WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2024

PHSACOR14T-PHYSICS (CC14)

STATISTICAL MECHANICS

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *two* questions from the rest

1. Answer any ***ten*** questions from the following: 2×10 = 20

(a) What is meant by the term 'equal a priori' probability?

(b) What is microcanonical ensemble? Write down the probability distribution function for microcanonical ensemble.

(c) Explain the statistical idea of entropy.

(d) What is partition function?

(e) What do you mean by phase space?

(f) What is density matrix?

(g) Find the ratio of the probability of a simple harmonic one-dimensional oscillator being in the first excited state to the probability of its being in the ground state.

(h) Three particles are to be distributed in four energy levels a, b, c, d. Calculate the number of macrostates and microstates when the particles are classical particles.

(i) State Pauli's exclusion principle.

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(j) Under what conditions do Bose-Einstein and Fermi-Dirac distribution approach Maxwell-Boltzmann distribution?

(k) How chemical equilibrium is defined?

(l) Sketch FD distribution function at absolute zero of temperature and finite non-zero temperature.

(m) State and explain Wien displacement law.

(n) From the knowledge of partition function z, write an expression for entropy s in ideal Fermi gas.

(o) What is Bose condensation?

2. (a) Show that average energy, $\langle E \rangle = -\dfrac{\partial \ln z}{\partial \beta}$ where $z = \sum_r e^{-\beta E_r}$ is the partition function. 2

(b) Consider a system consisting of N independent harmonic oscillators, whose Hamiltonian is given by,

$$H(p, q) = \frac{p_i^2}{2m} + \frac{1}{2}m\omega^2 q_i^2 \quad , \quad (i = 1, 2, \cdots, N)$$

(i) Calculate the partition function using canonical ensemble and show that Helmholtz free energy is given by $A = Nk_\beta T \ln\left(\frac{\hbar\omega}{k_\beta T}\right)$. 2+2

(ii) Find an expression for the entropy of the system. 2

(iii) Show that the internal energy of the system is $v = Nk_\beta T$. 2

3. (a) In a quantum-mechanical two-state system with energy levels $E_1 = 0$ and $E_2 = 0$, the probability of finding a particle in a state of energy E is proportional to Boltzmann factor $e^{-\beta E}$. Find the total energy and C_V for a system of N non-interacting particles. Find low and high temperature limits of C_V. 2+2+2

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(b) Find the number of classical states, $dS_E = g(E)dE$, in the energy interval between E and $E + dE$. 2

(c) State the principle of equipartition of energy. 2

4. (a) Plot and compare Fermi-Dirac, Bose-Einstein, and Maxwell-Boltzmann distribution function as a function of energy. 3

(b) Write the basic assumptions of Bose-Einstein statistics and derive the corresponding distribution function. 1+4

(c) State Liouville's theorem. When an ensemble is a stationary one? 2

5. (a) Calculate the Fermi energy at absolute zero. 3

(b) Write the chemical potential in terms of energy, Helmholtz's free energy and Gibb's free energy. 3

(c) Calculate the chemical potential for ideal gas. 4

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Answer Question No. 1 and any *two* questions from the rest

1. Answer any ***ten*** questions from the following: $2\times10 = 20$

(a) Define equilibrium state of a system.

(b) State the principle of equipartition of energy.

(c) Obtain the canonical partition function of a two level system having energies ε and 2ε. Also find the probability that the system is in the higher energy (2ε) state.

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(d) Define microstates and macrostates with examples.

(e) Distinguish between canonical and grand canonical ensemble.

(f) What do you mean by stationary ensemble?

(g) Write down the expression for the thermal wavelength of a particle of mass m at temperature T and show that it has the dimension of length.

(h) Why the elementary volume of a cell in phase space for quantum particle cannot be zero?

(i) Show that the average energy of a system of particles $\langle E\rangle = \dfrac{k_B T^2}{Z}\left(\dfrac{\partial Z}{\partial T}\right)_V$.

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(j) Determine the wavelength corresponding to the maximum emissivity of a blackbody at a temperature equal to 300 K. Take $b = 2898\,\mu\text{m.K}$

(k) Prove that total pressure of diffused radiation is $P_{\text{rad}} = \dfrac{1}{3}u$, u being the energy density of radiation.

(l) Draw the Fermi-Dirac distribution function at $T = 0\,\text{K}$ and $T > 0\,\text{K}$. Locate the Fermi energy in the diagram.

