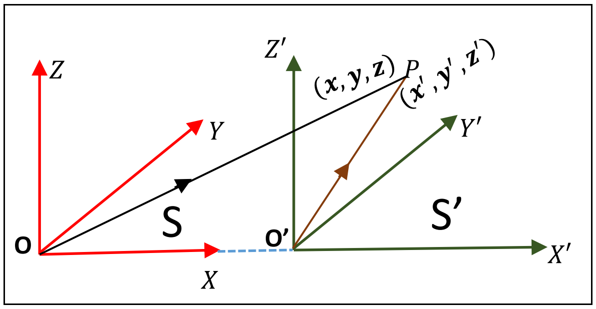
Galilean transformation.

The law of nature look exactly the same for all observers in the inertial frames, regardless of their state of relative velocity.



If the S' frame moving with a velocity v_0 along X-axis then the transformation equation becomes

$$x' = x - v_0 t; \quad y' = y; \quad z' = z; \quad t' = t$$

Differentiating both side with respect to t we get $\frac{dx'}{dt} = \frac{dx}{dt} - v_0, \frac{dy'}{dt} = \frac{dy}{dt}, \frac{dz'}{dt} = \frac{dz}{dt}$

Then transformation equation for velocity becomes

$$v'_x = v_x - v_0; \quad v'_y = v_y; \quad v'_z = v_z$$

Similarly differentiating both side w.r.t t we get $\frac{dv'_x}{dt} = \frac{dv_x}{dt}, \frac{dv'_y}{dt} = \frac{dv_y}{dt}, \frac{dv'_z}{dt} = \frac{dv_z}{dt}$

Then transformation equation for acceleration becomes

$$a'_x = a_x; \quad a'_y = a_y; \quad a'_z = a_z;$$

The transformation rule for partial derivatives operators for Galilean transformation.

[Note that: If $f = f(x', t')$; $x' = x'(x, t)$; $t' = t'(x, t)$ then according to chain rule for partial derivatives $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial f}{\partial t'}\frac{\partial t'}{\partial x}$ & $\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x'}\frac{\partial x'}{\partial t} + \frac{\partial f}{\partial t'}\frac{\partial t'}{\partial t}$]

As $\frac{\partial x'}{\partial x} = 1$; $\frac{\partial t'}{\partial x} = 0$; $\frac{\partial x'}{\partial t} = -v_0$; $\frac{\partial t'}{\partial t} = 1$ for Galilean Transformation. Then $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x'}$ or, $\frac{\partial}{\partial x} = \frac{\partial}{\partial x'}$

$$\& \frac{\partial f}{\partial t} = \frac{\partial f}{\partial x'}(-v_0) + \frac{\partial f}{\partial t'} \quad or, \quad \frac{\partial}{\partial t} = \frac{\partial}{\partial t'} - v_0\frac{\partial}{\partial x'} \Longrightarrow \frac{\partial}{\partial t} = \frac{\partial}{\partial t'} - \vec{v}_0.\vec{\nabla}$$

Then $\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial x'^2}$ & $\frac{\partial^2 f}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial f}{\partial t}\right) = \left(\frac{\partial}{\partial t'} - v_0\frac{\partial}{\partial x'}\right)\left(\frac{\partial f}{\partial x'}(-v_0) + \frac{\partial f}{\partial t'}\right) = -v_0\frac{\partial^2 f}{\partial t'\partial x'} + \frac{\partial^2 f}{\partial t'^2} + v_0^2\frac{\partial^2 f}{\partial x'^2} - v_0\frac{\partial^2 f}{\partial t'\partial x'} = \frac{\partial^2 f}{\partial t'^2} + v_0^2\frac{\partial^2 f}{\partial x'^2} - 2v_0\frac{\partial^2 f}{\partial t'\partial x'}$

Hence $\frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial x'^2}$ & $\frac{\partial^2}{\partial t^2} = \frac{\partial^2}{\partial t'^2} + v_0^2\frac{\partial^2}{\partial x'^2} - 2v_0\frac{\partial^2}{\partial t'\partial x'}$

Show that 'LENGTH' is invariant under Galilean transformation.

