PHYSICAL CHEMISTRY

VISCOSITY

(Viscosity of a gas)

The formula for the viscosity coefficient of a gas can be derived in way similar to that used for heat conduction. We imagine two very large parallel flat plates, one lying in the xy plane, the other at a distance Z above the xy-plane. We keep the lower plate stationary and pull the upper plate in the +x direction with a velocity U. The viscosity of the gas exerts a drag on the moving plate.

To keep the plate in uniform motion, a force must be applied to balance the viscous drag. Looking at the situation in another way, if the upper plate moves with a velocity U, the viscous force will tend to set the lower plate in motion. A force must be applied to the lower plate to keep it in place.

Again we suppose that the gas between the plates is made up of a series of horizontal layers. The layer next to the lower plate is immobile; as we move upward, each successive layer has a slightly larger component of velocity in the x direction, the topmost layer at the height Z having the velocity U. This type of flow, in which there is a regular gradation of velocity in passing from one layer to the next, is called laminar flow. The layer at the height 'z' has a velocity in the x direction given by u_z; ...

$$u_z = \frac{\partial u}{\partial z} \cdot z \quad \text{------ (I)}$$

At $z = Z$, $u = U$, so that

$$\frac{\partial u}{\partial z} = \frac{U}{Z} \quad \text{------ (II)}$$

If we observe a layer at the height z, we see that molecules enter this layer from the neighbouring layers. The molecules from the upper layers will bring extra x momentum to this layer, while those which come from below are deficient in 'x' momentum. There is therefore a net downward flow of 'x' momentum through the layer. Now we compute the rate of this flow through one square meter of the layer at a height 'z' (Fig-1).

The number of molecules passing downwards through one square meter per second is by equation - X (P-747)

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is. $\frac{1}{6} \tilde{N} \langle c \rangle$, and

as many come upward as come downward. The molecules that pass downward through the layer at 'z' carry x momentum appropriate to the layer in which they made their last collision, the layer at height $(z+\lambda)$, this 'x' momentum is

$$mu_{z+\lambda} = m\left(\frac{\partial u}{\partial z}\right)(z+\lambda).$$

So the momentum coming down through one square meter in one second is

$$(mu)\downarrow = \frac{1}{6}\tilde{N}\langle c\rangle m\left(\frac{\partial u}{\partial z}\right)(z+\lambda)$$

Similarly, the momentum coming up is

$$(mu)\uparrow = \frac{1}{6}\tilde{N}\langle c\rangle m\left(\frac{\partial u}{\partial z}\right)(z-\lambda).$$

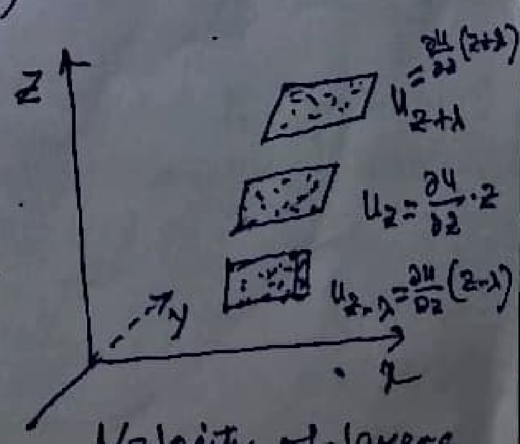
Since the molecules coming up adjusted their momentum in the layer at $(z-\lambda)$. The net downward flow of x momentum is

$$(mu)\downarrow - (mu)\uparrow = \frac{1}{3}\tilde{N}\langle c\rangle m\lambda\frac{\partial u}{\partial z}.$$

Since this quantity is independent of z, it must also be equal to the net 'x' momentum transferred in one second to one square meter of the lower plate. Since the momentum transfer in unit time is the force, the force acting in the 'x' direction on one square meter on the lower plate is $f_x = \frac{1}{3}\tilde{N}\langle c\rangle m\lambda\frac{\partial u}{\partial z}$ —(i)

To hold this plate stationary, we must apply an equal and opposite force f_{-x}, such that $f_x + f_{-x} = 0$

The viscosity coefficient, η is defined by $f_{-x} = -\eta\frac{\partial u}{\partial z}$ —(ii)



