P. R. Thakur Govt. College

Department of Physics

B.Sc. Semester-VI Internal Examination, 2024

Paper - **PHSACOR14T**

Statistical Mechanics

Time: 1 Hour **Full Marks: 20**

1. Answer any *five* questions from the following: 2 x 5=10

   (a) Define microcanonical ensemble. What is its significance in classical statistical mechanics?

   (b) Explain the concept of phase space in classical statistical mechanics.

   (c) State Liouville's theorem. Why is it important in classical statistical mechanics?

   (d) Write down Sackur-Tetrode equation in microcanonical ensemble.

   (e) Define density of states of a quantum mechanical system. Does it depend on the underlying statistics satisfied by the system of particles?

   (f) How many ways are there to distribute 2 identical bosons among 3 degenerate quantum states with same energy?

   (g) Show that for a system, in thermal equilibrium with a reservoir, the average energy is $\langle E \rangle = -\frac{\partial}{\partial \beta} ln Z$, where $Z = \sum_s e^{-\beta E(s)}$ is the partition function.

   (h) Draw the curve showing average occupancy vs energy for a system obeying Fermi-Dirac statistics.

Answer any *two* questions from the following: 5 x 2=10

2. What is ultraviolet catastrophe? Draw the Planck spectrum as a function of energy. State the Wien's displacement law. Why chemical potential of a photon is taken to be zero? 1+1+1+2

3. Consider a system of N particles in which the energy of each particle can assume two and only two distinct values 0 and ϵ. Denote n_0 and n_1 as the occupation numbers of the energy level 0 and ϵ, respectively. The total energy of the system is U.

   (a) Find the entropy of the system.

   (b) Find most probable values of n_0 and n_1.

   (c) Find temperature as a function of U and show it can be negative. 2+1.5+1.5

4. (a) The partition function Z of a system is given by $lnZ = aT^4V$ where a is a constant, T is the absolute temperature and V is the volume. Calculate the internal energy, pressure and entropy.

   (b) How is the probability of a system being in a particular microstate related to the partition function Z in the canonical ensemble?

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Statistical Mechanics

Time: 1 Hour **Full Marks: 20**

1. Answer any *five* questions from the following: 2 x 5=10

   (a) State Ergodic hypothesis in statistical mechanics.

   (b) A classical particle is free to move in a 3-dimensional box of volume V. If its energy is $\leq E$, find the volume of the phase space available to the particle.

   (c) Explain the statistical idea of entropy.

   (d) State the postulate of equal a priori probability.

   (e) State the conditions when it is mandatory to apply quantum statistics to a system of gas molecules.

   (f) How many ways are there to distribute 3 identical bosons among 3 degenerate quantum states with same energy?

   (g) Show that for a system, in thermal equilibrium with a reservoir, the average energy is $\langle E \rangle = -\frac{\partial}{\partial \beta} ln Z$, where $Z = \sum_s e^{-\beta E_s}$ is the partion function.

   (h) State spin-statistics theorem.

2. Answer any *two* questions from the following: 5 x 2=10

   (a) Calculate the number of microstates of a Einstein solid contaning N oscillators and q units of energy in the high temperature limit. Assume N and q are both large. Hence calculate its specific heat at the same limit. 3+2

   (b) Compute the partition function of a monatomic ideal gas containing N particles within a volume V at temperature T. Hence derive the expression for entropy of the gas. 3+2

   (c) i. What is Gibbs paradox? How is it resolved? 1+2

   ii. Consider a two-state paramagnet consisting of N dipoles that do not interact with each other. Each dipole can have energy $\pm\epsilon$. Show the maximum entropy of the paramagnet can be $Nkln2$. 2

   (d) Distinguish between microcanonical and canonical ensembles. Two states having energy difference 4.83×10^{-8}J occur with relative probability e^2. Calculate the temperature. Given $k = 1.38 \times 10^{-23}$ J K^{-1}. 2+3

P. R. Thakur Govt. College

Department of Physics

B.Sc. Semester-VI Internal Examination, 2022

Paper - **PHSACOR14T**

Statistical Mechanics

Time: 1 Hour **Full Marks: 20**

1. Answer any *five* questions from the following: 2 x 5=10

   (a) What do you mean by 'ultraviolet catastrophe'?

