WEST BENGAL STATE UNIVERSITY
B.Sc. Honours 5th Semester Examination, 2023-24

PHSACOR11T-PHYSICS (CC11)

QUANTUM MECHANICS AND APPLICATIONS

Time Allotted: 2 Hours　　　　　　　　　　　　Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any ***ten*** questions from the following: $2\times10=20$

(a) If an operator $\hat{A}$ is Hermitian, prove that the operator $\hat{B}=i\hat{A}$ is anti-Hermitian.

(b) Calculate the probability current density to $\psi=\frac{1}{r}e^{ikr}$ for a system having mass m.

(c) Two operators $\hat{A}$ and $\hat{B}$ have simultaneous eigenfunctions. Prove that $[\hat{A},\hat{B}]=0$.

(d) What is meant by the zero point energy of a linear harmonic oscillator?

(e) Show that the eigenfunctions belonging to different eigenvalues of a Hermitian operator are orthogonal.

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(f) What is the significance of Bohr magneton? What is its value in SI unit?

(g) Calculate Landé g-factor for atomic state $2P_{3/2}$.

(h) Why the magnetic field for Stern-Gerlach experiment should be inhomogeneous?

(i) What role does magnetic quantum number play in space quantization?

(j) Does $\tan(x)$ satisfy the properties of acceptable functions in quantum mechanics? — Explain.

(k) Show that the expectation values of $\hat{L}_x$ and $\hat{L}_y$ are zero for a quantum system which is an eigenstate of $\hat{L}_z$.

(l) Consider two states

$|\psi_1\rangle=2i|\phi_1\rangle+|\phi_2\rangle-a|\phi_3\rangle+4|\phi_4\rangle$ and $|\psi_2\rangle=3|\phi_1\rangle-i|\phi_2\rangle+5|\phi_3\rangle-|\phi_4\rangle$,

where $|\phi_1\rangle, |\phi_2\rangle, |\phi_3\rangle$ and $|\phi_4\rangle$ are orthonormal kets, and 'a' is constant. Find the value of 'a' so that $|\psi_1\rangle$ and $|\psi_2\rangle$ are orthogonal.

(m) Given $\hat{a}=\hat{p}_x-i\hat{x}^2$, $\hat{a}^\dagger=\hat{p}_x+i\hat{x}^2$. Find $[\hat{a},\hat{a}^\dagger]$ using the commutation relation of position and momentum operators.

(n) Only Hermitian operators are associated with physical quantities. — Why?

Turn Over

2. (a) What do you mean by degenerate wave functions? 1

(b) The normalized ground state wave function of a one-electron atom is given by $\psi(\bar{r})=\sqrt{\frac{z^3}{\pi a_0^3}}e^{-zr/a_0}$, where the notations have their usual meanings. Evaluate the probability that the electron in this state will be found at a distance greater than $\frac{2a_0}{z}$. 3

(c) What is the physical significance of a wave function? 3

(d) What do you understand by the wave function ψ of a moving particle? 1

(e) Consider a stationary state of energy E. Find how the wave function $\psi_E(t)$ for this state varies with time. 2

3. (a) For a given eigenfunction $\psi(x)=N\left(\frac{x}{x_0}\right)^n e^{-x/x_0}$, where N, n, x_0 are constants. Find the potential $V(x)$ and energy E using Schrödinger's equation. Consider $V(x)\to 0$ as $x\to\infty$. 3

(b) A particle moving along the x-axis, encounters a potential barrier of finite height and width. 1+3+3

(i) Draw a labelled diagram when height of barrier is more than energy of the particle of mass m.

(ii) Write the Schrödinger equation for different regions with appropriate boundary conditions.

(iii) Deduce the expression for the transmission coefficient.

4. (a) A particle is an infinite square well has as its initial wave function an equal mixture of first two stationary states, $\psi(x,0)=A[\psi_1(x)+\psi_2(x)]$. 2+2

Assuming $\psi_n(x)=\sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$, $E_n=\frac{n^2\pi^2\hbar^2}{2ma^2}$

(i) Normalize $\psi(x,0)$.

(ii) Find $\psi(x,t)$ and $|\psi(x,t)|^2$.

