DIFFRACTION OF LIGHT

LIGHT BENDING AROUND AN OBJECT

Diffraction is the slight bending of light as it passes around the sharp edge of an object. The amount of bending depends on the relative size of the wavelength of light to the size of the opening. If the opening is much larger than the light's wavelength, the bending will be almost unnoticeable. However, if the two are closer in size or equal, the amount of bending is considerable, and easily seen with the naked eye.

Diffraction can be usually being characterised by one of two types:

- **Fresnel diffraction**
- **Fraunhofer diffraction**

Fresnel Diffraction

In optics, the Fresnel diffraction equation for near-field diffraction is an approximation of the Kirchhoff–Fresnel diffraction that can be applied to the propagation of waves in the near field. It is used to calculate the diffraction pattern created by waves passing through an aperture or around an object, when viewed from relatively close to the object. **In this type of diffraction, the source or the screen (point of observation) or both are at a finite distance from the object/aperture.** This type of diffraction occurs when the distance of propagation of diffracted waves is short, resulting in a **Fresnel number greater than 1.** Diffracted waves become planar when the distance of propagation is increased.

Fresnel number is defined as $F = \frac{a^2}{\lambda L}$ **[where λ is the wavelength of light, a is the characteristic size ("radius") of the object/aperture, L is the distance of the screen from the object/aperture]**

Fraunhofer Diffraction

In optics, the Fraunhofer diffraction equation is used to model the diffraction of waves when the diffraction pattern is viewed at a long distance from the diffracting object, and also when it is viewed at the focal plane of an imaging lens. That means in this type of diffraction, **both the source and the screen are effectively at infinite distance from the obstacle or the aperture.**
Fresnel number less than 1.

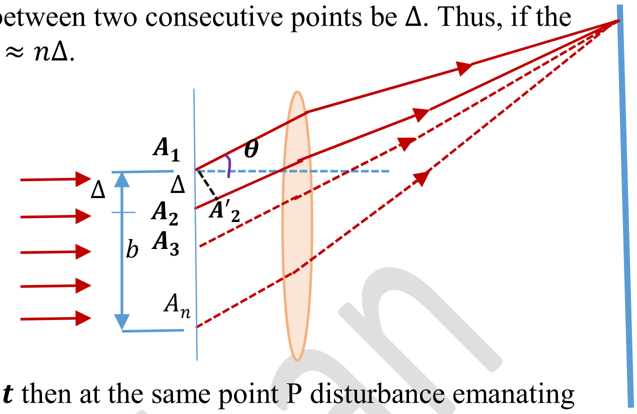
Comparison of Fresnel and Fraunhofer Diffraction

Characteristics	Fresnel Diffraction	Fraunhofer Diffraction
Surface of calculation	Fresnel diffraction occurs on spherical surfaces.	Fraunhofer diffraction patterns on flat surfaces.
Diffraction pattern	Shape and intensity of diffraction pattern change as the waves propagate downstream of the scattering source.	Shape and intensity remain constant.
Movement of diffraction pattern	Move along the corresponding shift in the object.	Remains in a fixed position.
Observation distance	Screen placed at finite distance from an object.	Infinite observation distance.
Wave fronts	Spherical or Cylindrical wave fronts.	Planar wave fronts.

Fraunhofer Diffraction single slit.

Let consider a slit of width b. We assume that the slit consists of a large number of equally spaced point sources and that each point on the slit is a source of Huygens secondary wavelets which interfere with the wavelets emanating from other points.

Let the point sources be at $A_1, A_2, A_3, A_4, A_5 \dots\dots$ and let the distance between two consecutive points be Δ. Thus, if the number of point sources be n, then $b = (n-1)\Delta$. Then for $n \to \infty$ $b \approx n\Delta$.



For an incident plane wave A_1, A_2 are in phase but for a point P on the screen (**receiving parallel rays making an angle θ with the normal to the slit**) due to different path lengths to the point P , the field produced by A_1 will differ in phase from the field produced by A_2.

