Ordinary laws of reflection and refraction.

Suppose that the XY plane forms the interface Z=0 between two linear homogeneous dielectric (non-conducting) media characterized by permittivities ϵ_1 & ϵ_2 and permeability's μ_1 & μ_2 respectively. Let a plane electromagnetic wave be incident on the interface at the point O. The wave is partly reflected and partly reflected. Let us represent the electric field associated with these waves by

$$\vec{E} = \overrightarrow{E_0} e^{j(\vec{k}.\vec{r}-\omega t)} \quad ; \quad \overrightarrow{E_1} = \overrightarrow{E_{10}} e^{j(\vec{k}_1.\vec{r}-\omega_1 t)} \quad ; \quad \overrightarrow{E_2} = \overrightarrow{E_{20}} e^{j(\vec{k}_2.\vec{r}-\omega_2 t)}$$

Where $\overrightarrow{E_0}$, $\overrightarrow{E_{10}}$ & $\overrightarrow{E_{20}}$ are amplitudes $\vec{k}$, $\vec{k}_1$& $\vec{k}_2$ are propagation vectors ω, ω_1 & ω_2 are the frequencies of the incident, reflected and refracted wave respectively.

Let us now apply the boundary condition that the tangent components of electric field are continuous across the interface Z=0.

Thus $\overrightarrow{(E_0)}_t e^{j(\vec{k}.\vec{r}-\omega t)} + \overrightarrow{(E_{10})}_t e^{j(\vec{k}_1.\vec{r}-\omega_1 t)} = \overrightarrow{(E_{20})}_t e^{j(\vec{k}_2.\vec{r}-\omega_2 t)}$

where the subscript t is used to denote tangential component.

The condition must be valid for all values of time and it immediately follows that $\omega = \omega_1 = \omega_2$...............(i)

Thus the frequency of electromagnetic wave does not change on reflection and refraction.

The condition (i) must also hold for all points on the interface so we must have $\vec{k}.\vec{r} = \vec{k}_1.\vec{r} = \vec{k}_2.\vec{r}$ at $Z = 0$

therefore $k_x = k_{1x} = k_{2x}$ and $k_y = k_{1y} = k_{2y}$

If the incident beam is assumed to be in the XZ plane then $k_y = 0$ hence $k_{1y} = k_{2y} = 0$ and both $\vec{k}_1$ and $\vec{k}_2$ lie in the XZ plane.

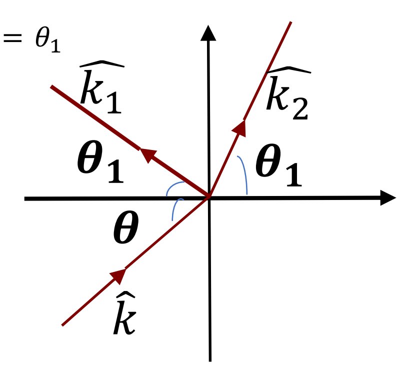
Since Z axis is normal to the interface we may conclude that incident, reflected, refracted waves and normal to the interface at the point of incident all lie in the same plane. From figure

$k_x = k \sin\theta$, $k_{1x} = k_1 \sin\theta_1$ And $k_{2x} = k_2 \sin\theta_2$

There are from the condition $k_x = k_{1x}$ we get $k \sin\theta = k_1 \sin\theta_1$.............(ii)

Since the vectors $\vec{k}$ & $\vec{k}_1$ lie in the same medium we can write for the phase velocity in medium 1

$\frac{\omega}{k} = \frac{\omega_1}{k_1} = \frac{1}{\sqrt{\mu_1 \epsilon_1}}$ As $\omega = \omega_1$ then $k = k_1$ hence from equation(ii) $\theta = \theta_1$



That means the angle of incident is equal to the angle of reflection.

