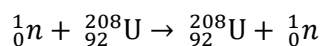
There are many ways of classifying nuclear reactions. Two commonly used ways to classify nuclear reactions are: 1. Reactions based on the reaction mechanism.

2. Reactions based on the mass of projectile.

### 1. Reactions Based on the Reaction Mechanism

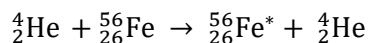
a). Elastic Scattering: In elastic scattering incident particle strikes the target nucleus and leaves the nucleus without any energy loss or in elastic scattering there is no loss or gain of energy of the incident projectile. The direction of the incident particle generally changes.

Examples:  ${}^4_2\text{He} + {}^{208}_{82}\text{Pb} \rightarrow {}^{208}_{82}\text{Pb} + {}^4_2\text{He}$



b). Inelastic Scattering: In inelastic scattering the incident particle loses some of its kinetic energy to the target nucleus. This raises the internal energy of the nucleus. The target nucleus is raised to the higher excited quantum state. Generally, this excited nucleus decays by  $\gamma$ -emission.

Examples:  ${}^1_1\text{H} + {}^{27}_{13}\text{Al} \rightarrow {}^{27}_{13}\text{Al}^* + {}^1_1\text{H}$

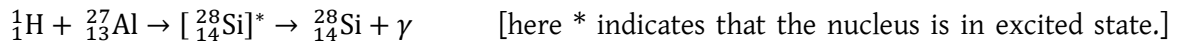


[ where \* on the residual or product nucleus indicates that the nucleus is in excited state.]

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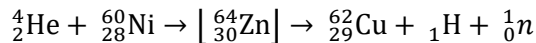
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c). Radiative Capture: In such reactions incident particle is captured by the nucleus with the emission of  $\gamma$ -rays. Examples:  ${}^4_2\text{He} + {}^{12}_6\text{C} \rightarrow [{}^{16}_8\text{O}]^* \rightarrow {}^{16}_8\text{O} + \gamma$

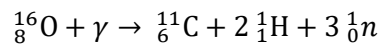


d) Compound Nuclear Reactions: The incident particle is absorbed by the target nucleus and a compound system is formed. This compound system lives for a long time  $\sim 10^{-14}$  to  $10^{-15}$  seconds (compared to the time  $\sim 10^{-21}$  to  $10^{-22}$  seconds taken by a low energy particle  $\sim 5\text{MeV}$  to cross a distance equivalent to nuclear diameter) and then a particle or group of particles is ejected.

Examples:  ${}^4_2\text{He} + {}^{27}_{13}\text{Al} \rightarrow [{}^{31}_{15}\text{P}] \rightarrow {}^{30}_{15}\text{P} + {}^1_0n$

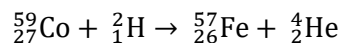


e) Photodisintegration: High energy  $\gamma$ -ray photons are absorbed by the nucleus and the nucleus ejects a particle or a group of particles. Example:  ${}^2_1\text{H} + \gamma \rightarrow {}^1_1\text{H} + {}^1_0n$

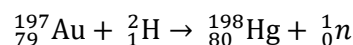


f) Direct Reactions: In some cases, fast moving incident particle while crossing through the nucleus or very close to the nucleus either picks up one or a few nucleons from the nucleus and keeps on moving in the forward direction or it loses one or a few nucleons to the nucleus and keep on moving in the forward direction. First type of reactions is known as pickup reactions while the second type of reactions are known as stripping reactions. The typical reaction time of direct reactions is of the order of  $10^{-21}$  to  $10^{-22}$  seconds (compared with reaction time of compound reactions which is  $\sim 10^{-14}$  to  $10^{-15}$  seconds).

