# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2021-22

## PHSACOR01T-PHYSICS (CC1)

## MATHEMATICAL PHYSICS-I

Time Allotted: 2 Hours | Full Marks: 40

*The figures in the margin indicate full marks.*
*Candidates should answer in their own words and adhere to the word limit as practicable.*
*All symbols are of usual significance.*

**Question No. 1 is compulsory and answer any *two* from the rest**

1. Answer any ***ten*** questions from the following: $2\times 10 = 20$

(a) Sketch: $f(\theta) = \sin\theta\cos\theta$ for $0 \le \theta \le 4\pi$.

(b) If $\vec{r}$ be the position vector of a point on a closed contour, prove that the line integral $\oint \vec{r}\cdot d\vec{r} = 0$.

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(c) If $\vec{A}$ and $\vec{B}$ are irrotational then prove that $\vec{A}\times\vec{B}$ is solenoidal.

(d) Find the Taylor series of the function $f(x) = \dfrac{1}{x^2+4}$ about the point $x = 0$.

(e) State the Uniqueness theorem of the solution of a differential equation for initial value problems.

(f) Find the volume of the parallelepiped whose edges are represented by $\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{B} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{C} = 3\hat{i} - \hat{j} + 2\hat{k}$.

(g) The position vector of a particle is $\vec{r}(t) = \cos(\omega t)\hat{i} + \sin(\omega t)\hat{j}$, where $\omega$ is a constant. Prove that, at every instant, its velocity and acceleration are perpendicular to each other.

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(h) Find the directional derivative of the scalar field $\phi(x,y,z) = 2x^2 + yz$ in the direction of the vector $\hat{i} + \hat{j}$ at the point $(0, 1, -1)$.

(i) Find the value of $k$ so that the average value of the function

$f(t) = \dfrac{3t^2}{8} + kt$ in the range $0 \le t \le 2$ vanishes.

(j) A particle moves along a curve, $x = 2t^2$, $y = t^2 - 4t$ and $z = 3t - 5$ where "$t$" is time. Find its component velocity at time $t = 1$ in the direction of vector $(\hat{i} - 2\hat{j} + 2\hat{k})$.

(k) Solve the following differential equation.

$$ye^y dx = (y^3 + 2xe^y)\,dy.$$

(l) Show that, $\vec{\nabla}\times\left(\dfrac{\vec{r}}{r^2}\right) = 0$.

(m) In a normal distribution, 31% of items are under 45 and 8% are over 64. Find the mean and the standard deviation of the distribution.

(n) An integer is chosen at random from the first 200 positive integers. What is the probability that the integer chosen is divisible by 6 or 8?

2. (a) Evaluate $\iint_R x(y-1)\,dxdy$, where R is the region bounded by the parabola $y = 1 - x^2$ and $y = 0$. 3+3+4

(b) Prove that for a scalar field $\phi$,

$$\oint_S \phi\, d\vec{S} = \int_V (\vec{\nabla}\phi)\, dV ,$$

where V is the volume bounded by the closed surface S.

(c) Solve: $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} + 6y = 2\sin(4x)$.

3. (a) State Green's theorem in a plane. 1+5+(1+3)

(b) Verify Gauss' divergence theorem for the vector field $\vec{F} = 4y\hat{i} - 2x\hat{j} + z^2\hat{k}$, where $V$ is the volume bounded by the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ together with the plane $z = 0$.

(c) State the condition of convergence of a Taylor series expansion. Find interval of $x$ for which the Taylor series of $\ln(1+x)$ about $x = 0$, converges.

4. (a) If $r = \sqrt{x^2 + y^2}$ and $z = \phi(r)$, show that $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = \dfrac{1}{r}\phi'(r) + \phi''(r)$. (2+2)+3+3

(b) Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube.

(c) If $\vec{A} = (y-2x)\hat{i} + (3x+2y)\hat{j}$, compute the circulation of $\vec{A}$ about a circle $C$ in the XY plane with center at the origin and radius 2, if $C$ is traversed in the positive direction.

5. (a) Find the area of the ellipse described by $x = a\cos\theta$, $y = b\sin\theta$. 3+3+1+3

(b) Prove $\vec{\nabla}\cdot\left(\dfrac{\vec{r}}{r^3}\right) = 0$, where $\vec{r}(\neq 0)$ is the position vector.

(c) What is meant by probability distribution function?

(d) A box ***A*** contains 2 white and 4 black balls. Another box ***B*** contains 5 white and 7 black balls. A ball is transferred from the box ***A*** to the box ***B***. Then a ball is drawn from the box ***B***. Find the probability that the drawn ball is white.