(b) Two operators $\hat{a}$ and $\hat{a}^\dagger$ are defined as 2

$$\hat{a}=\frac{1}{\sqrt{2m\hbar\omega}}(m\omega\hat{x}+i\hat{p}_x) \quad \text{and} \quad \hat{a}^\dagger=\frac{1}{\sqrt{2m\hbar\omega}}(m\omega\hat{x}-i\hat{p}_x)$$

Represent the position $\hat{x}$ and momentum $\hat{p}_x$ operators in terms of $\hat{a}$ and $\hat{a}^\dagger$.

(c) Show that parity operator has eigenvalues ± 1. 2

(d) Starting from radial equation of electron in the H atom show that the orbital angular momentum of electron is $L=\hbar\sqrt{l(l+1)}$, where l is orbital quantum number. 2

5. (a) For the n^{th} state of a linear harmonic oscillator, evaluate the uncertainty product $\Delta x\cdot\Delta p_x$. 2

(b) Why is Schrödinger equation not valid for relativistic particle? 2

(c) Define anomalous Zeeman effect. 2

(d) Discuss the general quantum mechanical theory of anomalous Zeeman effect to illustrate the Zeeman pattern for D_1 and D_2 lines of sodium. 4

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2022-23

PHSACOR11T-PHYSICS (CC11)

QUANTUM MECHANICS AND APPLICATIONS

Time Allotted: 2 Hours | Full Marks: 40

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Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any ***ten*** questions from the following: $2\times10=20$

(a) Explain the physical significance of energy time uncertainty relation.

(b) Give the physical interpretation of the wave function $\psi(x,t)$.

(c) What are stationary states in quantum mechanics?

(d) If the commutation relation between x and p is $[x,p]=i\hbar$. Find the commutation value of $[x^2, p]$.

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(e) What is the implication of the result: $[\hat{H}, \hat{L}]=0$?

(f) Consider the operator $\hat{Q}=i\dfrac{d}{d\phi}$ where ϕ is usual polar coordinates in two dimensions. Write down its eigenvalue equation and find its eigenvalues.

(g) What is normal Zeeman effect? Under what conditions it may be observed?

(h) What is Larmor precession of electron in an atom?

(i) Explain bound and unbound states in quantum mechanics.

(j) A wavefunction ψ is constructed as a linear combination of a set of orthonormal eigenfunctions ψ_n :

$$\psi=\sum_{n=1}^{\infty}c_n\psi_n$$

where c_n are constants. Show that if ψ is normalized then $\sum_{n=1}^{\infty}|c_n|^2=1$

(k) If the wavefunction of a particle trapped in space between $x=0$ and $x=L$ is given by $\psi(x)=A\sin\dfrac{2\pi x}{L}$, where A is a constant, for which value(s) of x will the probability of finding the particle be maximum?

(l) Electron configuration of Sodium is given by $1s^2\,2s^2\,2p^6\,3s^1$. Find the ground state term symbol of Sodium.

(m) What is Stark effect?

(n) A beam of spin $\frac{1}{2}$ particle is prepared in the state $|\psi\rangle = \frac{3}{\sqrt{34}}|+\rangle + i\frac{5}{\sqrt{34}}|-\rangle$; where $|+\rangle$ and $|-\rangle$ are eigen states of $\hat{S}_z$ with eigenvalues $+\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$ respectively. Find the average value in S_z measurement. 2

2. (a) If ψ_1 and ψ_2 are two eigen states with energy E_1 and E_2 respectively, check whether the state $(\psi_1 + \psi_2)$ is stationary or not. 3+2

(b) (i) Prove that the time rate of change of the expectation value of a dynamical variable satisfies the following relation

$$\frac{d}{dt}\langle \hat{A}\rangle = -\frac{i}{\hbar}\langle[\hat{A}, \hat{H}]\rangle + \left\langle\frac{\partial \hat{A}}{\partial t}\right\rangle,$$ where the symbols have their usual meanings.

(ii) Using the above relation show that the time rate of change of expectation value of momentum is equal to the average value of force. 2

(c) Prove that the parity of spherical harmonics $Y_{l,m}(\theta, \phi)$ is $(-1)^{\alpha}$ 1

(d) What do you mean by degenerate wavefunction?