Some of the pickup reactions are:  ${}^{45}_{21}\text{Sc} + {}^3_2\text{He} \rightarrow {}^{44}_{21}\text{Sc} + {}^4_2\text{He}$



And some of the stripping reactions are:  ${}^{63}_{29}\text{Cu} + {}^2_1\text{H} \rightarrow {}^{64}_{29}\text{Cu} + {}^1_1\text{H}$



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g) Heavy Ion Reactions: The reaction in which an incident particle is a **fast moving heavy ion** is known as heavy ion reaction. For example, when  $^{207}_{82}\text{Pb}$  is accelerated to high energy, i.e.  $^{207}_{82}\text{Pb}$  is a projectile

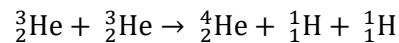
and is bombarded on say  $^{107}_{47}\text{Ag}$  target  $^{207}_{82}\text{Pb} + ^{107}_{47}\text{Ag} \rightarrow ^{106}_{47}\text{Ag} + ^{208}_{82}\text{Pb}$

Another such reaction is  $^{27}_{13}\text{Al} + ^{14}_7\text{N} \rightarrow ^{16}_8\text{O} + ^{25}_{12}\text{Mg}$

h) Radioactive Decay: Typical examples of radioactive decay are  $\alpha$ ,  $\beta$  and  $\gamma$  decay.

i) Fission Reactions: When heavy nuclei capture incident particle, they break into generally two fragments with the emission of few particles. The most common example is:  $^{235}_{92}\text{U} + ^1_0\text{n} \rightarrow ^{141}_{56}\text{Ba} + ^{92}_{36}\text{Kr} + ^1_0\text{n}$

j) Fusion Reactions: When two light nuclei combine to form a heavier nucleus, the reaction is known as fusion reaction. Example,  $^2_1\text{H} + ^2_1\text{H} \rightarrow ^3_2\text{He} + ^1_0\text{n}$



k) Elementary Particle Production Reactions: When very high energy ( $E > 300\text{MeV}$ ) incident particles like protons, etc, interact with protons or neutrons, a variety of elementary particles like Mesons, Pions, etc. are produced.

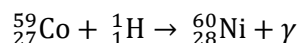
## 2. Reactions Induced by Particles:

2a) Particles of Mass 1 Mass 1 particles are **protons** and **neutrons**. We discuss both the cases as follows:

(i) Proton-induced reactions:

(a) (p,  $\gamma$ )-reactions: Radiative capture reactions have high probability for lighter elements and at low energy protons ( $E_p < 1\text{MeV}$ ). General form of this reaction is:  $^A_Z\text{X} + ^1_1\text{H} \rightarrow ^{A+1}_{Z+1}\text{Y} + \gamma$

Some specific cases are:  $^{27}_{13}\text{Al} + ^1_1\text{H} \rightarrow ^{28}_{14}\text{Si} + \gamma$



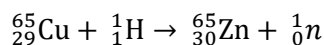
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(b) (p,n)-reactions: The probability of (p,n)-reactions increases with increase in energy of protons.

General form of this reaction is:  ${}_Z^AX + {}_1^1H \rightarrow {}_{Z+1}^AY + {}_0^1n$

Some typical cases are:  ${}_{11}^{23}\text{Na} + {}_1^1\text{H} \rightarrow {}_{12}^{23}\text{Mg} + {}_0^1n$

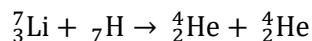


At higher energies (p,2n), (p,3n), etc. reactions occur.

(c) (p,α)-reactions: For majority of low Z nuclei, Q-value for (p,α)-reactions is positive, i.e. these reactions are **exothermic**. These reactions predominate, if proton energy is between 2MeV and 5MeV.