Again as $k_{1x} = k_{2x}$ $\quad k_1 \sin\theta_1 = k_2 \sin\theta_2$

$$\frac{k_1}{\omega}\sin\theta_1 = \frac{k_2}{\omega}\sin\theta_2$$

$$\frac{1}{v_1}\sin\theta_1 = \frac{1}{v_2}\sin\theta_2$$

$$\frac{c}{v_1}\sin\theta_1 = \frac{c}{v_2}\sin\theta_2$$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$ Which is well known Snell's law of refraction.

you may proceed another way

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{k_2}{k_1} = \frac{\frac{\omega}{k_1}}{\frac{\omega}{k_2}} = \frac{v_1}{v_2} = \frac{\frac{c}{v_2}}{\frac{c}{v_1}} = \frac{n_2}{n_1} \qquad n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Fresnel's equations:

The equations which relate the amplitudes of the reflected and refracted waves with that of incident wave are known as Fresnel's equations.

We know that $\vec{E}$vectors in a plane electromagnetic wave is perpendicular to the direction of propagation of the wave. However, it may lie in arbitrary directions in the plane normal to the direction of propagation. It is convenient to consider two extreme cases separately.

1. For which the $\vec{E}$ vector on the incident wave is perpendicular to the plane of incident ; this is called s polarization.
   (s for German word *senkrecht* meaning perpendicular).
2. For which the $\vec{E}$ vector on the incident wave is parallel to the plane of incident (the plane defined by $\vec{k}$ & $\hat{n}$); this is called $p-$ polarization (P for parallel).

The general case of arbitrary polarization can be obtained by a suitable linear combination of the two extreme results.

Case 1: $\vec{E}$ vector on the incident wave is perpendicular to the plane of incident

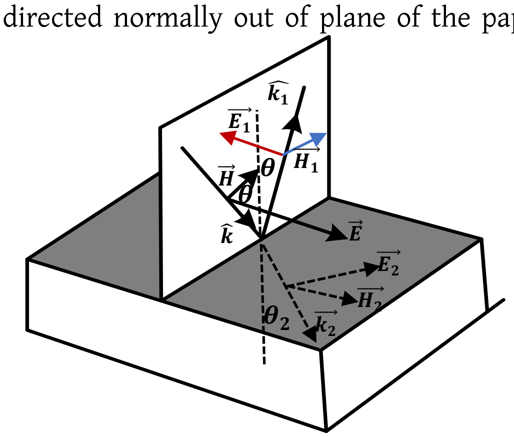
suppose the $\vec{E}$ vector of the incident wave lies perpendicular to the plane of incidence XZ plane as shown in figure as the media on two sides of interface Z=0 isotropic the $\vec{E}$ vector reflected and refracted waves will also be perpendicular to the plane of incident. Assume that $\vec{E}$ vectors are directed normally out of plane of the paper. $\vec{H}$,$\vec{H_1}$, $\vec{H_2}$ Lie in the plane of paper.

Let us represent the incident, reflected and refracted waves by

$$\vec{E} = \overrightarrow{E_0} e^{j(\vec{k}.\vec{r}-\omega t)} \quad , \quad \vec{H} = \frac{\vec{k}\times\vec{E}}{\omega\mu_1}$$

$$\overrightarrow{E_1} = \overrightarrow{E_{10}} e^{j(\vec{k}_1.\vec{r}-\omega_1 t)} \quad , \quad \overrightarrow{H_1} = \frac{\overrightarrow{k_1}\times\vec{E}_1}{\omega\mu_1}$$

$$\overrightarrow{E_2} = \overrightarrow{E_{20}} e^{j(\vec{k}_2.\vec{r}-\omega_2 t)} \quad , \quad \overrightarrow{H_2} = \frac{\overrightarrow{k_2}\times\vec{E}_2}{\omega\mu_2}$$



Since the electric vectors are all parallel to the interface z=0 , the continuity of the tangent component of $\vec{E}$ field gives us

$$E_0 e^{j(\vec{k}.\vec{r}-\omega t)} + E_{10} e^{j(\vec{k}_1.\vec{r}-\omega_1 t)} = E_{20} e^{j(\vec{k}_2.\vec{r}-\omega_2 t)}$$

The condition must be valid for all values of t and for all points on the interface. This lead to the cancellations of exponential factors. Hence $E_0 + E_{10} = E_{20}$