3. (a) The potential in a region is given as:

$$V(x) = 0 \text{ for } x < 0$$
$$= V_0 \text{ for } 0 \le x \le a$$
$$= 0 \text{ for } x > a$$

A particle of mass m and energy $E < V_0$ travelling from left to the right is incident on the potential barrier.

(i) Write down Schrodinger equations in three regions of the potential. 2

(ii) Write down appropriate boundary conditions. 2

(b) The wavefunction of a hydrogen atom is given by the following superposition of energy eigenfunctions $\psi_{nlm}(\vec{r})$ (n, l, m are the usual quantum numbers):

$$\psi(\vec{r}) = \sqrt{\frac{2}{7}}\psi_{100}(\vec{r}) - \frac{3}{\sqrt{7}}\psi_{210}(\vec{r}) + \frac{1}{\sqrt{14}}\psi_{322}(\vec{r})$$

(i) Determine the ratio of expectation value of the energy to the ground state energy. 2

(ii) What are the expectation value of $\hat{L}^2$ and $\hat{L}_z$ operators? 2

(iii) What is the probability that the atom is found in a state of even parity? 2

4. (a) Hamiltonian for the linear harmonic oscillator is given by $\hat{H} = \frac{1}{2}\hat{p}^2 + \frac{1}{2}m\omega^2\hat{x}^2$, where the symbols have usual meanings. Using the basic commutation relation between $\hat{x}$ and $\hat{p}$ show that,

(i) $[\hat{a}, \hat{a}^+] = 1$ and 2+2

(ii) the Hamiltonian is given by 3

$$H = (\hat{a}^{+}\hat{a} + 1/2)\hbar\omega$$

given that $\hat{a} = \left(\frac{m\omega}{2\hbar}\right)^{1/2}\left(\hat{x} + \frac{i}{m\omega}\hat{p}\right)$ and $\hat{a}^{+} = \left(\frac{m\omega}{2\hbar}\right)^{1/2}\left(\hat{x} - \frac{i}{m\omega}\hat{p}\right)$

Then find the normalized ground state wavefunction of linear harmonic oscillator.

(b) A particle constrained to move along x-axis in the domain $0 \le x \le L$ has a wavefunction $\psi(x) = \sqrt{\frac{2}{L}}\,\sin\left(\frac{n\pi x}{L}\right)$, where n is an integer. What is the expectation value of its momentum? 3

5. (a) State Moseley's Law. Derive this law from Bohr's theory. 1+4

(b) Considering the L-S coupling scheme for helium atom, find the spectroscopics terms for (i) $1s^1\,2s^1$ and (ii) $1s^1\,2p^1$ configurations. 3

(c) In a Stern-Gerlach experiment on turning on the magnetic field, the beam splits into seven components. What is the angular momentum of the atoms in the beam? 2

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2021-22

PHSACOR11T-PHYSICS (CC11)

QUANTUM MECHANICS AND APPLICATIONS

Time Allotted: 2 Hours

Full Marks: 40

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All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any ***ten*** questions from the following: $2\times10=20$

(a) What is meant by expectation value?

(b) Explain what is meant by spin-orbit coupling.

(c) If $L_{\pm}\,|1,m\rangle = C_{\pm}\,|1,m\pm1\rangle$, find $C_{\pm}$. Here $L_{\pm}=L_x \pm iL_y$.

(d) Find the value of the commutator $[\sin(x),\, p_x]$, where symbols have their usual meanings.

(e) The eigenvalue equations corresponding to two operators A and B are respectively given by $Af(x)=af(x)$ and $Bf(x)=bf(x)$, where a and b are the corresponding eigenvalues of the operators A and B. Prove that the operators A and B commute.

(f) The one-dimensional wave function is given by $\psi(x)=\sqrt{a}e^{-ax}$. Find the probability of finding the particle between $x=\frac{1}{a}$ and $x=\frac{2}{a}$.

(g) Consider a particle whose Hamiltonian matrix is $H=\begin{pmatrix} 2 & i & 0 \\ -i & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$. Is $|\lambda\rangle=\begin{pmatrix} i \\ 7i \\ -2 \end{pmatrix}$ an eigenstate of H?

(h) What is Larmor precession of electron in an atom?

(i) Two operators A and B have simultaneous eigen-functions. Show that $[A\cdot B]=0$.