General form of this reaction is:  ${}_Z^AX + {}_1^1H \rightarrow {}_{Z-1}^{A-3}Y + {}_2^4\text{He}$

Some specific cases are:  ${}_7^{14}\text{N} + {}_1^1\text{H} \rightarrow {}_6^{11}\text{C} + {}_2^4\text{He}$

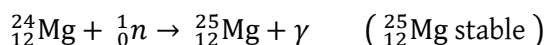


## (ii) Neutrons-induced reactions:

Neutrons have no electric charge and, therefore, can penetrate atomic nuclei without experiencing any Coulomb repulsion. So these reactions can occur at very low energies (~ 0.025eV). At these low energies reactions with charged particles are not possible. Some of the neutron induced reactions are:

(a) (n,γ)-reactions: These reactions have very high probability at energies ~ 0.025eV (called thermal energy). In most of (n,γ)-reactions, product nucleus formed is radioactive. The general form of these reactions is:  ${}_Z^AX + {}_0^1n \rightarrow {}_Z^{A+1}X + \gamma$

Some typical examples are:  ${}_{11}^{23}\text{Na} + {}_0^1n \rightarrow {}_{11}^{24}\text{Na} + \gamma$  ( $t_{1/2}$  of  ${}_{11}^{24}\text{Na} = 15$  hours)

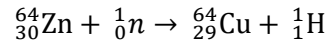


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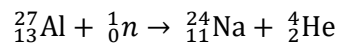
(b) (n,p)-reactions: Since these reactions are endothermic ( $Q$ -value is -ve) so they occur at relatively high energies. These reactions are represented by the relation  ${}_Z^AX + {}_0^1n \rightarrow {}_{Z-1}^AY + {}_1^1H$

Some examples are:  ${}_{13}^{27}Al + {}_0^1n \rightarrow {}_{12}^{27}Mg + {}_1^1H$



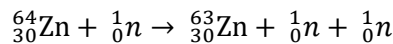
(c) (n, $\alpha$ )-reactions: General form of these reactions is:  ${}_Z^AX + {}_0^1n \rightarrow {}_{Z-2}^{A-3}Y + {}_2^4He$

Some typical examples are:  ${}_{30}^{64}Zn + {}_0^1n \rightarrow {}_{28}^{61}Ni + {}_2^4He$



d) (n,2n)-reactions: These reactions have - ve  $Q$ -value, so high energy neutrons are needed for these reactions. These reactions are represented by  ${}_Z^AX + {}_0^1n \rightarrow {}_Z^{A-1}X + {}_0^1n + {}_0^1n$

Typical examples are:  ${}_{13}^{27}Al + {}_0^1n \rightarrow {}_{13}^{26}Al + {}_0^1n + {}_0^1n$



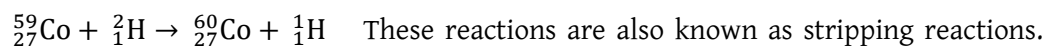
2b) Particles of Mass 2 Only projectile of mass 2 is **deuteron**. Deuteron is very loosely bound system

(binding energy) of deuteron  $\sim 2.2\text{MeV}$ ), so a large number of reactions are possible with deuterons and the most predominant being stripping and pickup reactions.

(i) Deuteron-induced reactions:

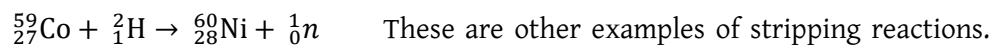
(a) (d,p)-reactions: The general (d,p)-reaction is:  ${}_Z^AX + {}_1^2H \rightarrow {}_Z^{A+1}X + {}_1^1H$

Examples are:  ${}_{13}^{27}Al + {}_1^2H \rightarrow {}_{13}^{28}Al + {}_1^1H$



(b) (d,n)-reactions: The general form of this reaction is:  ${}_Z^AX + {}_1^2H \rightarrow {}_{Z+1}^{A+1}X + {}_0^1n$

Examples are:  ${}_{13}^{27}Al + {}_1^2H \rightarrow {}_{14}^{28}Si + {}_0^1n$

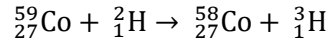


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(c) (d,t)-reactions: These reactions are represented as:  ${}_Z^AX + {}_1^2\text{H} \rightarrow {}_Z^{A-1}X + {}_1^3\text{H}$