In the **S frame** the distance between two points (x_1, y_1, z_1) & (x_2, y_2, z_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

In the **S' frame** this distance transformed to $\sqrt{(x'_2 - x'_1)^2 + (y'_2 - y'_1)^2 + (z'_2 - z'_1)^2} = \sqrt{(x_2 - vt - x_1 + vt)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ = the distance between two points in **S frame**

[Note that Hence the 'AREA' and 'VOLUME' is Galilean invariant]

Show that **'FORCE' is Galilean invariant or, Newton's law** is Galilean invariant

For Galilean transformation $x' = x - v_0 t$; $y' = y$; $z' = z$; $t' = t$ we have $a'_x = a_x$; $a'_y = a_y$; $a'_z = a_z$;

Hence $\vec{a'} = \vec{a}$ that means acceleration is Galilean invariant

We have $\vec{F} = m\vec{a}$ then $\vec{F'} = m'\vec{a'} = m\vec{a} = \vec{F}$ (proved) [as m is Galilean invariant.]

Show that **CHARGE DENSITY** is Galilean invariant.

Charge density $\rho = \frac{q}{V}$ as q is invariant according to charge conservation theorem and volume is Galilean invariant then charge density Galilean invariant.

Show that **CURRENT DENSITY is not** Galilean invariant.

As current density $\vec{J} = \rho\vec{v}$ as ρ is Galilean invariant but $\vec{v}$ is not as $v'_x = v_x - v_0$; $v'_y = v_y$; $v'_z = v_z$ i.e, $\vec{v}' = \vec{v} - \vec{v}_0$ then $\vec{J}' = \rho(\vec{v} - \vec{v}_0) = \vec{J} - \rho\vec{v}_0$

Check the invariance of **ELECTRIC FIELD STRENGTH** and **MAGNETIC INDUCTION VECTOR** from the pre-relativistic requirement of invariance of Lorentz force.

We have $\vec{F} = q\{\vec{E} + (\vec{v} \times \vec{B})\}$ then $\vec{F'} = q\{\vec{E'} + (\vec{v'} \times \vec{B'})\} = q\left\{\vec{E'} + \left((\vec{v} - \vec{v}_0) \times \vec{B'}\right)\right\}$

Hence $\vec{E'} + \left((\vec{v} - \vec{v}_0) \times \vec{B'}\right) = \vec{E} + (\vec{v} \times \vec{B})$ or, $\vec{E'} + \vec{v} \times \left(\vec{B'} - \vec{B}\right) = \vec{E} + \vec{v}_0 \times \vec{B'}$

Then the only possible solution of equ^n (i) not involving velocities in specific reference frame, and without restriction on the choice $\vec{v'}$ is $\vec{E'} = \vec{E} + \vec{v}_0 \times \vec{B'}$ & $\vec{B'} = \vec{B}$

Show that **MAGNETIC GAUSS'S LAW** is Galilean invariant

Here we have $\vec{\nabla}'.\overrightarrow{B'} = \vec{\nabla}.\vec{B} = 0$ hence solenoidal character of $\vec{B}$ is invariant under Galilean transformation.

Show that **FARADAY'S LAW** is Galilean invariant

$$\vec{\nabla}' \times \overrightarrow{E'} + \frac{\partial \overrightarrow{B'}}{\partial t'} = \vec{\nabla} \times (\vec{E} + \vec{v}_0 \times \vec{B}) + \frac{\partial \vec{B}}{\partial t} + (\vec{v}_0.\vec{\nabla})\vec{B} = \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \times (\vec{v}_0 \times \vec{B}) + (\vec{v}_0.\vec{\nabla})\vec{B}$$

As $\vec{v}_0$ is a constant vector then $\vec{\nabla} \times (\vec{v}_0 \times \vec{B}) = \vec{v}_0(\vec{\nabla}.\vec{B}) - \vec{B}(\vec{\nabla}.\vec{v}_0) + (\vec{B}.\vec{\nabla})\vec{v}_0 - (\vec{v}_0.\vec{\nabla})\vec{B} = 0 - 0 + 0 - (\vec{v}_0.\vec{\nabla})\vec{B} = -(\vec{v}_0.\vec{\nabla})\vec{B}$

$$\vec{\nabla}' \times \overrightarrow{E'} + \frac{\partial \overrightarrow{B'}}{\partial t'} = \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \text{ (proved)}$$

Show that GAUSS'S LAW IN ELECTROSTATIC is not Galilean invariant

$$\vec{\nabla}'.\overrightarrow{E'} - \frac{\rho'}{\epsilon_0} = \vec{\nabla}.\left(\vec{E} + \vec{v}_0 \times \overrightarrow{B'}\right) - \frac{\rho}{\epsilon_0} = \vec{\nabla}.\vec{E} - \frac{\rho}{\epsilon_0} + \vec{\nabla}.\left(\vec{v}_0 \times \overrightarrow{B'}\right) = \vec{\nabla}.\left(\vec{v}_0 \times \overrightarrow{B'}\right) \neq \vec{\nabla}.\vec{E} - \frac{\rho}{\epsilon_0} \text{ (PROVED)}$$