From figure path difference $A_2A'_2 = \Delta \sin\theta$ then corresponding phase difference is $\varphi = \frac{2\pi}{\lambda}\Delta\sin\theta$

Then if at the field point P disturbance emanating from A_1 is $\boldsymbol{a\cos\omega t}$ then at the same point P disturbance emanating from A_2 is $\boldsymbol{a\cos(\omega t - \varphi)}$

Then resultant field at point p due to whole slit

$\boldsymbol{E} = a\cos\omega t + a\cos(\omega t - \varphi) + a\cos(\omega t - 2\varphi) + a\cos(\omega t - 3\varphi) \dots\dots\dots\dots a\cos(\omega t - (n-1)\varphi)$

$$= a\frac{\sin\frac{n\varphi}{2}}{\sin\frac{\varphi}{2}}\cos\left(\omega t - \frac{n-1}{2}\varphi\right)$$

$$= E_\varphi \cos\left(\omega t - \frac{n-1}{2}\varphi\right)$$

where $E_\varphi = a\frac{\sin\frac{n\varphi}{2}}{\sin\frac{\varphi}{2}}$ now For $n \to \infty, \varphi \to 0$

then $\sin\frac{\varphi}{2} \to \frac{\varphi}{2} = \frac{\pi}{\lambda}\frac{b}{n}\sin\theta = \frac{\beta}{n}$ where $\beta = \frac{\pi}{\lambda}b\sin\theta$

then $E_\varphi = a\frac{\sin n\frac{\beta}{n}}{\frac{\beta}{n}} = na\frac{\sin\beta}{\beta} = A\frac{\sin\beta}{\beta}$. Hence now $E_\varphi = E(\beta) or, E(\theta)$

$$= A\frac{\sin\beta}{\beta}\cos(\omega t - \beta)$$

Hence corresponding intensity distribution is given by $\boldsymbol{I = E_\varphi E^*_\varphi = A^2\left(\frac{\sin\beta}{\beta}\right)^2 = I_0\frac{\sin^2\beta}{\beta^2}}$

[**Here I_0 is max. intensity, the intensity at $\theta = 0$**]

POSITION OF MINIMA AND MAXIMA

For principal maximum intensity:

As intensity $I = I_0\frac{\sin^2\beta}{\beta^2}$ then for $\beta = 0$ as $\lim_{\beta\to 0}\frac{\sin\beta}{\beta} \to 1$ then $I = I_0$ hence max. intensity.

Hence the intensity at $\beta = 0$ or $\beta = \frac{\pi}{\lambda}b\sin\theta = 0$ or, $\boxed{\boldsymbol{b\sin\theta = 0}}$ $\boldsymbol{\theta = 0}$ **is principal maximum intensity**

For minimum intensity:

As intensity $\boldsymbol{I = I_0\frac{\sin^2\beta}{\beta^2}}$ **then** intensity is zero when $\boldsymbol{\beta = m\pi}$ where $m = \pm1, \pm2, \pm3 \dots.$ **But $\boldsymbol{m \neq 0}$**

[As for $m = 0$ $\beta = 0$ and then $I \to I_0$ hence principal max. intensity.]

Hence condition for minimum is $\frac{\pi}{\lambda}b\sin\theta = m\pi$ or, $\boxed{\boldsymbol{b\sin\theta = m\lambda}}$ where $m = \pm1, \pm2, \pm3 \dots$

<u>MAXIMUN ALLOWED VALUE OF ORDER $\boldsymbol{m}$</u>

As first minimum occurs at $\theta = \pm\sin^{-1}\frac{\lambda}{b}$ second minimum occurs at $\theta = \pm\sin^{-1}\frac{2\lambda}{b}$ and so on.

Since $\sin\theta \leq 1$ then **maximum value of $\boldsymbol{m}$ is less than (and closest to) $\boldsymbol{\frac{b}{\lambda}}$**

For secondary maximum intensity:

As intensity $I = I_0\frac{\sin^2\beta}{\beta^2}$ then $\frac{dI}{d\beta} = I_0\left[\frac{2\sin\beta\cos\beta}{\beta^2} - \frac{2\sin^2\beta}{\beta^3}\right]$

For maximum intensity $\frac{dI}{d\beta} = 0$ then $\frac{\sin\beta}{\beta}\left[\frac{\beta\cos\beta - \sin\beta}{\beta^2}\right] = 0$

Then $\sin\beta\,[\beta\cos\beta - \sin\beta] = 0$ [as $\boldsymbol{\beta \neq 0}$ **for secondary maxima** as for principal maxima $\beta = 0$]