Typical examples are:  ${}_{13}^{27}\text{Al} + {}_1^2\text{H} \rightarrow {}_{13}^{26}\text{Al} + {}_1^3\text{H}$

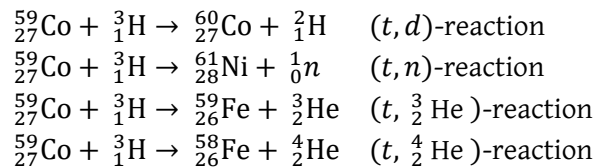


These reactions are also known as pick-up reactions. At high incident energies reactions such as

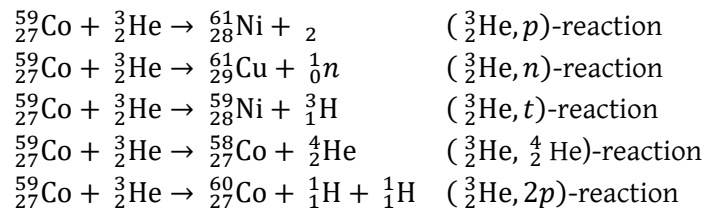
$(d, 2n)$ ,  $(d, 3n)$ ,  $(d, p\alpha)$ , etc. also become possible. However, the probability of  $(d, \gamma)$ -reaction is very small.

2c) Particles of Mass 3 In mass 3 category, we have 2 projectiles  ${}_1^3\text{H}$  and  ${}_2^3\text{He}$ .

Some of the reactions induced by  ${}_1^3\text{H}$  are:



The reactions induced by  ${}_2^3\text{He}$  are:

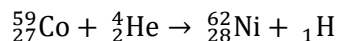


2d) Particles of Mass 4 The only projectile with mass 4 is  $\alpha$ -particle. The various reactions are as under.

Reactions Induced by  $\alpha$ -particle.

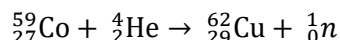
(a) ( $\alpha, p$ )-reactions: The general form of the reaction is:  ${}_Z^AX + {}_2^4\text{He} \rightarrow {}_{Z+1}^{A+3}Y + {}_1^1\text{H}$

Examples are:  ${}_{13}^{27}\text{Al} + {}_2^4\text{He} \rightarrow {}_{14}^{30}\text{Si} + {}_1^1\text{H}$



(b) ( $\alpha, n$ )-reactions: The general form of  $(\alpha, n)$ -reaction is:  ${}_Z^AX + {}_2^4\text{He} \rightarrow {}_{Z+2}^{A+3}Y + {}_0^1n$

Examples are:  ${}_{13}^{27}\text{Al} + {}_2^4\text{He} \rightarrow {}_{15}^{30}\text{P} + {}_0^1n$

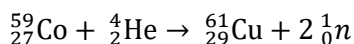


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(c) ( $\alpha, 2n$ )-reactions: The general form of this reaction is:  ${}^A_ZX + {}^4_2\text{He} \rightarrow {}^{A+2}_{Z+2}Y + 2 {}^1_0n$

Typical examples are:  ${}^{27}_{13}\text{Al} + {}^4_2\text{He} \rightarrow {}^{29}_{15}\text{P} + 2 {}^1_0n$



At higher energies reactions such as ( $\alpha, 3n$ ), ( $\alpha, 4n$ ), ( $\alpha, 5n$ ), etc. have been observed.

