

P. R. Thakur Govt. College
Department of Physics
 B.Sc. Semester-IV Internal Examination, 2023
 Paper - PHSACOR08T
 Mathematical Physics-III

Time: 1 Hour

Full Marks: 20

1. Answer any *five* questions from the following:

2 x 5=10

(a) Expand $f(z) = \frac{1}{z(z-1)}$ in a Laurent series in the open annulus $0 < |z-1| < 1$.

(b) Find eigenvalues of the matrix

$$R = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

(c) If $f(z) = u(x, y) + iv(x, y)$ is an analytic function with $u(x, y) = x^2 - y^2$, find $v(x, y)$.

(d) Calculate the residue of $f(z) = (z-1)^2 e^{-\frac{1}{(z-1)^2}}$ at $z = 1$.

(e) Evaluate $\oint_C f(z) dz$ for $f(z) = \frac{1}{z}$ where C is the circle $|z-2| = 1$.

(f) If H is a hermitian matrix, show that e^{iH} is unitary.

(g) For two $n \times n$ matrices A and B , prove that $\text{Tr}(AB) = \text{Tr}(BA)$.

(h) Prove that the characteristic polynomial of a matrix is preserved under similarity transformation.

2. Answer any *two* questions from the following:

5 x 2=10

(a)

Find the eigenvalues of $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$.

Find its normalised eigenvectors. Show that the transformation matrix P (such that $P^{-1}AP$ is diagonal) can be chosen to be orthogonal.

2+2+1

(b) Assuming the cartesian form of Cauchy-Riemann condition, show that its corresponding polar form may be written as

$$\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta}, \quad \frac{\partial v}{\partial r} = -\frac{1}{r} \frac{\partial u}{\partial \theta}.$$

(c) State Cayley-Hamilton theorem for a matrix. Verify the theorem for

$$A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$$

and hence find A^{-1} .

1+2+2

(d) An uncharged metal sphere of radius R is placed in an otherwise uniform electric field $\vec{E} = E_0 \hat{k}$. Find the potential outside the sphere.