(m) Find out the number of ways in which n identical bosons may be distributed among g energy levels.

(n) Helium has two isotopes, viz, He^3 and He^4. Classify these as fermions and bosons. Justify your conclusion.

(o) What do you mean by Bose-Einstein condensation?

2. (a) Consider a classical ideal gas consisting of N particles. The energy ε of a particle is given by $\varepsilon = cp$, where c is a constant and p is the magnitude of the momentum. Calculate (i) the partition function of the system, (ii) internal energy and (iii) specific heat C_V. 3+2+2

(b) What is Gibbs paradox? How this is solved? 3

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3. (a) What do you mean by partition function? 2

(b) Consider a system consisting of two particles each of which can be in any one of three quantum states of energies 0, $\varepsilon, 3\varepsilon$ respectively. The system is in contact with a heat reservoir at temperature $T = (k_B\beta)^{-1}$. Calculate partition for particles obeying 2+2+2

(i) MB statistics

(ii) BE statistics

(iii) FD statistics

(c) State Wiedemann-Franz law. 2

4. (a) Derive the Saha ionization equation in stellar atmosphere. 3

(b) State and explain the law of chemical equilibrium. 2

(c) Derive Stefan Boltzmann law from Planck's law of blackbody radiation and find the value of Stefan constant. In this case, what will be the heat capacity C_V? 3+1+1

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5. (a) State the symmetry requirement for a collection of identical particles under quantum regime. How does it lead to Pauli exclusion principle for fermions? 2+1

(b) Define Fermi energy and Fermi temperature. Explain the significance of Fermi energy at absolute zero and at any other temperature. 2+1

(c) Show that at $T = 0$, the average energy of an electron in a metal is $\frac{3}{5}E_F(0)$, where $E_F(0)$ denotes the Fermi energy at absolute zero temperature. 4

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2022

PHSACOR14T-PHYSICS (CC14)

STATISTICAL MECHANICS

Time Allotted: 2 Hours

Full Marks: 40

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Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *two* questions from the rest

1. Answer any ***ten*** questions from the following: $2\times10 = 20$

(a) Draw the phase space trajectory of 1-D simple harmonic oscillator.

(b) State Ergodic hypothesis in statistical mechanics.

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(c) What do you mean by ultraviolet catastrophe?

(d) In how many ways can two identical bosons be distributed in two energy states? Show the distribution diagrammatically.

(e) State Kirchoff's law and Stefan Boltzmann law.

(f) How does Sackur Tetrode equation resolve Gibbs paradox?

(g) How chemical equilibrium is defined?

(h) State the principle of equipartition of energy.

(i) Define microstates and macrostates.

(j) Explain the statistical idea of entropy.

(k) A spherical black body with radius R and at the temperature T (K) emits an energy E J/S. Another similar black body with radius $2R$ is at temperature $2T$ (K). What is the energy emitted by the second black body?

(l) State Saha ionization formula. What is its significance?

(m) Distinguish between canonical and grand-canonical ensembles.

(n) Two dices are rolled simultaneously. Enumerate the microstates and the macrostates.

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(o) Assuming a typical white dwarf star comprises a strongly degenerate electron gas, calculate the Fermi temperature of a typical white dwarf star.
(Given $m_e = 9.1\times10^{-31}$ kg, $k_B = 1.38\times10^{-23}$ JK^{-1}, $h = 6.627\times10^{-34}$ J.s and number density $N/V = 10^{36}$)

2. (a) Consider N independent, distinguishable, one dimensional quantum harmonic oscillators having energy spectrum $\varepsilon_n = \left(n + \frac{1}{2}\right)\hbar\omega$. Calculate the single particle partition function. Show that N oscillator partition function is given by $z = e^{-\frac{N}{2}\beta\hbar\omega}\{1 - e^{-\beta\hbar\omega}\}^{-N}$. 3

(b) Calculate internal energy U and C_V for the above system. 2+2

(c) Two states with energy difference 4.83×10^{-7} J occur with relative probability e^2. Calculate the temperature. Given $k = 1.38\times10^{-23}\,\text{J.K}^{-1}$. 3

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3. (a) State Liouville's theorem of ensemble theory. What information does it carry regarding the reversibility of a macroscopic process? 2+1