2 e) Reactions Induced by Heavy Ions Heavy ions are generally projectiles with mass greater than  $\alpha$ -

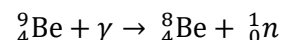
particles. Depending upon the energy of heavy ions large variety of reactions are possible. Some of the

heavy ion reactions are:

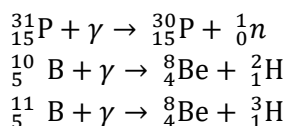
$$\begin{array}{l} {}^{27}_{13}\text{Al}({}^{14}_7\text{N}, {}^{16}_7\text{N}) {}^{25}_{13}\text{Al} \\ {}^{27}_{13}\text{Al}({}^{14}_7\text{N}, {}^{14}_8\text{O}) {}^{27}_{12}\text{Mg} \\ {}^{27}_{13}\text{Al}({}^{12}_6\text{C}, \alpha, 2n) {}^{33}_{17}\text{Cl} \\ {}^{197}_{79}\text{Au}({}^{12}_6\text{C}, 4n) {}^{205}_{85}\text{At} \\ {}^{238}_{92}\text{U}({}^{14}_7\text{N}, \alpha, 5n) {}^{243}_{97}\text{Bk} \end{array}$$

Generally, heavy ion reactions have been used to synthesize elements with atomic number  $> 92$ .

2 f) Reactions Induced by  $\gamma$ -rays The first reactions induced by  $\gamma$ -rays are:  ${}^2_1\text{H} + \gamma \rightarrow {}^1_1\text{H} + {}^1_0n$



Gamma rays used for these reactions were 2.614 MeV from  ${}^{208}\text{Tl}$ . When  $\gamma$  rays of very high energy are available, number of reactions induced by  $\gamma$ -rays are observed. Some of these reactions are:



These reactions induced by  $\gamma$ -rays are also known as photodisintegrations or photonuclear reactions.

Dominant differences between nuclear decay and nuclear reactions are shown in Table

| Nuclear decay                                                          | Nuclear reactions                                                                                                                            |
|------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------|
| 1. It is a spontaneous process (i.e. no external aid is needed)        | 1. It is not a spontaneous process (i.e. external aid is needed)                                                                             |
| 2. Only $\alpha$ -, $\beta$ - and $\gamma$ -emissions are possible.    | 2. In nuclear reactions all possible particles like $p, n, {}^2\text{H}, {}^3\text{He}, {}^4\text{He}, {}^6\text{Li}$ , etc. can be emitted. |
| 3. Mass of the daughter nucleus either remains same or decreases.      | 3. Mass of the product nucleus can also increase.                                                                                            |
| 4. Energy of the emitted particles or $\gamma$ -rays is generally low. | 4. Energy of the emitted particles or $\gamma$ -rays can be very high.                                                                       |

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## Collision between subatomic particles

The collision between a projectile and a target nucleus can be analyzed from the point of view of an observer at rest in the laboratory. This is known as **Laboratory frame of reference or the L-system**.

Alternatively, the collision may also be analyzed from the point of view of an observer at rest w.r.t. the center of mass of the colliding system, known as **center of mass system or the C-system**.

### 1. Elastic collision in L-system (non-relativistic).

We consider the elastic collision between a particle of mass  $M_1$ , velocity  $v_1$  & K.E  $E_1$  (in the  $L$ -system) and a stationary target of mass  $M_2$  (velocity  $v_2 = 0$  & K.E  $E_2 = 0$ ). The particles fly apart after collision from the point of collision with velocities  $v'_1$  and  $v'_2$  with K.E  $E'_1$   $E'_2$  at angles  $\theta_1$  and  $\theta_2$  w.r.t. the direction of incidence as shown in Fig. The conservation of momentum along and perpendicular to the incident direction gives

$$p'_1 \cos \theta_1 + p'_2 \cos \theta_2 = p_1 \dots \dots \dots (i)$$

$$p'_1 \sin \theta_1 + p'_2 \sin \theta_2 = 0 \dots \dots \dots (ii)$$

$$\text{Squaring and adding (i) and (ii), we get } p_1'^2 = p_1^2 + p_2'^2 - 2p_1 p'_1 \cos \theta_1 \dots \dots \dots (iii)$$

