P. R. Thakur Govt. College

Department of Physics

B.Sc. Semester-I Internal Examination, 2022-23

Paper - **PHSACOR01T** Department of Physics

Mathematical Physics P. R. Thakur Govt. College

Time: 1 Hour **Full Marks: 20**

1. Answer any *five* questions from the following: 2 x 5=10

   (a) Plot $f(x) = 1 + \sin^2 x$ in the range $-2\pi \leq x \leq 2\pi$.

   (b) If $\vec{A} = 2\hat{i} + \hat{j} - 3\hat{k}$ and $\vec{B} = \hat{i} - 2\hat{j} + \hat{k}$, find a vector of magnitude 5 perpendicular to both $\vec{A}$ and $\vec{B}$.

   (c) Find the Taylor series of the function $f(x) = \sin(2x)$ around the point $x = 0$.

   (d) Find the value of the constant a so that the vector field $\vec{F} = (axy - z^3)\hat{i} + (a-2)x^2\hat{j} + (1-a)xz^2\hat{k}$ is irrotational.

   (e) Find the directional derivative of the scalar field $\phi(x, y, z) = 2xyz + e^{3z}$ in the direction of the vector $\hat{i} - 2\hat{j}$ at $(1, -1, 2)$.

   (f) For a vector field $\vec{A}$, prove that $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$.

   (g) If $\vec{A}(t)$ has constant magnitude, prove that $\vec{A}(t)$ and $\frac{d\vec{A}}{dt}$ are perpendicular.

   (h) Find the unit normal to the surface $z = x^2 + y^2$ at the point $(1, -1, 2)$.

2. Answer any *two* questions from the following: 5 x 2=10

   (a) Show that the equation $2x\frac{dy}{dx} + 3x + y = 0$ is either exact or can be made exact and solve it. 1+4

   (b) i. Prove that $\vec{\nabla} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{A}) - \vec{A} \cdot (\vec{\nabla} \times \vec{B})$.

   ii. Evaluate the surface integral of $f(x, y)$ over the rectangle $0 \leq x \leq a$, $0 \leq y \leq b$ for the function

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$$f(x, y) = \frac{x}{x^2 + y^2}$$

   (c) i. Prove that a rectangle having maximum area that can be inscribed within a given circle is a square.

   ii. Evaluate $\oint_C \vec{A} \cdot d\vec{r}$ where $\vec{A} = (y - 2x)\hat{i} + (3x + 2y)\hat{j}$ and C is a circle in the X-Y plane with center at the origin and radius 2 and is traversed in the positive direction. 2+3

   (d) i. Why is complementary function (CF) added to the particular integral (PI) to construct the general solution of a inhomogeneous ODE?

   ii. Solve $\frac{d^2f}{dt^2} + 6\frac{df}{dt} + 9f = e^{-t}$ subject to the conditions $f(t) = 0$ and $\frac{df}{dt} = \lambda$ at $t = 0$. 1+4

P. R. Thakur Govt. College

Department of Physics

B.Sc. Semester-I Second Internal Examination, 2022-23

Paper - **PHSACOR01T**

Mathematical Physics - I

Time: 1 Hour **Full Marks: 20**

1. Answer any *five* questions from the following: 2 x 5=10

   (a) Plot $f(x) = 1 + \sin^2 x$ in the range $-2\pi \leq x \leq 2\pi$.

   (b) Determine a unit vector perpendicular to the plane containing the vectors $\vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$.

   (c) Find the Taylor series of the function $f(x) = \sin^2 x$ around the point $x = 0$.

   (d) Find the value of the constant a so that the vector field $\vec{F} = (axy - z^3)\hat{i} + (a-2)x^2\hat{j} + (1-a)xz^2\hat{k}$ is irrotational.

   (e) Find the directional derivative of the scalar field $\phi(x, y, z) = 2xyz + \sin(x)e^{3z}$ in the direction of the vector $\hat{i} - 2\hat{j}$ at $(0, -1, 2)$.

   (f) For a scalar field ϕ, prove that $\vec{\nabla} \times (\vec{\nabla}\phi) = 0$.

   (g) If $\vec{A}(t)$ has constant magnitude, prove that $\vec{A}(t)$ and $\frac{d\vec{A}}{dt}$ are perpendicular.

   (h) Find the unit normal to the surface $x^2 + y^2 + z^2 = a^2$ at the point $(\frac{a}{\sqrt{2}}, -\frac{a}{\sqrt{2}}, 0)$.