**N.B.** : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2020, held in 2021

## PHSACOR01T-PHYSICS (CC1)

## MATHEMATICAL PHYSICS I

Time Allotted: 2 Hours — Full Marks: 40

*The figures in the margin indicate full marks.*
*Candidates should answer in their own words and adhere to the word limit as practicable.*
*All symbols are of usual significance.*

**Question No. 1 is compulsory and answer any *two* from the rest.**

1. Answer any ***ten*** questions from the following: 2×10=20



(a) Sketch: $f(\theta)=1+\frac{1}{2}\sin^2\theta$ for $0\le\theta\le 2\pi$.

(b) Show that $f(x)=\frac{|x|}{x}$ is discontinuous at $x=0$, where $f(0)=0$.

(c) If $d\varphi(x,\ y)=M(x,\ y)\,dx+N(x,\ y)\,dy$, where $\varphi$ is a well-behaved function of its arguments, then show that

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$$

(d) Solve: $\frac{dy}{dx}+2xy=4x$

(e) Determine a unit vector perpendicular to the plane of $\vec{A}=2\hat{i}-6\hat{j}-3\hat{k}$ and $\vec{B}=4\hat{i}+3\hat{j}-\hat{k}$.

(f) The position vector $\vec{r}$ of any arbitrary point on the surface satisfies the equation $|\vec{r}|=k$, a constant. Identify the geometry of the surface and justify your answer.

(g) $\vec{r}$ is the position vector of an arbitrary point in a three-dimensional space. Using Cartesian coordinate system, find gradient of $1/|\vec{r}|$ at any point away from the origin.



(h) For a vector field $\vec{F}(x,\ y,\ z,\ t)$ show that $dF=(d\vec{r}\cdot\vec{\nabla})\,\vec{F}+\frac{\partial\vec{F}}{\partial t}dt$.

(i) Show that $\vec{\nabla}\times\vec{r}f(r)=0$ where $r$ is the magnitude of position vector of any arbitrary point in three-dimensional space ($f(r)$ being differentiable everywhere).

(j) $\phi$ is a scalar function satisfying the equation $\nabla^2\phi=0$. Show that $\vec{\nabla}\phi$ is both solenoidal and irrotational.

(k) Show that $\oiint_S d\vec{S}=0$.

(l) A dice is thrown. What is the probability that the number obtained is a prime number?

(m) There are five green and seven red balls. Two balls are selected one by one without replacement. Find the probability that the first is green and the second is red.

(n) What is meant by a probability distribution function? Cite an example.

2. (a) Prove the identity $\vec{\nabla}.(\vec{A}\times\vec{B}) = \vec{B}.(\vec{\nabla}\times\vec{A}) - \vec{A}.(\vec{\nabla}\times\vec{B})$. Hence show that $(\vec{A}\times\vec{r})$ is solenoidal if $\vec{A}$ is irrotational. 3+1

(b) If $f(x, y, z) = 0$, then show that $\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$. 4

(c) A multiple-choice test consists of 100 questions. Answer to each question has four possible options among which only one is correct. If a student answers all the questions by guessing at random, then what is the expected number of correct answers given by him? 2

3. (a) Solve: $\frac{y}{x^2} + 1 + \frac{1}{x}\frac{dy}{dx} = 0$ 2

(b) Determine the constant $a$ so that the following vector is solenoidal: 4

$$\vec{V} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+az)\hat{k}$$

(c) Calculate the mean and the variance of a binomial distribution. 2+2

4. (a) The relativistic sum $w$ of two velocities $u$ and $v$ in the same direction is given by 4

$$\frac{w}{c} = \frac{\frac{u}{c} + \frac{v}{c}}{1 + \frac{uv}{c^2}}$$

If $u/c = v/c = 1 - \alpha$, where $0 \leq \alpha \leq 1$, find $w/c$ in powers of $\alpha$. Show terms only up to $\alpha^3$.

(b) Find the directional derivative of $\varphi(x, y, z) = x^2 y + xz$ at $(1, 2, -1)$ along the direction of $\vec{A} = 2\hat{i} - 2\hat{j} + \hat{k}$. 3

(c) Use Green's theorem on a plane to show that the area bounded by a simple closed curve $C$ is $\frac{1}{2}\oint_C (x\,dy - y\,dx)$. 3

5. (a) An integer $N$ is chosen at random with $1 \leq N \leq 100$. What is the probability that $N$ is a perfect square? 2

(b) Obtain the complementary function of the differential equation 3

$$\frac{d^2 y}{dx^2} + 5\frac{dy}{dx} + 4y = \cos 2x$$

(c) Evaluate the line integral of $\vec{A}(x, y, z) = x^2\hat{i} + y^2\hat{j} - z^2\hat{k}$, from the origin to $(a, b, c)$, along the path given parametrically by $x = at^2$, $y = bt$, $z = c\sin(\pi t/2)$. Does the result depend on the path? Justify your answer. 3+2