   (b) State Stefan-Boltzmann law.

   (c) A blackbody at a temperature of 1646K has the wave-length corresponding to the maximum emission (λ_m) equal to 1.78 micron. Find the temperature of the moon (assumed to be a blackbody) if λ_m for the moon is 14 micron.

   (d) State the postulate of equal a priori probability.

   (e) Under what condition(s), is it mandatory to treat a system of ideal gas quantum mechanically?

   (f) How many ways are there to distribute 5 identical bosons among 3 degenerate quantum states having same energy value?

   (g) If a system is in thermal and diffusive equilibrium with a reservoir at temperature T and chemical potential μ, prove the mean square number of particles in the system is

$$\overline{N^2} = \frac{(KT)^2}{\mathcal{Z}} \frac{\partial^2 \mathcal{Z}}{\partial \mu^2},$$

   where $\mathcal{Z}$ is the grand partion function.

   (h) State spin-statistics theorem.

2. Answer any *two* questions from the following: 5 x 2=10

   (a) An ideal gas containing N particles at $T = 300$K obeys the M-B statistics. Calculate (i) the average thermal energy of a molecule in the system in eV, (ii) the internal energy of the system, and (iii) its heat capacity at constant volume. What is Gibbs Paradox? 3+2

   (b) Define phase-space. Draw the phase-space trajectory of a simple harmonic oscillator with energy E. Distinguish between microstates and macrostates. 1+2+2

   (c) Derive the expression for average occupancy at an energy level for a quantum system obeying Fermi-Dirac statistics. From this formula, define Fermi energy for the system. 4+1

   (d) Derive the dependence of Fermi energy of a degenerate electron gas on the number density of electrons. Draw the curve showing average occupancy vs energy for a system obeying Bose-Einstein (BE) statistics. Is there any upper bound on chemical potential for a sytem obeying BE statistics? 3+1+1

P. R. Thakur Govt. College

Department of Physics

B.Sc. Semester-VI Internal Examination, 2021

Paper – **PHSACOR14T**

Statistical Mechanics

Time: 1 Hour **Full Marks: 20**

1. Answer any five questions from the following: 2×5=10

(a) A system of three particles has energy levels with energies 0, 1, 2, 3 units. The total energy of the system is 3 units. List the accessible microstates, if the particles are indistinguishable.

(b) Write down the postulate of equal a priori probabilities.

(c) An ideal gas, containing N particles at $T = 300\ K$, obeys the *MB*-statistics. Calculate the average thermal energy of a molecule of the system.

(d) Find the Fermi energy at $T = 0\ K$ for metallic silver containing one free electron per atom. The density of silver is 10.5 g/c.c and atomic weight = 108.

(e) Find the phase trajectory for a particle of mass m and charge $-e$, moving under the influence of Coulomb force of attraction towards a fixed charge $+e$ at a distance r.

(f) There are 10 identical particles, each of mass m, to be accommodated in a cubical box of side L. What is the lowest energy of the system if the particles obey *BE*-statistics.

(g) State Stirling's formula for the factorial of a very large number.

(h) Define the term 'density of states'.

2. Answer any two questions from the following: 5×2=10

(a) What do you mean by phase space? Discuss the nature of the trajectory in the phase space of a simple pendulum. Show that the area enclosed by the trajectory is equal to the product of the total energy E and the time period T. 1+2+2

(b) Using the BE-statistics, deduce Planck's law for black body radiation and hence derive Wien's displacement law.

(c) What is Fermi energy? Does it depend on temperature? Sketch the FD-distribution function at absolute zero at any finite non-zero temperature. What are 'degenerate' and 'non-degenerate' distribution functions? 1+1+1+2

(d) N particles are distributed among three states having energies 0, kT, and $2KT$. If the total equilibrium energy of the system is $2000kT$, calculate the value of N. What are Bosons and Fermions? 3+2