Show that **AMPERE'S LAW** is **not** Galilean invariant

$$\vec{\nabla}' \times \overrightarrow{B'} - \frac{1}{c^2}\frac{\partial \overrightarrow{E'}}{\partial t'} - \mu_0\vec{J'} = \vec{\nabla} \times \vec{B} - \frac{1}{c^2}\left(\frac{\partial\left(\vec{E} + \vec{v}_0 \times \overrightarrow{B'}\right)}{\partial t} + (\vec{v}_0.\vec{\nabla})\left(\vec{E} + \vec{v}_0 \times \overrightarrow{B'}\right)\right) - \mu_0(\vec{J} - \rho\vec{v}_0)$$

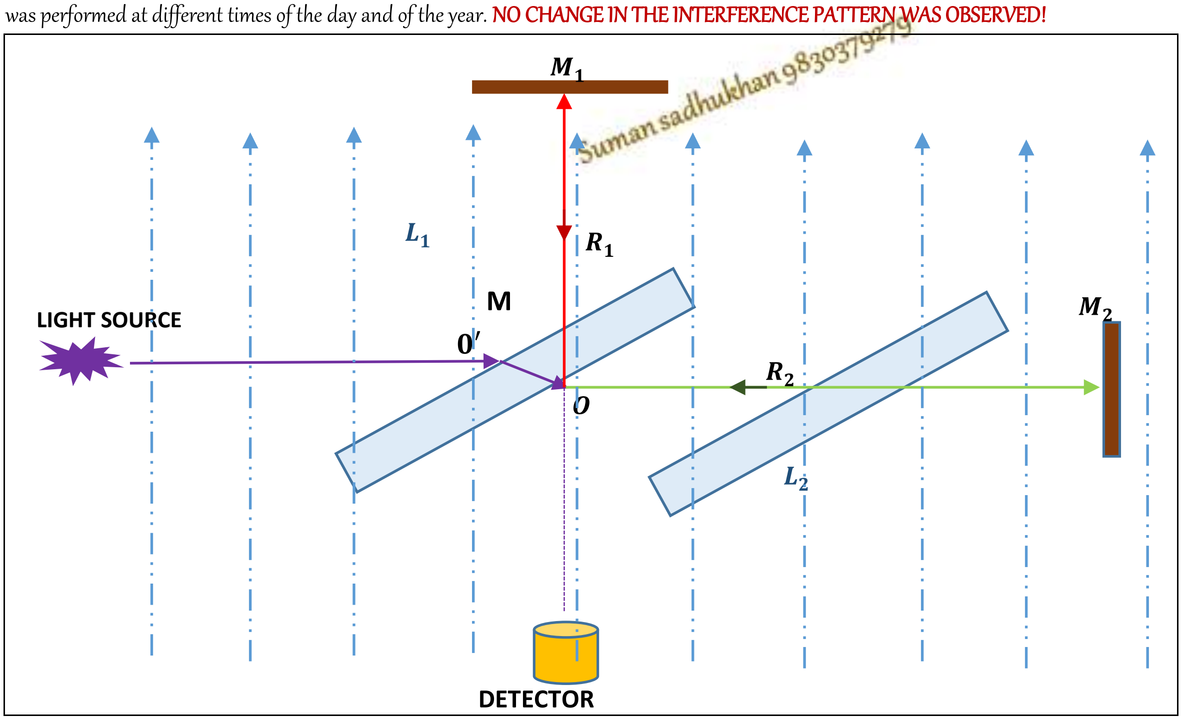
$$= \vec{\nabla} \times \vec{B} - \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} - \mu_0\vec{J} - \frac{1}{c^2}\vec{v}_0 \times \left[\frac{\partial}{\partial t} + (\vec{v}_0.\vec{\nabla})\right]\vec{B} - \frac{1}{c^2}(\vec{v}_0.\vec{\nabla})\vec{E} + \mu_0\rho\vec{v}_0 \neq \vec{\nabla} \times \vec{B} - \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} - \mu_0\vec{J} \text{ (PROVED)}$$