The continuity of tangential components of $\vec{H}$ fields across the interface z=0 gives

$$-H_0 \cos\theta + H_{10}\cos\theta_1 = -H_{20}\cos\theta_2$$

Now $\vec{H} = \frac{\vec{k}\times\vec{E}}{\omega\mu_1} = \sqrt{\frac{\epsilon_1}{\mu_1}}\ \frac{\vec{k}\times\vec{E}}{k}$ that means $H_0 = \sqrt{\frac{\epsilon_1}{\mu_1}} E_0$

similarly $H_{10} = \sqrt{\frac{\epsilon_1}{\mu_1}} E_{10}$ and $H_{20} = \sqrt{\frac{\epsilon_2}{\mu_2}} E_{20}$

Therefore $-\sqrt{\frac{\epsilon_1}{\mu_1}} E_0 \cos\theta + \sqrt{\frac{\epsilon_1}{\mu_1}} E_{10}\cos\theta_1 = -\sqrt{\frac{\epsilon_2}{\mu_2}} E_{20}\cos\theta_2$

Or, $(-E_0 + E_{10})\sqrt{\frac{\epsilon_1}{\mu_1}} E_0 \cos\theta = -\sqrt{\frac{\epsilon_2}{\mu_2}} E_{20}\cos\theta_2$ as $\theta = \theta_1$

Then $(-E_0 + E_{10})a = -bE_{20}$ ....(i) $\sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta = a$ $\sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta_2 = b$

$E_0 + E_{10} = E_{20}$..................................(ii)

Multiplying equation (ii) by $-a$

$E_0(-2a) = (-b-a)\,E_{20}$ or, amplitude of transmission coefficient $t_\perp = \frac{E_{20}}{E_0} = \frac{2a}{a+b} = \frac{2\sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta}{\sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta + \sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta_2}$

Multiplying equation (ii) by b

$E_0(b-a) + (b+a)E_{10} = 0$ or, amplitude of reflection coefficient $r_\perp = \frac{E_{10}}{E_0} = \frac{a-b}{a+b} = \frac{\sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta - \sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta_2}{\sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta + \sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta_2}$

As most of the dialectics are essentially nonmagnetic we can write $\mu_1 = \mu_2 \approx \mu_0$ again $n_1 = \sqrt{\frac{\epsilon_1}{\epsilon_0}}$ $n_2 = \sqrt{\frac{\epsilon_2}{\epsilon_0}}$

Where n_1, n_2 are the refractive index of these two medium respectively. Then we have

$r_\perp = \frac{E_{10}}{E_0} = \frac{n_1\cos\theta - n_2\cos\theta_2}{n_1\cos\theta + n_2\cos\theta_2}$ $t_\perp = \frac{E_{20}}{E_0} = \frac{2n_1\cos\theta}{n_1\cos\theta + n_2\cos\theta_2}$

Now as from Snell's law $n_1\sin\theta = n_2\sin\theta_2$

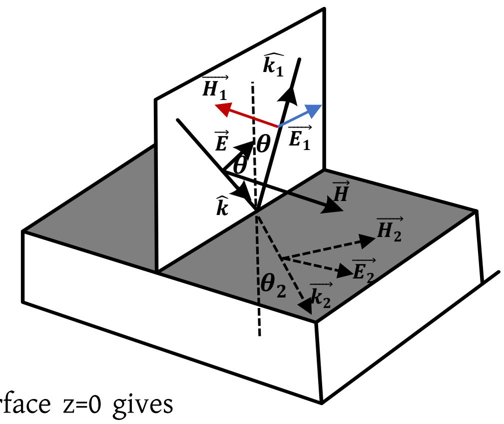
Fresnel's equation can be rewritten as

$$r_\perp = \frac{E_{10}}{E_0} = \frac{\frac{n_2\sin\theta_2}{\sin\theta}\cos\theta - n_2\cos\theta_2}{\frac{n_2\sin\theta_2}{\sin\theta}\cos\theta + n_2\cos\theta_2} = -\frac{\sin(\theta-\theta_2)}{\sin(\theta+\theta_2)}$$

$$t_\perp = \frac{E_{20}}{E_0} = \frac{2\frac{n_2\sin\theta_2}{\sin\theta}\cos\theta}{\frac{n_2\sin\theta_2}{\sin\theta}\cos\theta + n_2\cos\theta_2} = \frac{2\cos\theta\sin\theta_2}{\sin(\theta+\theta_2)}$$