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# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2019

## PHSACOR01T-PHYSICS (CC1)

Time Allotted: 2 Hours — Full Marks: 40

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*Candidates should answer in their own words and adhere to the word limit as practicable.*
*All symbols are of usual significance.*

### Answer Question No.1 and any *two* questions from the rest

1. Answer any ***ten*** questions from the following: $2\times10=20$

(a) Prove that the series $x+\frac{x^2}{2}+\frac{x^3}{3}+\cdots$ is convergent for $0<x<1$.

(b) Find the general solution of the equation



$$2x\frac{dy}{dx}+y=2xe^{5/2}$$

(c) A function $f(x)$ is defined by

$$\begin{aligned} f(x) &= \cos x \;\text{ for } x\geq 0 \\ &= -\cos x \;\text{ for } x<0 \end{aligned}$$

Is $f(x)$ continuous at $x=0$? Give reasons.

(d) If $f(x)=|x|$, show that $f(0)$ is a minimum although $f'(0)$ does not exist.

(e) Prove $\nabla^2 f(r)=\frac{d^2 f}{dr^2}+\frac{2}{r}\frac{df}{dr}$

(f) Calculate the Laplacian of the scalar field $\ln(x^2+y^2)$.

(g) Expand $f(x)=\frac{1}{x-2}$ in a Taylor series about the point $x=1$.

(h) Wronskian of two functions is $\omega(t)=t\sin^2 t$. Are the functions linearly independent or linearly dependent? Explain.

(i) If $\vec{A}=2\hat{i}+\hat{j}-3\hat{k}$ and $\vec{B}=\hat{i}-2\hat{j}+\hat{k}$, find a vector of magnitude 5 perpendicular to both $\vec{A}$ and $\vec{B}$.



(j) Show that $\vec{\nabla}\phi$ is a vector perpendicular to the surface $\phi(x,y,z)=c$ where $c$ is a constant.

(k) Find an expression for $ds^2$ in curvilinear co-ordinates $u, v, w$. Then determine $ds^2$ for the special case of an orthogonal system.

(l) An integer is chosen at random from the first 200 positive integers. What is the probability that the integer chosen is divisible by 6 or 8?

(m) A distribution function is given by $f(x)=\dfrac{1}{\pi\sqrt{A^2-x^2}}$ where $x$ is the random variable. Find the value of $\langle|x|\rangle$. (Assume '$A$' is a constant).

(n) Show that, for large number of trials, Binomial distribution yields Poisson distribution.

1+3

2. (a) Check if the following differential equation $\dfrac{dy}{dx}-y\tan x=e^{x}\sec x$ is exact. Hence, solve the equation. 2+1

(b) Show that $\vec{A}\cdot(\vec{B}\times\vec{C})$ is an absolute value equal to the volume of a parallelepiped with sides $\vec{A}, \vec{B}$ and $\vec{C}$. Hence, find the condition for these vectors to be coplanar. 3

(c) Find the extrema of $f(x,y)=5x-3y$ subject to the constraint $x^2+y^2=136$.

6

3. (a) Solve: $\dfrac{d^2y}{dx^2}+3\dfrac{dy}{dx}+2y=e^{2x}\sin x$. 4

(b) Show that $r^n\vec{r}$ is an irrotational vector for any value of $n$, but is a solenoidal only if $n=-3$.

$2\frac{1}{2}+2\frac{1}{2}+1$

4. (a) Suppose that $X$ is exponentially distributed with $\lambda=3$.

(i) What is $P\{X>2\}$?

(ii) What is $P\{X>5\,|\,X>3\}$?

(iii) What did you notice about these two answers? Is it a coincidence? 4

(b) A person makes steps of length '$l$' is just likely to step forwards as backwards. Prove that after $n$ steps in this random walk, the person will have gone forward a distance $rl$ with a probability $\left(\frac{1}{2}\right)^n\, {}^nC_{\frac{n+r}{2}}$.

4

5. (a) Find the expression of Laplacian in cylindrical coordinate system. 3

(b) Prove that $\oint u\nabla v\cdot d\lambda=-\oint v\nabla u\cdot d\lambda$ 3

(c) Evaluate $\oint_C y^3dx-x^3dy$ where $C$ is the positively oriented circle of radius 2 centered at the origin.