Show that THE CHARGE CONTINUITY EQUATION is invariant under Galilean transformation

$$\vec{\nabla}'.\vec{j}' + \frac{\partial \rho'}{\partial t'} = \vec{\nabla}.\vec{j} - \vec{\nabla}.\rho\vec{v}_0 + \left(\frac{\partial}{\partial t} - \vec{v}_0.\vec{\nabla}\right)\rho = \vec{\nabla}.\vec{j} - \frac{\partial \rho}{\partial t} \text{ (proved)}$$

THE MICHELSON-MORLEY EXPERIMENT

Note that - Sound waves need a medium though which to travel. In 1864 James Clerk Maxwell showed that light is an electromagnetic wave. Therefore it was assumed that there is an ether which propagates light waves. This ether was assumed to be everywhere and unaffected by matter. This ether could be used to determine an absolute reference frame (with the help of observing how light propagates through the ether). Note. The Michelson-Morley experiment (circa 1885) was performed to detect the Earth's motion through the ether as follows: The viewer will see the two beams of light which have travelled along 1 different arms display some interference pattern. If the system is rotated, then the influence of the "ether wind" should change the time the beams of light take to travel along the arms and therefore should change the interference pattern. The experiment was performed at different times of the day and of the year. NO CHANGE IN THE INTERFERENCE PATTERN WAS OBSERVED!



The beam of light from laboratory source S is split by the partially silvered mirror M into two coherent beam $R_1(\text{reflected through } M)$& $R_2(\text{transmitted through } M)$. R_1 reflected back to M by M_1 mirror and R_2 reflected back to M by M_2 mirror. Then this two beam interfere at O. The interference pattern is constructive or destructive depending on the phase (path) difference between the two light. The half silvered mirror is inclined at $45°$ to the beam direction. M_1 & M_2 are very nearly at right angle. We shall observe a fringe system in the telescope.

<u>Let compute the phase difference between two light beams R_1 & R_2</u>

This phase difference arise due to (i) Different path length (L_1 & L_2) travelled.

(ii) Different speed of travel due to **existence of either wind** (let it's velocity w.r.t laboratory is $\boldsymbol{v}$)

The time taken for beam R_1 from M to M_1 and back to M is $t_1 = \frac{L_1}{c-v} + \frac{L_1}{c+v} = L_1\left(\frac{2c}{c^2-v^2}\right) = \frac{2L_1}{c}\left(\frac{1}{1-v^2/c^2}\right)$

The time taken for beam R_2 from M to M_2 and back to M is $t_2 = \frac{2\left[L_2{}^2+\left(\frac{vt_2}{2}\right)^2\right]^{\frac{1}{2}}}{c}$ then $t_2 = \frac{2L_2}{c}\left(\frac{1}{\sqrt{1-v^2/c^2}}\right)$

Note that the calculation t_2 is made in either frame and t_1 in laboratory frame. In classical mechanics time is absolute.

The difference in transit time is $\Delta t = t_2 - t_1 = \frac{2}{c}\left(\frac{L_2}{\sqrt{1-v^2/c^2}} - \frac{L_1}{1-v^2/c^2}\right)$

Now suppose the instrument is rotated by an angle $90°$ then $\Delta t' = t'_2 - t'_1 = \frac{2}{c}\left(\frac{L_2}{1-v^2/c^2} - \frac{L_1}{\sqrt{1-v^2/c^2}}\right)$

Hence the rotation changes the time difference by $\Delta t'$-$\Delta t = \frac{2}{c}\left(\frac{L_2+L_1}{1-v^2/c^2} - \frac{L_1+L_2}{\sqrt{1-v^2/c^2}}\right)$

$$= \frac{2(L_1+L_2)}{c}\left[1 + v^2/c^2 - 1 - \frac{1}{2}v^2/c^2 \dots\dots\right] \approx \frac{(L_1+L_2)}{c}\frac{v^2}{c^2}$$

Therefore rotation should cause a shift in fringe pattern.

Let ΔN represent the number of fringes moving past the crosshairs as the pattern shift. Then $\Delta N = \frac{\Delta t' - \Delta t}{T} \approx \frac{(L_1+L_2)}{cT}\frac{v^2}{c^2} = \frac{(L_1+L_2)}{\lambda}\frac{v^2}{c^2}$

As earth rotating with a speed of 30km/s then if Michelson Morley take $L_1 = L_2 = 22m$ $\lambda = 5.5 \times 10^{-7}m$ $\frac{v}{c} = 10^{-4}$

Then $\Delta N = 0.4$ or shift of fourth tenths a fringe.

Experimental results: there was no fringe shift at all.

Suman sadhukhan 9830379279