WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 3rd Semester Examination, 2021-22

PHSACOR05T-PHYSICS (CC5)

MATHEMATICAL PHYSICS-II

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
Answers must be precise and to the point to earn credit.
All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any ***ten*** questions from the following: $2\times 10 = 20$

(a) How will you change the function $f(x)$ in the interval $(-\pi,\pi)$ to $(-l,l)$ in Fourier series?

(b) Sketch the periodic extension of $f(t)=0$ for $t<0$, $f(t)=1$ for $t>0$, if the fundamental interval is $(-1, 1)$.

(c) Let $F(x,y,y')=2x+xy'+(y')^2+y$, then obtain the corresponding Euler Lagrange equations that follow from Hamilton's principle.

(d) Prove that if the Lagrangian of a conservative system does not contain time explicitly, the total energy is conserved.

(e) Prove that $P_{2m+1}(0)=0$.

(f) Find the nature and location of the singularities of the following differential equation $(1-x^2)y''-2xy'+l(l+1)y=0$.

(g) For integer n Bessel function of first kind are given as,

$$J_n(x)=\sum_{j=0}^{\infty}\frac{(-1)^j(x/2)^{2j+n}}{j!\Gamma(n+j+1)}$$

Show that $J_{-n}(x)=(-1)^n J_n(x)$.

(h) State Hamilton's principle.

(i) What do you understand by degrees of freedom of a dynamical system?

(j) Determine the Wronskian for the differential equation $y''+y=0$.

(k) If $H_n(x)$ are Hermite polynomials then evaluate $\int_{-\infty}^{\infty} H_2(x)H_3(x)e^{-hx^2}dx$.

(l) Using Generating function of $H_n(x)$ i.e. $e^{-t^2+2tx} = \sum_{n=0}^{\infty} H_n \frac{t^n}{n!}$,

show that $H_n(-x) = (-1)^n H_n(x)$

(m) With the help of Legendre transformation, determine the Hamiltonian corresponding to the Lagrangian $L = \frac{1}{m}\dot{x}^2$, where m is a constant.

(n) Consider $y = \sum_{r=0}^{\infty} a_k x^{k+m}$. Obtain the indicial equation and it's roots for the Legendre's differential equation.

2. (a) Write the differential equation obeyed by Legendre polynomials. Show that 3

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2-1)^n$$

4

(b) If $f(x) = 0$ for $0 < x < \pi/2$

$= \pi - x$ for $\pi/2 < x < \pi$,

Then show that $f(x) = \frac{\pi}{4} - \frac{2}{\pi}\left(\frac{1}{1^2}\cos 2x + \frac{1}{3^2}\cos 6x + \ldots\right)$

(c) Solve the differential equation 3

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \text{ and obtain } \phi(x,y)$$

Given $\phi(x,y) = \phi(0,y) = 0$

3. (a) Using the generating function for the Legendre polynomials $P_n(x)$ to show that 2+2

Prove that $\int_{-1}^{1} [P_n(x)]^2 dx = \frac{2}{2n+1}$

and hence show that $\int_{-1}^{1} P_3^2(x)\, dx = \frac{2}{7}$

(b) Show that the Bessel function 3

$$J_n(x) = \sum_{j=0}^{\infty} \frac{(-1)^j (x/2)^{2j+n}}{j!\,\Gamma(n+j+1)}$$

satisfy the Bessel's equation $x^2 y'' + xy' + (x^2 - n^2)y = 0$.

(c) Apply Legendre Transformation on the Internal energy function U(S,V) to obtain Gibbs free energy G(T,P). 3

4. (a) Consider the differential equation $2x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - (x+1)y = 0$, obtain a series solution in powers of x using Frobenius method. 5

(b) Derive Hamilton's canonical equations of motion. Obtain Hamilton's equation for a particle in a central force field. 2+3

5. (a) A charge $+q$ is distributed uniformly along the z-axis from $z=-a$ to $z=+a$. 4
Show that the electric potential at a point $\vec{r}$ is given by

$$V(r,\theta)=\frac{q}{4\pi\varepsilon_0 r}\left[1+\frac{1}{3}\left(\frac{a}{r}\right)^2 P_2(\cos\theta)+\cdots\cdots\right],$$

where $P_n(\cos\theta)$ are Legendre polynomial.

(b) Using Hamilton's canonical equations of motion, show that the Hamiltonian $(H(x,p,t))$ is conserved provided $\frac{\partial H}{\partial t}=0$. 3

(c) If $u=f(r)$ and $x=r\cos\theta$, $y=r\sin\theta$, prove that 3

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f''(r)+\frac{1}{r}f'(r).$$

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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B.Sc. Honours 3rd Semester Examination, 2020, held in 2021

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Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any ***ten*** questions from the following: $2\times 10 = 20$

(a) State Hamilton's principle.