(b) A system has two energy states E and $3E$, the lower level is 6 fold degenerate and the upper level is 2 fold degenerate. If there are N particles, calculate the fraction of molecules at the upper level. 3

(c) Show that the density of state g for molecules obeying Maxwell Boltzmann distribution is 4

$$g(p)dp = \frac{4\pi p^2 dp}{h^3}$$

4. (a) Starting from Fermi-Dirac distribution law derive the expression for energy distribution of free electrons in metal. 4

(b) Calculate the Fermi energy at absolute zero. 3

(c) Evaluate the temperature at which there is one percent probability that a state with an energy 0.5V above the Fermi Energy will be occupied by an electron. 3

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5. (a) Write the chemical potential in terms of energy, Helmholtz's free energy and Gibb's free energy. 3+4+3

(b) Calculate the chemical potential for ideal gas.

(c) State law of mass action and Saha Ionization formula.

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2021

PHSACOR14T-PHYSICS (CC14)

STATISTICAL MECHANICS

Time Allotted: 2 Hours

Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *two* questions from the rest

1. Answer any ***ten*** questions from the following: 2×10 = 20

(a) Explain the statistical idea of entropy.

(b) What is meant by the term 'equal a priori probability'?

(c) Draw a phase space trajectory of a simple harmonic oscillator of energy E.

(d) A classical particle is free to move in a cube of side l. If its energy $\leq E$ find the volume of the phase space available to it.

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(e) What is ergodic hypothesis?

(f) What do you mean by 'ultraviolet catastrophe'?

(g) Find differences among microcanonical, canonical and grand canonical ensembles.

(h) What is the most probable kinetic energy $\tilde{\varepsilon}$ corresponding to Maxwellian velocity distribution?

(i) State Pauli's exclusion principle.

(j) What are distinguishable and indistinguishable particles?

(k) What is Bose-Einstein condensation?

(l) Show that the volume element

$$d\tau = \prod_{i=1}^{3N} (dq_i dp_i)$$

of the phase space remains invariant under a canonical transformation.

(m) State and explain Wien's displacement law.

(n) Prove that total pressure of diffused radiation is $P_{\text{rad}} = \frac{1}{3}u$, u being the energy density of radiation.

(o) From the knowledge of partition function Z, write an expression for entropy S in ideal Fermi gas.

Answer any *two* questions from the following 10×2 = 20

2. (a) Distinguish between microstates and macrostates. 2
   (b) Two dices are rolled simultaneously. Write the number of microstates and number of macrostates. 2
   (c) What is meant by a stationary ensemble? Give one example of a stationary ensemble. 2
   (d) Write all possible microstates of two quantum harmonic oscillators having total energy 4ε, ε being the spacing between the energy levels. Neglect the zero point energy. 2
   (e) State the principle of equipartition of energy. 2

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3. (a) Show that average energy, $\langle E\rangle = -\dfrac{\partial \ln Z}{\partial \beta}$ where $Z = \sum_r e^{-\beta E_r}$ is the partition function. 2
   (b) Consider a system consisting of N independent harmonic oscillators, whose Hamiltonian is given by,

$$H(p,q) = \frac{p_i^2}{2m} + \frac{1}{2}m\omega^2 q_i^2,\ (i = 1, 2, \ldots, N).$$

   (i) Calculate the partition function for the system using canonical distribution and show that Helmholtz free energy is given by $A = Nk_BT \ln\left(\dfrac{\hbar\omega}{k_BT}\right)$. 2+2
   (ii) Find an expression for the entropy of the system. 2
   (iii) Show that the internal energy of the system is $U = Nk_BT$. 2

4. (a) Plot and compare Fermi-Dirac, Bose-Einstein, and Maxwell-Boltzmann distribution function as a function of energy. 3
   (b) Show that at $T = 0$, the average energy of an electron in a metal is $\frac{3}{5}E_F$ where, E_F denotes the Fermi energy. 4
   (c) What is Gibbs paradox and how is it resolved? 3

5. (a) What is Bose-Einstein statistics? What are the basic postulates used? Derive an expression $n_i = g_i/(e^{\alpha}e^{\beta E_i} - 1)$ for the most-probable distribution of the particles of a system obeying B.E. statistics, hence deduce Planck's blackbody radiation formula. 1+1+3+3
   (b) Consider N non-interacting two level system with energy $\pm\varepsilon$. Show that the maximum entropy is $Nk_B \ln 2$. 2

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