Case 2: $\vec{E}$ vector on the incident wave is parallel to the plane of incident

suppose the $\vec{E}$ vector of the incident wave lies parallel to the plane of incidence XZ plane as shown in figure as the media on two sides of interface Z=0 isotropic the $\vec{E}$ vector reflected and refracted waves will also be in the plane of incident. Assume that $\vec{E}$ vectors are in the plane of the paper. $\vec{H}$ $\vec{H_1}$ $\vec{H_2}$ Are perpendicular to the plane of incident or a plane of paper.



Let us represent the incident, reflected and refracted waves by

$$\vec{E} = \overrightarrow{E_0} e^{-j(\vec{k}.\vec{r}-\omega t)} \quad , \quad \vec{H} = \frac{\vec{k}\times\vec{E}}{\omega\mu_1}$$

$$\overrightarrow{E_1} = \overrightarrow{E_{10}} e^{-j(\vec{k}_1.\vec{r}-\omega_1 t)} \quad , \quad \overrightarrow{H_1} = \frac{\overrightarrow{k_1}\times\vec{E}_1}{\omega\mu_1}$$

$$\overrightarrow{E_2} = \overrightarrow{E_{20}} e^{-j(\vec{k}_2.\vec{r}-\omega_2 t)} \quad , \quad \overrightarrow{H_2} = \frac{\overrightarrow{k_2}\times\vec{E}_2}{\omega\mu_2}$$

The continuity of tangential components of $\vec{E}$ fields across the interface z=0 gives

$$E_0\cos\theta - E_{10}\cos\theta = E_{20}\cos\theta_2$$

The continuity of tangential components of $\vec{H}$ fields across the interface z=0 gives

$$H_0 + H_{10} = H_{20}$$

As before $H_0 = \sqrt{\frac{\epsilon_1}{\mu_1}}E_0$, $H_{10} = \sqrt{\frac{\epsilon_1}{\mu_1}}E_{10}$, $H_{20} = \sqrt{\frac{\epsilon_2}{\mu_2}}E_{20}$

therefore $\sqrt{\frac{\epsilon_1}{\mu_1}}(E_0 + E_{10}) = \sqrt{\frac{\epsilon_2}{\mu_2}}E_{20}$.....................................(I) $\times \cos\theta$

$$E_0 \cos\theta - E_{10}\cos\theta = E_{20}\cos\theta_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (II) \times \sqrt{\frac{\epsilon_1}{\mu_1}}$$

$E_0\ 2\sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta = \left(\sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta + \sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_2\right)E_{20}$ $\therefore$ amplitude of transmission coefficient $t_\parallel = \frac{E_{20}}{E_0} = \frac{2\sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta}{\sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta + \sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_2}$

$\sqrt{\frac{\epsilon_1}{\mu_1}}(E_0 + E_{10}) = \sqrt{\frac{\epsilon_2}{\mu_2}}E_{20}$.....................................(I) $\times \cos\theta_2$

$E_0\cos\theta - E_{10}\cos\theta = E_{20}\cos\theta_2$........................(II) $\times -\sqrt{\frac{\epsilon_2}{\mu_2}}$

$$\left(\sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_2 - \sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta\right)E_0 + \left(\sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_2 + \sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta\right)E_{10} = 0$$

$\therefore$ amplitude of reflection coefficient $r_\parallel = \frac{E_{10}}{E_0} = \frac{\left(\sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta - \sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_2\right)}{\sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta + \sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_2}$

As most of the dialectics are essentially nonmagnetic we can write$\mu_1 = \mu_2 \approx \mu_0$ $n_1 = \sqrt{\frac{\epsilon_1}{\epsilon_0}}$ and $n_2 = \sqrt{\frac{\epsilon_2}{\epsilon_0}}$

Where n_1, n_2 are the refractive index of these two medium respectively. Then we have

$$r_{\parallel} = \frac{E_{10}}{E_0} = \frac{n_2 \cos\theta - n_1 \cos\theta_2}{n_2 \cos\theta + n_1 \cos\theta_2} \qquad t_{\parallel} = \frac{E_{20}}{E_0} = \frac{2\, n_1 \cos\theta}{n_2 \cos\theta + n_1 \cos\theta_2}$$