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2. Answer any *two* questions from the following: 5 x 2=10

   (a) Solve: $(4x + 3y^2)dx + 2xydy = 0$.

   (b) i. Prove that $\vec{\nabla} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{A}) - \vec{A} \cdot (\vec{\nabla} \times \vec{B})$.

   ii. Evaluate the surface integral of $f(x, y)$ over the rectangle $0 \leq x \leq a$, $0 \leq y \leq b$ for the function

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$$f(x, y) = \frac{x}{x^2 + y^2}$$

   (c) i. Prove that a rectangle having maximum area that can be inscribed within a given circle is a square.

   ii. Evaluate $\oint_C \vec{A} \cdot d\vec{r}$ where $\vec{A} = (y - 2x)\hat{i} + (3x + 2y)\hat{j}$ and C is a circle in the X-Y plane with center at the origin and radius 2 and is traversed in the positive direction. 2+3

   (d) i. Why is complementary function (CF) added to the particular integral (PI) to construct the general solution of a inhomogeneous ODE?

   ii. Solve $\frac{d^2 f}{dt^2} - 2\frac{df}{dt} + f = e^t$ subject to the conditions $f(t) = 0$ and $\frac{df}{dt} = \lambda$ at $t = 0$. 1+4

P. R. Thakur Govt. College

B.Sc Honours 1st Semester 1st Internal Examination, 2019

PHSACOR01T-PHYSICS (CC1)

MATHEMATICAL PHYSICS

Time: 1 Hour | Full Marks: 20

1. Answer *any five* questions: $2 \times 5 = 10$

   (a) Show that the function $f(x) = |x|$ is not differentiable at the point $x = 0$.

   (b) Plot the function $f(x) = \frac{\sin(x)}{x}$ in the interval $-2\pi \leq x \leq 2\pi$.

   (c) Prove that for any three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ in $\mathbf{R}^3$,
   $\vec{A}\cdot(\vec{B}\times\vec{C}) + \vec{B}\cdot(\vec{C}\times\vec{A}) + \vec{C}\cdot(\vec{A}\times\vec{B}) = 0$.

   $4xe^{-y}y^3 + xe^{-y}z$

   $12xe^{-y}y^2 - 4xe^{-y}y^3 - xe^{-y}z$.

   (d) Prove that the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ does not converge.

   (e) Find the Taylor series expansion for the function $f(x) = e^x$. Hence find its radius of convergence.

   (f) Calculate the directional derivative of a scalar field $\phi(x, y, z) = xe^{-y}(4y^3 + z)$ in the direction of the vector $\vec{t} = 3\hat{j} - \hat{k}$.

   (g) For a scalar field ϕ and a vector field $\vec{A}$, prove that $\vec{\nabla}\cdot(\phi\vec{A}) = \vec{\nabla}\phi\cdot\vec{A} + \phi(\vec{\nabla}\cdot\vec{A})$.

   (h) State Green's theorem in a plane.

   (i) Obtain an expression for gradient operator $(\vec{\nabla})$ in plane polar coordinates (ρ, ϕ).

2. Answer *any two* questions: $5 \times 2 = 10$

   (a) Verify Stokes' theorem for $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

   (b) The pressure at a point (x, y, z) on a unit sphere is given by $P(x, y, z) = 1 + xy + yz$. By using the method of Lagrange multipliers, find the maximum pressure on the sphere.

   (c) Using Stokes' theorem prove that, for a scalar field ϕ

   $$\oint_C \phi d\vec{r} = -\int_S \vec{\nabla}\phi \times d\vec{s},$$

   where S is any open surface with boundary C. Hence prove that $\oint_C d\vec{r} = 0$ for any closed contour C.

$\vec{\nabla}\phi\cdot\hat{t}$

$= \frac{\partial\phi}{\partial y}\hat{t}_y + \frac{\partial\phi}{\partial z}\hat{t}_z$

$= xe^{-y}(12y^2 - 4y^3 - z)\frac{3}{\sqrt{10}} - \frac{1}{\sqrt{10}}xe^{-y}$

$= \frac{xe^{-y}}{\sqrt{10}}\left[3(12y^2 - 4y^3 - z) - 1\right]$

$\frac{\partial\phi}{\partial x} = e^{-y}(4y^3 + z)$

$\frac{\partial\phi}{\partial z} = xe^{-y}$.