(b) A cart with a pair of wheels having radius R connected through an axle of length l is rolling without slipping on an inclined plane with an angle of inclination θ. Determine the number of degrees of freedom.

(c) Lagrangian of a particle of mass m is $L=\frac{1}{2}m\dot{q}^2-\frac{1}{2}Kq^2-Kq\dot{q}t$, where K is a constant. Show that the particle is moving freely.

(d) What is the fundamental difference between Lagrange's equation and Hamilton's canonical equations for the same system?

(e) Suppose a second-order linear homogeneous ordinary differential equation in x has a power series solution like $\sum_{r=0}^{\infty} a_r x^{k+r}$, where k is a constant. Can it satisfy the recurrence relation $a_{r+2}(k+r+2)(k+r+1)+a_r(k+r+1)=0$ for $a_0, a_1=0$ simultaneously? Explain.

(f) "Hermite polynomials are orthogonal to each other in the range [0, 1] with weight function $\exp(-x^2)$." Is the statement true? Justify your answer.

(g) Mention orthogonality conditions required to determine the Fourier Coefficients.

(h) $J_0(x)$ and $J_1(x)$ are solutions of the Bessel differential equations of order 0 and 1 respectively. Show that J_0 and J_1 are linearly independent of each other.

(i) Let $F(x, y, y')=2x+xy'+y'^2+y$, then obtain the corresponding Euler-Lagrange equations that follow from Hamilton's principle.

(j) What is a convex function? Give an example.

(k) Apply Legendre Transformation on the Internal energy function $U=U(S,V)$ to obtain Helmholtz free energy $F=F(T,V)$.

(l) Prove that homogeneity of time for an isolated system leads to conservation of energy.

(m) Show that the generalized momenta conjugate to cyclic coordinates are conserved.

(n) Show that for Legendre polynomial $P_n(x)$ we can have $P_n(-x)=(-1)^n P_n(x)$.

2. (a) If $f(x)$ is any square-integrable function in the range $-\pi<x<\pi$ such that $\int_{-\pi}^{+\pi}[f(x)]^2 dx$ is finite, show that $\lim_{m\to\infty} a_m = 0$, $\lim_{m\to\infty} b_m = 0$ for the Fourier coefficients a_m and b_m. 3

(b) Suppose that the following differential equation refers to a problem of the 2D steady flow of heat: 5

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

Solve for u using separation of variables with given boundary conditions:

$u(0,\, y)=u(a,\, y)=u(x,\, \infty)=0$ and $u(x,\, 0)=\sin\frac{\pi x}{a}$, where a is the length of the boundary in x-direction.

(c) If ψ is a solution of Laplace's equation, show that $\partial\psi/\partial z$ is also a solution. 2

3. (a) Prove that the solutions of the equation of motion for one dimensional simple harmonic oscillator are linearly independent. 3

(b) Expand the following function in a Fourier series $f(x)=\begin{cases} -\pi, & -\pi<x<0 \\ x, & 0<x<\pi \end{cases}$. 3+2

Hence show that $\frac{\pi^2}{8}=1+\frac{1}{9}+\frac{1}{25}+\frac{1}{49}+\cdots$

(c) Can we apply the method of separation of variables to solve the given equation 2

$$xy^2\frac{\partial^2 u}{\partial x^2}+x^2 y\frac{\partial^2 u}{\partial y^2}=x+y$$

4. (a) Using the Rodrigue's formula, determine $P_3(x)$. 2

(b) Show that Legendre's equation has regular singularities at $x=-1$, 1 and ∞. 2

(c) Starting from Hamilton's principle, establish Euler-Lagrange's equation of motion for any bilateral holonomic system having n-number of degrees of freedom and without any non-potential force. 4

(d) A ring is sliding on a smooth coaxial horizontal cylinder that rotates about a vertical axis with a constant angular velocity. Find the number of holonomic constraints and degrees of freedom for this system. 2

5. (a) Check if the Frobenius power series solution is applicable for the differential equation: $x^4 y''+y=0$. Justify your answer. 2

(b) Expand the function $\dfrac{1}{|\vec{r}-\vec{r}'|}$ in a power series of $\dfrac{r'}{r}$, using Legendre polynomials, for $\dfrac{r'}{r}<1$. Hence identify the quadrupole term in the electrostatic potential $V(\vec{r})$ due a point charge Q at $\vec{r}'$. 2+1

(c) The Lagrangian $L(q_i, \dot{q}_i, t)$ undergoes a gauge transformation: 3+2

$$L' = L + \frac{dF(q_i, t)}{dt}$$

Prove that (i) the Euler-Lagrange equation of motion is invariant under this transformation but (ii) the generalized momenta change to $p_i + \dfrac{\partial F}{\partial q_i}$.

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