P. R. Thakur Govt. College

Department of Physics

B.Sc. Semester-III Internal Examination, 2023-24

Paper - **PHSACOR05T**

Mathematical Physics - II

Time: 1 Hour **Full Marks: 20**

1. Answer any *five* questions from the following: 2 x 5=10

(a) State the Dirichlet criteria for a function to expand it in Fourier series.

(b) At which value does the Fourier series of a function converge at a point of finite discontinuity?

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(c) What is cyclic coordinate? Show that for a cyclic coordinate the corresponding canonical momentum is conserved.

(d) Define Legendre transform of a function $f(x)$.

(e) Write down the Lagrangian of a particle moving under a central force.

(f) Prove that the angular momentum is conserved in every central force.

(g) Suppose $f = f(y(x), y'(x), x)$ satisfies Euler-Lagrange equation. In the special case, if f does not explicitly depend on x, prove that $f - y'\frac{\partial f}{\partial y'}$ is constant.

(h) Is there any real advantage of the Hamiltonian formalism over the Lagrangian formalism? State clearly.

2. Answer any *two* questions from the following: 5 x 2=10

(a) Expand the following function in Fourier series:

$$f(t) = \begin{cases} -1 & (-\frac{T}{2} < t < 0) \\ 1 & (0 \leq t < \frac{T}{2}) \end{cases}$$

(b) From variational principle, show that the shortest path on a plane joining two given points is a straight line.

(c) Using the method of separation of variables, solve:

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$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0,$$

subject to the boundary conditions: $\Phi(0, y) = \Phi(a, y) = \Phi(x, \infty) = 0$ and $\Phi(x, 0) = \sin\left(\frac{\pi x}{a}\right)$, inside an infinitely long rectangular strip along y-direction having width a.

(d) Consider the differential equation $y'' - 2xy' + 2ny = 0$. Show that $x = 0$ is an ordinary point where as there is an essential singularity at $x \to \infty$. 2+3

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B.Sc. Semester-III Internal Examination, 2022-23

Paper - **PHSACOR05T**

Mathematical Physics - II

Time: 1 Hour **Full Marks: 20**

1. Answer any *five* questions from the following: 2 x 5=10

(a) Sketch the periodic extension of

$$f(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } t \geq 0 \end{cases}$$

by choosing the fundamental interval to be $(-1, 1)$.

(b) What is holonomic constraint? Give an example of it.

(c) Derive the functional equation: $\Gamma(x+1) = x\Gamma(x)$.

(d) Write the Lagrangian of a particle moving under a central force.

(e) Assuming the rotational invariance (isotropy) of space, show that the total angular momentum of a mechanical system is conserved.

(f) Prove the following Poisson bracket relation: $\{p_i, L_j\} = \epsilon_{ijk}p_k$, where p_i is the i^{th} component of linear momentum and L_j, the j^{th} component of angular momentum.

(g) Define Beta function $B(x, y)$. Show this function is symmetric w.r.t. interchange of x and y, i.e. $B(y, x) = B(x, y)$.

(h) Evaluate for $r > -1, a > 0$, $\int_0^\infty dt e^{-at^2} t^r$ using Gamma function.

2. Answer any *two* questions from the following: 5 x 2=10

(a) Find the Fourier series of $f(x) = x^2$ for $0 < x \leq 2$. From this result and using Parseval's theorem, prove $\Sigma_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$. 3+2

(b) From variational principle, show that the shortest path on a plane joining two given points is a straight line.

(c) Using the method of separation of variables, solve:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0,$$

subject to the boundary conditions: $\Phi(0, y) = \Phi(a, y) = \Phi(x, \infty) = 0$ and $\Phi(x, 0) = \sin\left(\frac{\pi x}{a}\right)$, inside an infinitely long rectangular strip along y-direction having width a.

(d) Suppose f and g are two conserved quantities for a system. Prove that their Poisson bracket $\{f, g\}$ is conserved.

The Lagrangian of a particle of mass m is given by $L = \frac{1}{2}m\dot{q}^2 - \frac{1}{2}\alpha q^2 - \alpha q\dot{q}t$, where α is a constant. Show that the particle is moving freely.

3+2

P. R. Thakur Govt. College

B.Sc Honours 3rd Semester 1st Internal Examination, 2019

PHSACOR05T-PHYSICS (CC5)

Mathematical Physics-II

Time: 1 Hour Full Marks: 20

1. Answer *any five* questions: $2 \times 5 = 10$

   (a) Write down the Dirichlet criteria for the expansion of a function as a Fourier series.

   (b) Write down the Parseval's theorem for a complex Fourier series in mathematical form.

   (c) What is the predicted value from the Fourier series of a function at a point of finite discontinuity?

   (d) Show that the sufficient condition for the linear independence of two functions defined over an interval is that their Wronskian is non-zero over that interval.

   (e) Check if there is any singularity at the point $x = \pm 1$ of the differential equation $(1-x^2)y'' - 2xy' + [l(l+1) - \frac{m^2}{1-x^2}]y = 0$

   (f) Show that the differential equation $(1-x^2)y'' - xy' + \nu^2 y = 0$ has a regular singularity at $x \to \infty$.

   (g) Find the indicial equation of the equation $x^2 y'' + xe^x y' + (x^3 - 1)y = 0$ if the solution is required near $x = 0$.

2. Answer *any two* questions: $5 \times 2 = 10$

   (a) Find the Fourier series of the function $f(x) = x$ in the range $-\pi < x < \pi$.

   (b) Find the power series solution of the differential equation $4xy'' + 2y' + y = 0$ about the point $x = 0$.

   (c) Given one root as $y_1(x) = \frac{x}{(1-x)^2}$ of the differential equation $x(x-1)y'' + 3xy' + y = 0$, find the other linearly indepndent root by the Wronskian method.

$+x^2 \frac{l(l+1)}{(1-x^2)} - \frac{(1+x)^2 m^2}{(1-x^2)^2}$

$= \frac{l(l+1)\cancel{(1+x)}}{(1-x)} - \frac{m^2}{(1-x)^2}$

$\to -1 \qquad -\frac{m^2}{4}.$

$(x-1)^2 \left[\frac{l(l+1)}{(1-x^2)} - \frac{m^2}{(1-x^2)^2}\right]$

$= \frac{(1-x)^2 l(l+1)}{(1-x^2)} - \frac{m^2(1-x)^2}{(1-x^2)^2}$

$= \frac{(1-x)}{(1+x)} l(l+1) - \frac{m^2}{(1+x)^2}$

$\to -\frac{m^2}{4}.$