P. R. Thakur Govt. College

Department of Physics

B.Sc. Semester-V Internal Examination, 2024-25

Paper - **PHSACOR11T**

Quantum Mechanics

Time: 1 Hour **Full Marks: 20**

1. Answer any *five* questions from the following: 2 x 5=10

   (a) Show that every eigenvalue of a Hermitian operator is real.

   (b) Write down the boundary conditions for a wavefunction and its first derivative at a finite potential discontinuity.

   (c) If $\hat{A}$ and $\hat{B}$ are Hermitian operators, prove that their commutator $[\hat{A}, \hat{B}]$ is anti-Hermitian.

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   (d) Prove that the number operator $N = a^\dagger a$ is Hermitian.

   (e) For the Hamiltonian $\hat{H} = \frac{\hat{P}^2}{2m} + V(\hat{X})$, evaluate $[\hat{X}, [\hat{X}, \hat{H}]]$.

   (f) What is Lermor precession of electrons in an atom?

   (g) What is the Stark effect?

   (h) In a Stern-Gerlach experiment on turning on the magnetic field, the beam splits into five components. What is the angular momentum of the atoms in the beam?

Answer any *two* questions from the following: 5 x 2=10

2. Starting from $[a, a^\dagger] = 1$, show that

   (a) $[N, a^\dagger] = a^\dagger$.

   (b) $[N, a] = -a$. 3+2

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3. For a particle in a 1-d box of width a, the wavefunction is given by :

$$\Phi_n(x) = \begin{cases} \sqrt{\frac{2}{a}}\sin(\frac{n\pi x}{a}), & \text{if } 0 \leq x \leq a \\ 0 & \text{otherwise.} \end{cases}$$

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   Calculate $\langle \hat{X} \rangle$ and $\langle \hat{P} \rangle$. 3+2

4. Draw the ground state and first excited state wavefunctions for a particle trapped in a 1-d box of width a. 3+2

5. Explain the term "spin-orbit coupling". Calculate possible angles between $\vec{L}$ and $\vec{S}$ for a 'd' electron in a one-electron atom. 2+3

P. R. Thakur Govt. College
Department of Physics
B.Sc. Semester-V Internal Examination, 2023-24

Paper - PHSACOR11T

Quantum Mechanics & Applications

Time: 1 Hour **Full Marks: 20**

1. Answer any *five* questions from the following: 2 x 5=10

   (a) Prove that eigenvectors corresponding to two different eigenvalues of a Hermitian operator are orthogonal.

   (b) The wave-function of a particle is given by $\psi(x) = \sqrt{a}e^{-a|x|}$ with $a > 0$. Find the probabilty to find the particle between $x = 1/a$ and $x = 2/a$.

   (c) Starting from the Schroedinger equation, prove that $\frac{d}{dt}\langle\psi(t)|\psi(t)\rangle = 0$ for an arbitrary quantum state $|\psi(t)\rangle$.

   (d) Prove that the probability current density $\vec{J}(\vec{r},t)$ associated with the probability density $\rho(\vec{r},t) = |\psi(\vec{r},t)|^2$ is a real vector.

   (e) Show that $\frac{d}{dx}$ is an anti-Hermitian operator on the space of normalizable functions.

   (f) Show that the commutator of two Hermitian operators is anti Hermitian.

   (g) What is Larmor precession of an electron in an atom?

   (h) Suppose $[\hat{A},\hat{B}] = 0$ and $\hat{A}|\psi\rangle = \lambda|\psi\rangle$. If λ is a non-degenerate eigenvalue then prove that $|\psi\rangle$ is also an eigenvector of $\hat{B}$.

Answer any *one* question from the following:

2. (a) Prove that the wavefunction of a particle moving in 1-dimension under a even potential, i.e. $V(-x) = V(x)$, is either even or odd. 3

   (b) Prove that for bound states in 1-dimension, there is no degenerate eigenvalue of the Hamiltonian for a particle moving under an arbitrary potential. 3

   (c) Calculate the expectation value of the position of a particle (in the n^{th} energy eigenstate) of mass m trapped inside a 1-dimensional infinite square well of width L. 4

3. (a) Consider the step potential given by

$$V(x) = \begin{cases} V_0, & (x \geq 0) \\ 0, & (x < 0). \end{cases}$$

   If a beam of particles each having total energy E ($> V_0$) is incident on this step from the left, obtain the boundary conditions on the wave-function and hence find the fraction of particle reflected by the step. 2+3