Elastic collision in the  $L$ -frame. Applying the law of conservation of energy, we have  $E_1 = E'_1 + E'_2$ ,

$$\text{or, } \frac{p_1^2}{2M_1} = \frac{p_1'^2}{2M_1} + \frac{p_2'^2}{2M_2} \dots \dots \dots (iv)$$

$$\text{Substituting for } p_2'^2 \text{ from (iii), we get } \frac{p_1^2}{2M_1} = \frac{p_1'^2}{2M_1} + \frac{p_1^2}{2M_2} + \frac{p_1'^2}{2M_2} - \frac{2p_1 p'_1}{2M_2} \cos \theta_1$$

$$\text{or, } p_1'^2 \left(1 + \frac{M_2}{M_1}\right) - 2p_1 p'_1 \cos \theta_1 + p_1^2 \left(1 - \frac{M_2}{M_1}\right) = 0.$$

$$\text{Putting } r = M_2/M_1, \text{ the above equation becomes } p_1'^2(1+r) - 2p_1 p'_1 \cos \theta_1 + p_1^2(1-r) = 0.$$

$$\text{In terms of energies, we get } E'_1(1+r) - 2\sqrt{E_1 E'_1} \cos \theta_1 + E_1(1-r) = 0$$

$$\text{or, } \frac{E'_1}{E_1}(1+r) - 2\sqrt{\frac{E'_1}{E_1}} \cos \theta_1 + (1-r) = 0 \dots \dots \dots (v)$$

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The equation above is a quadratic equation in  $\sqrt{\frac{E'_1}{E_1}}$ .

The solution can be written as  $\sqrt{\frac{E'_1}{E_1}} = \frac{\cos \theta_1 \pm \sqrt{\cos^2 \theta_1 + (r^2 - 1)}}{r + 1}$  .....(vi)

If the target is heavier, then  $r > 1$ . Since  $\sqrt{\frac{E'_1}{E_1}} = \frac{p'_1}{p_1}$  we have to choose the sign in (vi), so that  $p'_1/p_1 > 0$ .

So, we take the plus sign. Squaring (vi),  $\frac{E'_1}{E_1} = \frac{2\cos^2 \theta_1 + (r^2 - 1) + 2\cos \theta_1 \sqrt{\cos^2 \theta_1 + r^2 - 1}}{(r + 1)^2}$ .

So, the ratio of energy received by  $M_2$  to the incident particle energy becomes

$$\frac{E_1 - E'_1}{E_1} = \frac{2(r + 1 - \cos^2 \theta_1) - 2\cos \theta_1 \sqrt{\cos^2 \theta_1 + r^2 - 1}}{(r + 1)^2}$$

The energy received is maximum for  $\theta_1 = \pi$ , then for  $r \gg 1$  (i.e.,  $M_2 \rightarrow M_1$ ) is  $\frac{E_1 - E'_1}{E_1} = \frac{4r}{(r + 1)^2} = \frac{4}{r} = \frac{4M_1}{M_2}$ .

But, actually all values of  $\theta_1$  are possible in this case. This type of scattering is observed when  $\beta$ -particles are scattered by nuclei as they pass through matter. The energy received by the nucleus in such collisions is very small ( $r = M_{\text{nuc}}/m_e > 1$ ).

When  $r < 1$  (as in the collision of a nuclear particle with an electron), the solution of (vi) can be real if

$$\cos^2 \theta \geq 1 - r^2$$

Both (+) and (−) signs are possible in (vi).

Evidently,  $\sin \theta_1 \leq r$  or,  $\theta_1 < \pi/2$ , since  $r < 1$ . For  $r \ll 1$ ,  $\theta_1 \rightarrow 0$ .

Thus, a heavy particle scattered by a very light target, such as an electron, goes on almost undeviated.