Then $[\beta\cos\beta - \sin\beta] = 0$ [as $\boldsymbol{\sin\beta \neq 0}$ **for secondary maxima** as for minima $\sin\beta = 0$]

Hence condition for secondary maxima is $\boxed{\boldsymbol{\beta = \tan\beta}}$

Relative intensities

The intensity of central maxima is $I_c = I_0$

The intensity of 1st secondary maxima is

$$I_1 = I_0 \frac{\sin^2\frac{3\pi}{2}}{\left(\frac{3\pi}{2}\right)^2} = \frac{4}{9\pi^2} I_0$$

The intensity of 2nd secondary maxima is

$$I_2 = I_0 \frac{\sin^2\frac{5\pi}{2}}{\left(\frac{5\pi}{2}\right)^2} = \frac{4}{25\pi^2} I_0$$

Width of central maximum

The angle of diffraction θ_1 for the first minimum on the either side of central maxima occur for $m = \pm 1$

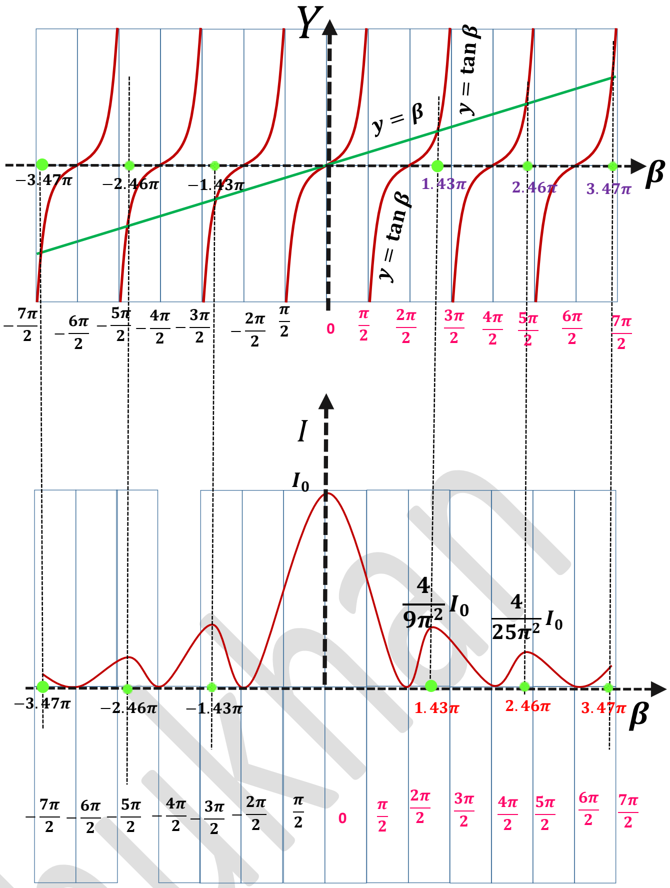
That means $b\, sin\theta_1 = \pm\lambda$ then $\theta_1 \approx \pm\frac{\lambda}{b}$

Hence angular width of central maxima is then $2\theta_1 \approx 2\frac{\lambda}{b}$

What happens when slit is narrower?

Then the ratio $\frac{\lambda}{b}$ increases in the limit of $b \to \lambda$, $\sin\theta_1 = 1$

then $\theta_1 = 90°$, the central maxima fills the whole space diffraction pattern disappears.



Difference between single slit diffraction and double slit interference

For diffraction pattern the central maxima is brightest, having $\frac{\lambda}{a}$ as the angular half width , **narrower the slit broader the central maxima** the subsidiary maximum is on either side on it situated non-symmetrically and are rapidly diminishing intensity.

In an interference pattern, the maxima and minima are **equidistant** and **equally wide** and all **maxima** have the **same intensity.** The fringe width increases with decreasing separation between the slits.

Fraunhofer diffraction at a circular aperture

A circular aperture of diameter '*D*' is shown as *AB* in Fig1 A plane wave front WW' is incident normally on this aperture. Every point on the plane wave front in the aperture acts as a source of secondary wavelets. The secondary wavelets spread out in all directions as diffracted rays in the aperture. These diffracted secondary wavelets are converged on the screen *SS'* by keeping a convex lens (*L*) between the aperture and the screen. The screen is at the focal plane of the convex lens. Those diffracted rays traveling normal to the plane of aperture [i.e., along CP_0] are get converged at P_0.