Now as from Snell's law $n_1 \sin\theta = n_2 \sin\theta_2$

Fresnel's equation can be rewritten as

$$r_{\parallel} = \frac{E_{10}}{E_0} = \frac{n_2 \cos\theta - \frac{n_2 \sin\theta_2}{\sin\theta}\cos\theta_2}{n_2 \cos\theta + \frac{n_2 \sin\theta_2}{\sin\theta}\cos\theta_2} = \frac{\sin(2\theta) - \sin(2\theta_2)}{\sin(2\theta) + \sin(2\theta_2)} = \frac{2\cos(\theta+\theta_2).\sin(\theta-\theta_2)}{2\cos(\theta-\theta_2).\sin(\theta+\theta_2)} = \frac{\tan(\theta-\theta_2)}{\tan(\theta+\theta_2)}$$

$$t_{\parallel} = \frac{E_{20}}{E_0} = \frac{2\frac{n_2 \sin\theta_2}{\sin\theta}\cos\theta}{n_2 \cos\theta + \frac{n_2 \sin\theta_2}{\sin\theta}\cos\theta_2} = \frac{4\cos\theta \sin\theta_2}{\sin(2\theta) + \sin(2\theta_2)} = \frac{4\cos\theta \sin\theta_2}{2\cos(\theta-\theta_2).\sin(\theta+\theta_2)} = \frac{2\cos\theta \sin\theta_2}{\cos(\theta-\theta_2).\sin(\theta+\theta_2)}$$

Physical implications of Fresnel's equation

various important aspects of electromagnetic waves can be described by using Fresnel's equation. In particular, we are interested in polarization effects, phase changes, total internal reflection, reflectance and transmittance.

Polarisation by reflection

we find that $r_{\parallel} = 0$ for $\theta + \theta_2 = \frac{\pi}{2}$, that means that if $\theta + \theta_2 = \frac{\pi}{2}$ then electric field polarised parallel to the plane of incident is not reflected at all. In this condition $r_{\perp} \neq 0$ that means electric field polarised normal to plane of incident is partly reflected. Thus, an un-polarised light consisting of both types of $\vec{E}$ fields incident at an angle $\theta = \theta_P$ satisfying the condition $\theta_P + \theta_2 = \frac{\pi}{2}$ we'll be plane polarised normal to the plane of incidence. This angle θ_P for which $r_{\parallel} = 0$ known as Brewster angle or angle of polarisation. Snell's law in this case gives us

$$\frac{n_2}{n_1} = \frac{\sin\theta}{\sin\theta_2} = \frac{\sin\theta_P}{\sin(\frac{\pi}{2} - \theta_P)} = \tan\theta_P.$$

Thus, the tangent of the angle of polarisation is equal to the refractive index of the reflecting media related to the incident medium. This is known as Brewster law.

Amplitude coefficients and phase change

Fresnel's equation show that amplitude coefficients depend on the value of incident angle θ. Let us to examine the dependence over that in the range of values of θ. For nearly normal incidence θ and θ_2 tends to zero.

$$[r_{\perp}]_{\theta=0} = [r_{\parallel}]_{\theta=0} = \frac{n_1 - n_2}{n_1 + n_2}$$

$$[t_{\perp}]_{\theta=0} = [t_{\parallel}]_{\theta=0} = \frac{2n_1}{n_1 + n_2}$$

As an example for air-glass interface $n_1 = 1$ $n_2 = 1.5$ $[r_\perp]_{\theta=0} = [r_\parallel]_{\theta=0} = -0.2$

$[t_\perp]_{\theta=0} = [t_\parallel]_{\theta=0} = 0.8$

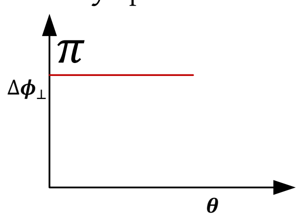
for normal incidences the plane of incidence becomes undefined and any distinction between parallel and perpendicular components vanishes. The difference in signs of $r_\parallel$& $r_\perp$ arises only because $\vec{E}$ and $\vec{E_1}$ are antiparallel when θ tends to zero, where as $\vec{E}$ and $\vec{E_1}$ Point in the same direction for grazing incident when θ tends to 90° then