(j) Can the principal quantum number for an electron in a hydrogen atom be zero? Explain your answer.

(k) Justify the statement that the probability current density cannot be directly measured.

(l) What is Lande g-factor? Obtain an expression for it in terms of 1, s and j.

(m) Show that if a quantum particle has the wave function $\psi=e^{ikz}$, the z-component of its angular momentum is zero.

(n) Can Lithium $(Z=3)$ give rise to normal Zeeman effect? Justify your answer.

2. (a) What is stationary state? If ψ_1 and ψ_2 are two eigen states with energy E_1 and E_2 respectively, check whether the state $(\psi_1+\psi_2)$ is stationary or not. 1+2

(b) Does a stationary state evolve with time? Explain your answer. 2

(c) If $\psi_l^m(\vec{r},t)$ be the simultaneous eigenfunctions of the angular momentum operator L and L_z, what are the eigenvalue equations corresponding to the operators L^2 and L_z? 3+2

3. (a) The initial state of a two level system is a superposition of the ground state, $|E_1\rangle$, and first excited state, $|E_2\rangle$ (symbols bearing usual meaning), as follows: $|\Psi\rangle = 3|E_1\rangle + 2|E_2\rangle$. 2+2

(i) Find the possible results of energy measurement with their corresponding probabilities.

(ii) Find the average value of energy. Will it be time dependent?

(b) Explain the origin of spin-orbit interaction. 3

(c) What is meant by space quantization? What role does magnetic quantum number play in space quantization? Explain in the light of vector atom model. 3

4. (a) A particle of mass m and momentum p is incident from left on the potential step of height V_0. Calculate the probability that the particle is scattered backward by the potential if $\frac{p^2}{2m} < V_0$. 4

(b) Calculate the expectation value of the potential energy of the electron in the 1s state of H-atom. The wave function of the 1s-electron of H-atom is given by $\psi_{100} = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$ where a_0 is the Bohr radius $= 4\pi\varepsilon_0\hbar^2/me^2$, with usual meanings of symbols. 3

(c) Determine stating reasons whether each of the following functions is acceptable or not as a state function over the indicated intervals. 3

(i) $\sqrt{\frac{2}{l}}\sin\frac{n\pi x}{l}$ in the range -0 to $+l$

(ii) $\sin^{-1}x$ in the range +1 to – 1.

5. (a) Find the energy of n^{th} state of a linear harmonic oscillator with mass m and frequency ω. Show that the average potential energy of the n^{th} state of a linear harmonic oscillator is half of the energy of the oscillator in this state. 3+3

(b) Discuss the goal of Stern-Gerlach experiment. Why is it necessary to apply an inhomogeneous magnetic field in this experiment? 3+1

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2020, held in 2021

PHSACOR11T-PHYSICS (CC11)

QUANTUM MECHANICS AND APPLICATIONS

Time Allotted: 2 Hours | Full Marks: 40

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Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any ***ten*** questions from the following: 2×10 = 20

(a) Consider a system whose Hamiltonian is given by $\hat{H} = \alpha(|\phi_1\rangle\langle\phi_2| + |\phi_2\rangle\langle\phi_1|)$, where α is a real number having appropriate dimension and $|\phi_1\rangle, |\phi_2\rangle$ are normalized eigenstates of an Hermitian operator $\hat{A}$ that has no degenerate eigenvalue. Determine if $|\phi_1\rangle, |\phi_2\rangle$ are eigenstates of $\hat{H}$.

(b) State Heisenberg uncertainty principle.

(c) Find the lowest energy of an electron confined to move in a one dimensional box of length 1 Å.

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[Given, $m = 9.11\times10^{-31}\,\text{kg}$, $\hbar = 1.05\times10^{-34}\,\text{Js}$, $1\,\text{eV} = 1.6\times10^{-19}\,\text{J}$]

(d) Consider the operator $\hat{Q} = i\frac{d}{d\phi}$, where ϕ is plane polar azimuthal angle in two dimensions. Write down its eigenvalue equation and find its eigenvalues.

(e) What is Larmor precession of an electron in an atom?

(f) For one dimensional bound state motion of a particle of mass m, prove that the expectation value of its kinetic energy is given by $\langle K\rangle = \frac{\hbar^2}{2m}\int_{-\infty}^{\infty}\frac{\partial\psi}{\partial x}\frac{\partial\psi^*}{\partial x}dx$.