Velocity of layers in a flowing gas

The viscosity coefficient is the force that must be applied to hold the lower plate stationary, if the velocity gradient $\frac{\partial u}{\partial z}$ is unity and the plate has unit area. Comparing the equation: $f_x = \frac{1}{3}\tilde{N}\langle c\rangle m\lambda \frac{\partial u}{\partial z}$ with $f_x = -\eta \frac{\partial u}{\partial z}$, we see that

$$\eta = \frac{1}{3}\tilde{N}\langle c\rangle m\lambda \quad \text{— (iii)}$$

If the density of the gas is ρ, then $\rho = \tilde{N}m$, and so

$$\eta = \frac{1}{3}\rho\langle c\rangle\lambda \quad \text{— (iv)}$$

Again the numerical factor $\frac{1}{3}$ is not quite correct since the flow of the gas produces a non-equilibrium situation. For elastic sphere, the factor should be $\frac{1}{2}$.

The unit of viscosity coefficient is 1 Newton second per square meter ($N s m^{-2}$) = 1 Pascal second (Pa-s)
$= 1\ kg\ m^{-1} s^{-1}$ (In CGS 1 poise = $1 g\ cm^{-1} s^{-1}$)
$= 10^{-1}\ kg\ m^{-1} s^{-1}$

end Ans

The Coefficient of Viscosity depends on the products $\tilde{N}\lambda$ and so is independent of pressure, eqn (iii)

* $J = LX$ Linear law

The general equation for transport.

If any physical quantity is transported, the amount transported through unit area in unit time is the number of molecules passing through unit area in unit time multiplied by the amount of the physical quantity carried by each molecule.

for any transport $j = N'q$. —— (i)

Where 'j' is the flow / m^2. sec, N' is the no. of carriers passing through one square meter in one second, and 'q' is the amount of physical quantity possessed by each carrier. By calculating N' and q, we obtain the value of j. We begin with N'.

How many molecules pass the base, $1\,m^2$, of the parallelpiped in fig -1 in unit time?

If all the molecules were moving downward with an average velocity $\langle c \rangle$, then each travels a distance $\langle c \rangle dt$ in the time interval dt.

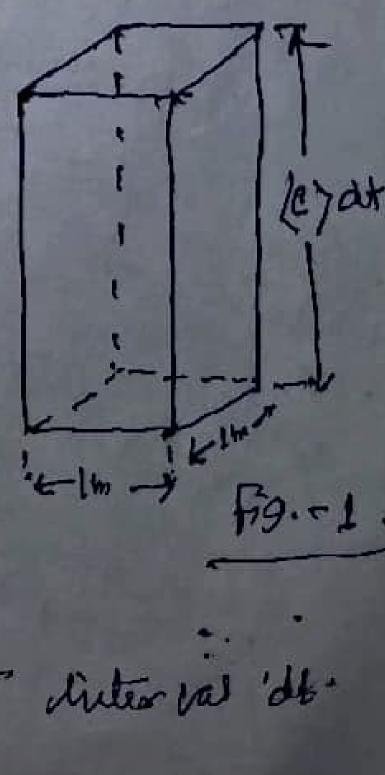


Fig.-1.

Therefore, all the molecules in the parallelpiped of height $\langle c \rangle dt$ will pass the bottom face in the interval 'dt'.

The volume of the parallelpiped is $\langle c \rangle\, dt$ m^3; if $\tilde{N}$ is the number of molecules per cubic metre, then the number crossing the base in dt is $\tilde{N}\langle c \rangle\, dt$. In unit time the number crossing $1\,m^2$ area is $n' = \tilde{N}\langle c \rangle$

The expression for the flow, Eqn (i) $[j = N'q]$ becomes $j = \tilde{N}\langle c \rangle q$. — (ii)

Which is applicable to any transport process; the flow is equal to the product of the number of carriers per unit volume, the average velocity in the direction of the flow, and the amount of the physical quantity carried by each ~~molecule~~.

If not all, but only a fraction, α, of the molecules are moving downward, then the expression of the right side of eqn (ii), $(j = \tilde{N}\langle c \rangle q.)$ must be multiplied by that fraction!

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$$j = \alpha \tilde{N}\langle c \rangle q.$$

End

Abe 10/3/22