   (b) Explain the term spin-orbit coupling. Calculate the possible orientations of the total angular momentum $\vec{J}$ corresponding to $j = \frac{3}{2}$ with respect to the magnetic field along z-axis. 2+3

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B.Sc. Semester-V Internal Examination, 2022-23

Paper - **PHSACOR11T**

Quantum Mechanics & Applications

Time: 1 Hour **Full Marks: 20**

1. Answer any *five* questions from the following: 2 x 5=10

   (a) State and explain the Heisenberg uncertainty principle.

   (b) A particle constrained to move along x-axis in the domain $0 \leq x \leq L$ has a wavefunction $\sqrt{\frac{2}{L}} \sin(\frac{n\pi x}{L})$, where n is an integer. What is the expectation value of its momentum?

   (c) Prove that eigenvectors corresponding to two different eigenvalues of a Hermitian operator are orthogonal.

   (d) For one dimensional bound state motion of a particle of mass m, prove that the expectation value of its kinetic energy is given by
   $$\langle \hat{K} \rangle = \frac{\hbar^2}{2m} \int_{-\infty}^{\infty} \frac{\partial \psi}{\partial x} \frac{\partial \psi^*}{\partial x} dx.$$

   (e) For a 1-dimensional harmonic oscillator, if $|\lambda\rangle$ denotes an eigenvector of the number operator $\hat{N}$, prove that $a^\dagger|\lambda\rangle$ is also an eigenvector of $\hat{N}$ with eigenvalue $\lambda + 1$.

   (f) Show that the commutator of two Hermitian operators is anti-Hermitian.

   (g) Calculate expectation value of potential energy $\langle \hat{V} \rangle$ in the n^{th} energy eigenstate of harmonic oscillator, where $V(x) = \frac{1}{2}m\omega^2 x^2$.

   (h) Suppose $[\hat{A}, \hat{B}] = 0$ and $\hat{A}|\psi\rangle = \lambda|\psi\rangle$. If λ is a non-degenerate eigenvalue then prove that $|\psi\rangle$ is also an eigenvector of $\hat{B}$.

2. Answer any *two* questions from the following: 5 x 2=10

   (a) i. Prove that the wavefunction of a particle moving in 1-dimension under a symmetric potential, i.e. $V(-x) = V(x)$, is either even or odd.

   ii. Show that the eigenvalue of the number operator $\hat{N} = a^\dagger a$ for a a linear harmonic oscillator in 1-dimension can take on only non-negative real values. 3+2

   (b) i. For any observable $\hat{A}$ that does not depend on time explicitly, prove that $\frac{d}{dt}\langle \hat{A} \rangle = \frac{1}{i\hbar}\langle [\hat{A}, \hat{H}] \rangle$, where $\hat{H}$ is the Hamiltonian of the system.

   ii. Prove that the probability current density $\vec{J}(\vec{r}, t)$ associated with the probability density $\rho(\vec{r}, t) = |\psi(\vec{r}, t)|^2$ is a real vector. 3+2

   (c) Define ground state of a linear harmonic oscillator in 1-dimension. Find its normalized ground state wavefunction using the lowering operator $\hat{a} = \frac{1}{\sqrt{2m\omega\hbar}}(m\omega\hat{X} + i\hat{P})$. 1+4

   (d) For a quantum system, show that the expectation value of the position operator obeys
   $$\frac{d^2}{dt^2}\langle \hat{X} \rangle = -\frac{\langle V'(\hat{X}) \rangle}{m}$$
   where m is the mass of a particle moving in 1-dimension and is subject to the potential $V(x)$. Here $'$ denotes differentiation w.r.t. X.

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B.Sc. Semester-V Internal Examination, 2022

Paper - **PHSACOR11T**

Quantum Mechanics & Applications

Time: 1 Hour **Full Marks: 20**

1. Answer any *five* questions from the following: 2 x 5=10

   (a) Show that $\frac{d}{dx}$ is an anti-Hermitian operator on the space of normalizable functions.

   (b) Prove that the eigenvectors corresponding to different eigenvalues of a Hermitian operator are orthogonal.