(g) If $\psi_1(x,t)$ and $\psi_2(x,t)$ are both solutions of the time dependent Schrödinger equation for the motion of a particle with potential energy $V(x)$, prove that the linear combination $\psi(x,t) = A_1\psi_1(x,t) + A_2\psi_2(x,t)$ is also a solution, where A_1 and A_2 are constants.

(h) Calculate the probability current density of a quantum mechanical system of mass m and described by the state function $\psi(r) = \frac{1}{r}e^{ikr}$.

(i) A particle constrained to move along x-axis in the domain $0 \le x \le L$ has a wavefunction $\psi(x) = \sqrt{\frac{2}{L}} \sin(n\frac{\pi x}{L})$, where n is an integer. What is the expectation value of its momentum?

(j) Plot the wavefunctions of the ground state and the 1[st] excited state for a particle within an infinite square well potential of width a. Also plot the probability density for those states.

(k) Find the probability of finding an electron within Bohr radius for the ground state of hydrogen atom. Given that the ground state wavefunction is $\psi_{1S} = \sqrt{\frac{1}{\pi a^3}} \exp\left(-\frac{r}{a}\right)$, where a is the Bohr radius.

(l) Find the degeneracy of the n-th energy eigenstate for the electron in a hydrogen atom neglecting its spin.

(m) In a Stern-Gerlach experiment, on turning on the magnetic field, the beam splits into seven components. What is the angular momentum of the atoms in the beam?

(n) Find the uncertainty in the measurement of S_z on a system prepared in a state $|\psi\rangle = \frac{(|\uparrow\rangle + |\downarrow\rangle)}{\sqrt{2}}$ where $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigenvectors of the operator $\hat{S}_z$ with eigenvalues $+\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$ respectively.

2. A particle of mass m is in a normalized state $\psi(x) = Ae^{-a\left[\frac{mx^2}{\hbar} + it\right]}$, where A and a are constants.

(a) Find A. 2

(b) For what potential energy function $V(x)$ does $\psi(x)$ satisfy Schrödinger equation. 2

(c) Calculate the expectation values of x and p_x. 3+3

3. (a) Starting from the time-dependent Schrödinger equation in one dimension, derive the equation of continuity of probability. 4

(b) If a dynamical variable is represented by the quantum mechanical operator $\hat{A}$ that does not depend on time explicitly, prove $\frac{d}{dt}\langle\hat{A}\rangle = \frac{i}{\hbar}[\hat{H}\hat{A} - \hat{A}\hat{H}]$, where $\hat{H}$ is the Hamiltonian operator. 3

(c) If $\{|\vec{r}\rangle\}$ and $\{|\vec{p}\rangle\}$ represent the bases in position and momentum representations respectively, then prove that $\langle\vec{p}\,|\,\vec{r}\rangle = \langle\vec{r}\,|\,\vec{p}\rangle^* = \frac{1}{(2\pi\hbar)^{3/2}} e^{-i(\vec{p}.\vec{r})/\hbar}$ 3

4. (a) Ground state eigenfunction of a one dimensional quantum oscillator is 3

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{\left\{-\frac{m\omega}{2\hbar}x^2\right\}}.$$

Show that its ground state energy is $E_0 = \frac{1}{2}\hbar\omega$

(b) Write down the Schrödinger equation in spherical polar co-ordinates corresponding to the hydrogen atom problem. Write down the radial part of the wave function. 2

(c) What is space quantization of orbital angular momentum? 2

(d) Find the normalized ground state wavefunction of linear harmonic oscillator using the operator $\hat{a} = \frac{m\omega\hat{x} + i\hat{p}}{\sqrt{2m\omega\hbar}}$. 3

5. (a) Explain the term 'spin-orbit coupling'. 2

(b) In a many electron atom, the orbital, spin and total angular momenta are denoted by $L=2$, $S=1$ and $J=2$. Find the angle between $\vec{L}$ and $\vec{S}$ using the vector atom model. 2

(c) Derive an expression for the magnetic moment for an electron moving in a circular orbit. Hence show that the ratio of the orbital magnetic moment to its angular momentum is $\frac{e}{2m}$. 3+3

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