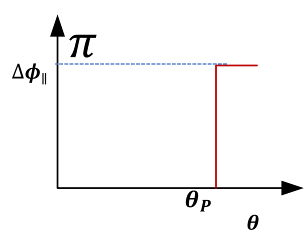
$$r_\parallel = r_\perp = -1 \qquad t_\parallel = t_\perp = -0$$

For $n_2 > n_1$ from Snell's law $\theta_2 < \theta$ and hence from $r_\perp = -\frac{\sin(\theta-\theta_2)}{\sin(\theta+\theta_2)}$ $r_\perp$ is negative for all values of θ . The significance of negative sign is that the component of electric field perpendicular to the plane of incident suffers a phase change $\Delta\phi_\perp = \pi$ when reflected from a surface back by an optically denser medium. In contrast from $r_\parallel = \frac{\tan(\theta-\theta_2)}{\tan(\theta+\theta_2)}$

Shows that for $n_2 > n_1$ that is $\theta > \theta_2$, $r_\parallel$ starts out the positive at θ= 0 and gradually decreases until it becomes zero at the polarizing angle $\theta = \theta_P$. As θ increases beyond θ_P $r_\parallel$ becomes progressively more negative and reaches the value -1 at $\theta = 90°$. Thus for $n_2 > n_1$ the electric field polarised parallel to the plane of incident $r_\parallel$ is positive that is no phase change $\Delta\phi_\parallel = 0$ for $\theta < \theta_P$. And $r_\parallel$ is negative that means phase change $\Delta\phi_\parallel = \pi$ for $\theta < \theta_P$.

$t_\parallel$ and $t_\perp$ are always positive that means that refracted waves do not suffer any phase change





Here it is important to mention that we have chosen $\vec{E}, \vec{H}$, $\vec{k}$ vectors to have the same relative orientation when looking down the propagation vector. The $\vec{E}$ field normal to the plane of incident are said to be in phase, if parallel;

π-out of phase, if antiparallel. The $\vec{E}$**field** parallel to the plane of incident is said to be in phase if their z-components are parallel, and said to be out of phase if z-components are anti-parallel. Note that a phase difference between two

$\vec{E}$fields at the same as that between two associated $\vec{H}$fields. So we need only to look at the vectors normal to the plane of incidence , way that they be $\vec{E}$ *or* $\vec{H}$ to determine the relative phase of accompanying fields in the plane of incidence.

Total internal reflection.

Suppose we consider the case of internal reflection which the incident medium is more dense. i.e., $n_1 > n_2$.

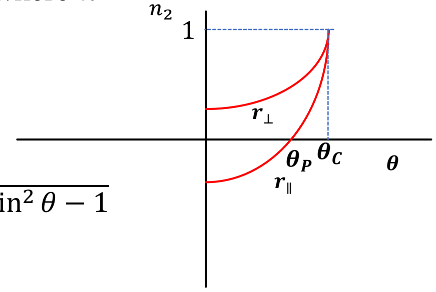
In this case $\theta_2 > \theta$ and equation $r_\perp = -\frac{\sin(\theta-\theta_2)}{\sin(\theta+\theta_2)}$ $r_\perp$ is positive for all values of θ. Hence $r_\perp$ starts from the value

$\frac{n_1-n_2}{n_1+n_2}$ At $\theta = 0$ and then increases with θ and attains the value +1 for $\theta = \theta_c$ where θ_c is called critical angle. $\theta_c$is the value of the angle of incidence for which the angle of refraction $\theta_2 = 90°$.

In contrast from $r_\parallel = \frac{\tan(\theta-\theta_2)}{\tan(\theta+\theta_2)}$ Shows that for $n_1 > n_2$ that is $\theta < \theta_2$, $r_\parallel$ starts from a negative value $-\frac{n_1-n_2}{n_1+n_2}$

At $\theta = 0$ and then increases with θ becomes zero at polarising angle $\theta + \theta_P$ and attains the value +1 for $\theta = \theta_c$.