   (c) Suppose $[\hat{A}, \hat{B}] = 0$ and $\hat{A}$ has a non-degenerate eigenvalue λ associated with the eigenvector $|\phi\rangle$. Show that $|\phi\rangle$ is also an eigenvector of $\hat{B}$.

   (d) Show that the commutator of two Hermitian operators is an anti-Hermitian operator.

   (e) Suppose the operator $\hat{A}$ has no explicit time dependence. Prove that $\frac{d}{dt}\langle\hat{A}\rangle(t) = \frac{1}{i\hbar}\langle[\hat{A}, \hat{H}]\rangle$, where $\hat{H}$ is the Hamiltonian operator and $\langle\ \rangle$ denotes the expectation value.

   (f) Show that the number of atomic states (considering electron spin) with a given quantum number n is $2n^2$.

   (g) Derive the expression for the energy shift in Normal Zemann effect and mension the selection rule.

   (h) Define Larmor frequency.

2. Answer any *two* questions from the following: 5 x 2=10

   (a) Prove that there is no degeneracy in the energy eigenvalue for a particle forming bound states in 1-dimension.

   (b) Normalised eigenfunction of a quantum particle constrained to move within a 1-dimensional box of length a is given by

   $$\Phi_n(x) = \begin{cases} \sqrt{\frac{2}{a}}\sin(\frac{n\pi x}{a}) & (0 \leq x \leq a) \\ 0 & (x < 0\ \&\ x > a). \end{cases}$$

   Calculate the uncertainty product $(\Delta\hat{X}\Delta\hat{P})_n$ in the n^{th} eigenstate of the Hamiltonian.

   (c) Normalise the eigenfunction $\psi(x) = Ae^{-k|x|}$, where $k(> 0)$ is a constant. Plot $\psi(x)$ vs x. What is the dimension of A? 3+1+1

   (d) How many spectral lines appear in the Zeeman splitting of $^2D_{3/2} \rightarrow {}^2P_{1/2}$ transition of Sodium Explain. 5

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B.Sc. Semester-V Internal Examination, 2021

Paper - PHSACOR11T

Quantum Mechanics & Applications

Time: 1 Hour **Full Marks: 20**

1. Answer any *five* questions from the following: 2 x 5=10

   (a) Show that eigenvalues of a Hermitian operator are all real.
   (b) Write down the boundary conditions for a wavefunction and its first derivative at a finite potential discontinuity.
   (c) Starting from the Schrodinger equation, prove that the norm of a state vector is preserved for all future time.
   (d) Find the eigenvalues of the parity operator $\mathcal{P}$ defined by $\mathcal{P}\phi(x) = -\phi(x)$.
   (e) Can there be energy eigenvalue of a quantum system which is less than the minimum value of the potential. Justify.
   (f) If $H = \frac{1}{2m}P^2 + V(X)$, compute the value of $[X, [X, H]]$.
   (g) Consider a single electron atom with orbital angular momentum $L = \sqrt{2}\hbar$. What are the possible values of a measurement of z-component of L?
   (h) Can Lithium ($z = 3$) give rise to normal Zeeman effect? Justify.
   (i) Find out the different states in L-S coupling scheme for a two-electron atom with $l_1 = 2$ & $l_2 = 1$.

2. Answer any *two* questions from the following: 5 x 2=10

   (a) If a particle in 1-dimension is subject to a potential that is even, i.e., $V(-x) = V(x)$, show that all eigenfunctions of the Hamiltonian will have a definite parity, i.e., either even or odd in x.
   (b) Find the energy eigenvalue and eigenfunction of a particle forming bound state by an attractive δ-function potential given by $V(x) = -\lambda\delta(x)$, where $\lambda > 0$ denotes the strength of the potential. 3+2
   (c) For a linear harmonic oscillator in 1-dimension, the Hamiltonian H may be written as $H = (a^\dagger a + \frac{1}{2})\hbar\omega$ where a and $a^\dagger$ are the ladder opearators. If the energy eigenkets are denoted by $\{|n\rangle\}$, where $n = 0, 1, 2, \ldots$, find the uncertainty product $(\Delta X \Delta P)_n$ in the n^{th} energy eigenstate. The symbols have their usual meaning.
   (d) Discuss the theory of anomalus Zeeman effect with special reference to Zeeman pattern for D_1 & D_2 lines of sodium. 2+1+2