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WEST BENGAL STATE UNIVERSITY
B.Sc. Honours 1st Semester Examination, 2018

# PHSACOR01T-PHYSICS (CC1)

## MATHEMATICAL PHYSICS-I

Time Allotted: 2 Hours | Full Marks: 40

*The figures in the margin indicate full marks.*
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*All symbols are of usual significance.*

**Answer Question No. 1 and any *two* questions from the rest**

1. Answer any ***ten*** questions from the following: $2\times10=20$

(a) Plot $f(x)=xe^{-x}$ for $x\geq 0$. Indicate any extremum within this range.

(b) Test whether $\cos(x)$ and $\sin(x)$ are linearly independent within the interval $-\infty<x<\infty$

(c) Expand $f(x,y)=xy$ in a Taylor series around (1, 1).

(d) Verify that $y=e^{t^2}\int_0^t e^{-s^2}ds+e^{t^2}$ is a solution of the differential equation $\frac{dy}{dt}-2ty=1$

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(e) What do you mean by a regular singular point of a second order linear differential equation?

(f) Prove that the volume of a sphere with radius '$a$' is $\frac{4}{3}\pi a^3$

(g) Find the value of $\varepsilon_{ijk}\varepsilon_{ijk}$, where $\varepsilon_{ijk}$ = Levi-Civita symbol and assume Einstein's summation convention.

(h) Given a vector $\vec{A}=2x\hat{i}-3y\hat{j}+5z\hat{k}$. Verify that $l^2+m^2+n^2=1$, where the $(l,\, m,\, n)$ are the directional cosines along three mutually perpendicular directions $(\hat{i},\,\hat{j},\,\hat{k})$.

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(i) Find the directional derivative of the function $\Phi(x,\, y,\, z)=2xy+z^2$ in the direction of the vector $\vec{i}+2\vec{j}+2\vec{k}$ at the point (1, –1, 3).

(j) Find $\nabla^2\gamma^4$ using cartesian coordinate where $\vec{\gamma}$ is the position vector of an arbitrary point.

(k) Using Gauss's theorem prove that

$$\oiint_s d\vec{s}=0$$

(l) Show that $\vec{\nabla}\times(\vec{r}f(r))=0$, where $\vec{r}$ is a position vector and $f(r)$ is differentiable.

(m) A dice is rolled and a coin is tossed simultaneously. Find the probability that the dice shows an odd number and the coin shows a head.

(n) If $x$ is a random variable obeying Poisson distribution of probability, find the mean of $x^2$.

2. (a) Calculate the extreme values of the function $f(x, y, z)=xyz$, subject to the constraint $x^2+y^2+z^2=3$, using Lagrange's multipliers. 4+(3+2)+1

(b) Show that,

(i) $\vec{\nabla}\times(\vec{\nabla}\times\vec{A})=\vec{\nabla}(\vec{\nabla}\cdot\vec{A})-\nabla^2\vec{A}$ and

(ii) $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A})=0$

(c) What is meant by a random variable?

3. (a) Show that the following differential equation is exact, 2+(1+3+1)+3

$$2xy\frac{dy}{dx}+y^2-2x=0$$

(b) What is meant by linear independence of three vectors? Find the necessary and sufficient condition for three vectors $\vec{A}, \vec{B}\ \&\ \vec{C}$ (in three dimension) being linearly independent in terms of their scalar triple product. State the geometrical significance of the result.

(c) For an exhaustive set of events $\{A_1, A_2, \ldots, A_n\}$ which are mutually exclusive and $B$ is another event defined on the same space, show that

$$P(A_j\,|\,B)=\frac{P(B\,|\,A_j)P(A_j)}{\sum_{i=1}^{n}P(B\,|\,A_i)P(A_i)}$$

4. (a) Solve the differential equation, 3+(2+3)+2

$$\frac{d^2y}{dx^2}+5\frac{dy}{dx}+4y=\cos(2x)$$

(b) State Gauss' divergence theorem.

Prove the vector integral theorem, $\int\vec{\nabla}\phi\,dV=\oint\phi\,d\vec{S}$, where $\phi(\vec{r})$ denotes a well behaved scalar field and $\vec{S}$ is the closed surface enclosing a volume $V$.

(c) Calculate the mean of a normal distribution, $f(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$ using the fact that the mean and variance of standard normal variate $z$ is 0 and 1 respectively, given that the probability density function of $z$ is, $g(z)=\frac{1}{\sqrt{2\pi}}e^{\frac{-z^2}{2}}$

5. (a) Find the expression of Laplacian in Spherical polar co-ordinate system. 5+(1+3)+1

(b) State the condition of convergence of a Taylar series expansion. Find interval of $x$ for which the Taylor Series of $\ln(1+x)$ about $x=0$, converges.

(c) What is meant by probability distribution function?

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