$\frac{\partial\phi}{\partial y} = x\frac{\partial}{\partial y}e^{-y}(4y^3 + z)$

$= -xe^{-y}(4y^3 + z) + xe^{-y}\{12y^2$

$\cancel{= xe^{-y}(12y^2 + z - 4y^3 - z)}$ $= xe^{-y}(12y^2 - 4y^3 - z)$.

P. R. Thakur Govt. College

B.Sc Honours 1st Semester 2nd Internal Examination, 2019

PHSACOR01T-PHYSICS (CC1)

MATHEMATICAL PHYSICS

Time: 1 Hour

Full Marks: 20

1. Answer *any five* questions: $2 \times 5 = 10$

(a) Show that the function $f(x) = x|x|$ is differentiable at the point $x = 0$.

(b) Plot the function $f(x) = 1 + \sin^2 x$ in the interval $-\pi \leq x \leq \pi$.

(c) Prove that for any three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ in $\mathbf{R}^3$,
$\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A}) = \vec{C} \cdot (\vec{A} \times \vec{B})$.

(d) Prove that the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

(e) Find the Taylor series expansion for the function $f(x) = \cos x$ about the point $x = 0$.

(f) Calculate the directional derivative of a scalar field $\phi(x, y, z) = xe^{-y}(y^2 + z)$ in the direction of the vector $\vec{t} = \hat{i} + 3\hat{j} - \hat{k}$.

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(g) For a scalar field ϕ and a vector field $\vec{F}$, prove that $\vec{\nabla} \times (\phi\vec{F}) = \vec{\nabla}\phi \times \vec{F} + \phi(\vec{\nabla} \times \vec{F})$.

(h) State Green's theorem in a plane.

J

(i) Calculate the Jacobian of the coordinate transformation:
$x = \rho\cos(\phi)$, $y = \rho\sin(\phi)$, from Cartesian to plane polar coordinates.

2. Answer *any two* questions: $5 \times 2 = 10$

(a) Verify Stokes' theorem for $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

(b) The temperature at a point (x, y) on a unit circle is given by $T(x, y) = 1 + xy$. By using the method of Lagrange, multipliers, find the temperature of the two hottest point on the circle.

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(c) Using divergence theorem prove that, for a scalar field ϕ

$$\oint_S \phi d\vec{S} = -\int_V \vec{\nabla}\phi dV,$$

where V is any volume enclosed by the surface S. Hence prove that $\oint_S d\vec{S} = 0$ for any closed surface C.

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P R. Thakur Govt. College

First Internal Examination – 2018

Subject - PHYSICS

Paper - Core-T1

Full Marks: 20

Time 1 Hour

1. Answer any **five** from the following: $5 \times 2 = 10$

   (a) What is absolutely convergent series?

   (b) Check whether the two functions $f(x) = x$ and $g(x) = x^2$ are linearly independent.

   (c) Check the continuity of the function $f(x) = \frac{1}{x+a}$ at the point $x = a \quad (\neq 0)$.

   (d) Determine whether the following series converges: $\sum_{n=0}^{\infty} \frac{1}{n!}$.

   (e) Evaluate the limit:

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   $$\lim_{x\to\infty} \frac{x^n}{e^x},$$

   where n is a finite, positive integer.

   (f) Show that the differential $df = x^2dy - (y^2 + xy)dx$ is not exact, but $dg = \frac{1}{xy^2}df$ is exact.

   (g) The temperature of a point (x, y) on a unit circle is given by $T(x, y) = 1 + xy$. Find the temperature of the two hottest points on the circle.

   (h) Check the order and degree of the following differential equation:
   $\frac{d^3y}{dx^3} + x^2(\frac{dy}{dx})^{1/3} + xy = 0$.

Answer any **two** from the following:

$5 \times 2 = 10$

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2. (a) Solve: $\frac{dy}{dx} = \frac{y}{x} + \tan(\frac{y}{x})$.

   (b) Find the Wronskian of the functions: $f_1(x) = \sin(x)$; $f_2(x) = \cos(x)$; $f_3(x) = e^x$.

   (c) Find the values of α and β that make $(\frac{1}{x^2+2} + \frac{\alpha}{y})dx + (xy^\beta + 1)dy$ an exact differential.

   (d) Expand $f(x) = \cos(x)$ as a Taylor series about $x = 0$ retaining first five terms.