The energy received by the target nucleus for  $\theta = 0$  is  $\frac{E_1 - E'_1}{E_1} = \frac{2r \pm 2\sqrt{r^2}}{(r + 1)^2} = \begin{cases} \frac{4r}{(r + 1)^2} \\ 0 \end{cases}$

In this case, the energy given to the target is  $E_1 - E'_1 = \frac{4M_2 E_1}{M_1} < E_1$

Hence, the energy loss by the incident particles in such a collision is very small as compared to  $E_1$ .

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## 2. Elastic collision in the C-system (non-relativistic).

We consider the elastic collision of two particles in the C-system [Fig].

Let the particle  $M_2$  be at rest in the L-system ( $v_2 = 0$ ). The velocity of the centre of mass (CM) in the L-system is  $v_c = \frac{M_1 v_1 + M_2 v_2}{M_1 + M_2} = \frac{M_1 v_1}{M_1 + M_2}$  ( $\because v_2 = 0$ ).

Hence the velocities of  $M_1$  and  $M_2$  in the C-system before collision are

$$V_1 = v_1 - v_c = \frac{M_2 v_1}{M_1 + M_2} \quad \& \quad V_2 = v_2 - v_c = -\frac{M_1 v_1}{M_1 + M_2} \quad \text{respectively,}$$

The corresponding momenta are respectively:

$$P_1 = M_1 V_1 = \frac{M_1 M_2 v_1}{M_1 + M_2} = \mu v_1 \quad \& \quad P_2 = M_2 V_2 = -\frac{M_1 M_2 v_1}{M_1 + M_2} = -\mu v_1$$

[where  $\mu = \frac{M_1 M_2}{M_1 + M_2}$  is the reduced mass.]

So, the two particles have equal and opposite sign before collision and as such their total momentum is

$P_1 + P_2 = 0$ . According to the law of conservation of momentum, the two particles must have total momentum zero after collision.  $\therefore P'_1 + P'_2 = P_1 + P_2 = 0$ , [where prime quantities denotes momentum after collision.]

Sums of the KE before and after collision are respectively:

$$E_1 + E_2 = \frac{p_1^2}{2M_1} + \frac{p_2^2}{2M_2} = \frac{p^2}{2\mu} \quad \& \quad E'_1 + E'_2 = \frac{p'^2_1}{2M_1} + \frac{p'^2_2}{2M_2} = \frac{p'^2}{2\mu} \quad [\text{As } |P_1| = |P_2| = |P| \text{ and } |P'_1| = |P'_2| = |P'|.]$$

Now from law of conservation of energy,  $E'_1 + E'_2 = E_1 + E_2$ . So, we get  $P' = P$ .

$\therefore$  the magnitude of the momenta of the particles before and after collision are equal:  $P_1 = P_2 = P'_1 = P'_2$ .

**Thus, we conclude that there is no change of KE and momenta of the particles after collision in the C-system.**

Fig. shows the vector diagram of momentum of the particles. The two particles fly apart from the point of collision with equal and opposite momenta, so that  $\theta_1 + \theta_2 = \pi$ .

$$\text{The K.E of CM } E_c = \frac{1}{2}(M_1 + M_2)v_c^2 = \frac{1}{2}(M_1 + M_2)\left(\frac{M_1 v_1}{M_1 + M_2}\right)^2 = \frac{1}{2} M_1 v_1^2 \left(\frac{M_1}{M_1 + M_2}\right) = \left(\frac{M_1}{M_1 + M_2}\right) \mathcal{E}_1$$

[where  $\mathcal{E}_1$  is the KE of  $M_1$  in the L-system  $\mathcal{E}_1 = \frac{1}{2} M_1 v_1^2$  ]

Note that the energy  $E_c$  is not available for the production of any inelastic collision (e.g., reaction).

The total amount of energy **available** for any inelastic collision (e.g., reaction) is

$$\mathcal{E}_1 - E_c = \mathcal{E}_1 - \frac{M_1}{M_1 + M_2} \mathcal{E}_1 = \frac{M_2}{M_1 + M_2} \mathcal{E}_1 = \frac{1}{2} \mu v_1^2.$$

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