The critical angle is given by Snell's law $\frac{n_2}{n_1} = \frac{\sin\theta_c}{\sin\frac{\pi}{2}}$ $\quad n\sin\theta_c = 1$ $\quad$ where $n = \frac{n_1}{n_2}$



For any angle of incidence θ, $\frac{n_2}{n_1} = \frac{\sin\theta}{\sin\theta_2}$ Then $n\sin\theta = \sin\theta_2$

Hence $\cos\theta_2 = \sqrt{1-\sin^2\theta_2} = \sqrt{1-n^2\sin^2\theta}$

When θ> θ_c $\quad n\sin\theta_c > 1$ $\quad \cos\theta_2$ Becomes imaginary $\quad \cos\theta_2 = j\sqrt{n^2\sin^2\theta - 1}$

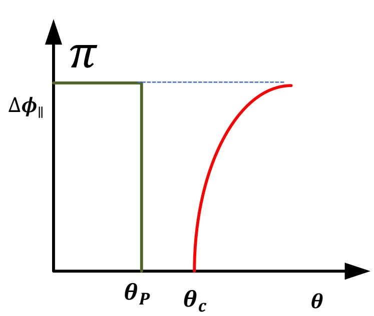
We know examine Fresnel's equation for θ> θ_c using the values of $\sin\theta_2$, $\cos\theta_2$

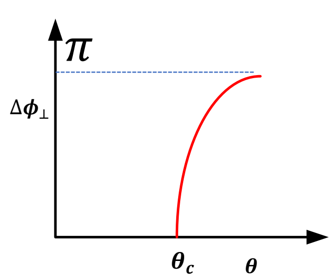
$$r_\perp = \frac{E_{10}}{E_0} = \frac{\text{n}\cos\theta-\cos\theta_2}{\text{n}\cos\theta+\cos\theta_2} = \frac{\text{n}\cos\theta-j\sqrt{n^2\sin^2\theta-1}}{\text{n}\cos\theta+j\sqrt{n^2\sin^2\theta-1}}$$

obviously the magnitude of the amplitude reflection coefficient $|r_\perp| = 1$. $r_\perp$is complex with implies a change of phase on reflection. This change of phase depends on the angle of incident greater than θ_c but magnitude of $r_\perp$remains 1.

Similarly we can show that for θ> θ_c the coefficient $r_\parallel$ is complex and$|r_\parallel| = 1$. Again phase change depends on the angle of incidence greater than $\theta_c$but magnitude of $r_\perp$remains 1. Thus both the component of electric field is totally reflected back into the denser medium. This is well-known phenomenon of total internal reflection. It plays an important role in fibre optics.

Phase change in internal reflection.





evanescent wave

in the total internal reflection the phase difference between the incident and reflected wave is not π. As a result, some resultant $\vec{E}$–fields exist at the denser side of the boundary. In order to satisfy boundary conditions there must be some disturbance in the rarer medium. The wave function of the transmitted

$\vec{E}$–fields is $\vec{E_2} = \vec{E_{20}} e^{j(\vec{k}_2.\vec{r}-\omega t)}$

where $\vec{k}_2.\vec{r} = k_{2x}.x+k_{2z}.z$ as there is no y component.

Now $k_{2x} = k_2 \sin\theta_2$ $k_{2z} = k_2 \cos\theta_2$

Then $\vec{E_2} = \vec{E_{20}} e^{j(k_2 \sin\theta_2.x+k_2\cos\theta_2.z-\omega t)} = \vec{E_{20}} e^{j\left(k_2 n \sin\theta.x+k_2\sqrt{1-n^2\sin^2\theta}.z-\omega t\right)} = \vec{E_{20}} e^{-\beta z} e^{j(k_2 n \sin\theta.x-\omega t)}$

Where $\beta = k_2\sqrt{n^2\sin^2\theta - 1}$

The above equation shows that for $\theta > \theta_c$ that refracted wave advanced in the X direction as a surface or evanescent wave. But it's amplitude decays exponentially with the depth of penetration into the second medium. Although some fields exist on the rarer side of the interface.