All these waves travel some distance to reach P_o and there is no path difference between these rays. Hence a bright spot is formed at P_o known **Airy's disc**. P_o corresponds to the central maximum.

Next consider the secondary waves traveling at an angle θ with respect to the direction of CP_o. All these secondary waves travel in the form of a cone and hence, they form a diffracted ring on the screen. The radius of that ring is x and its centre is at P_o. Now consider a point P_1 on the ring, the intensity of light at P_1 depends on the path difference between the waves at A and B to reach P_1. The path difference is $BD = AB \sin\theta = d\sin\theta$.

The diffraction due to a circular aperture is similar to the diffraction due to a single slit. Hence, the intensity at P_1 depends on the path difference $D \sin\theta$.

If the path difference is an integral multiple of λ then intensity at P_1 is minimum.

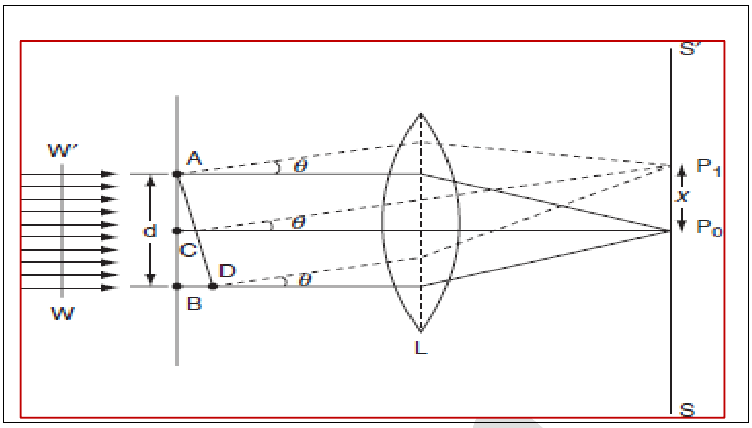
On the other hand, if the path difference is in odd multiples of $\frac{\lambda}{2}$, then the intensity is maximum.

Condition for **minima** is $\boldsymbol{D\sin\theta = n\lambda}$........... (1)

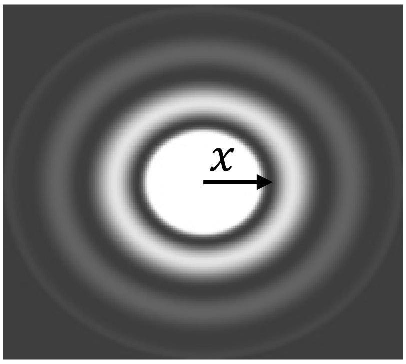
Condition for **maxima** is $\boldsymbol{D\sin\theta = \frac{2n+1}{2}\lambda}$......... (2)

where n = 1, 2, 3, …. etc. n = 0 for central maximum.

The Airy disc is surrounded by alternate bright and dark concentric rings, called the Airy's rings.



The intensity of the dark ring is zero and the intensity of the bright ring decreases as we go radially from P_0 on the screen. If the collecting lens(L) is very near to the circular aperture or the screen is at a large distance from the lens, then $\boldsymbol{\sin\theta \approx \theta = \frac{x}{f}}$ … … … … …(3) **Where f is the focal length of the lens.**



Airy's disc

Also from the condition for first secondary minimum [from equation (1)]

$\boldsymbol{\sin\theta \approx \theta = \frac{\lambda}{D}}$ … … … … …(4)

Then from equation (3) and (4) we have, $\boldsymbol{\frac{x}{f} = \frac{\lambda}{D}}$ or, $x = \frac{f\lambda}{D}$

But according to Airy, the exact value of x is $\boxed{\boldsymbol{x = \frac{1.22\, f\lambda}{D}}\text{.........(5)}}$

Using equation (5) the radius of Airy's disc can be obtained. Also from equation (5) we know that the radius of Airy's disc is inversely proportional to the diameter of the aperture. Hence by decreasing the diameter of aperture, the size of Airy's disc increases.

Rayleigh criterion:

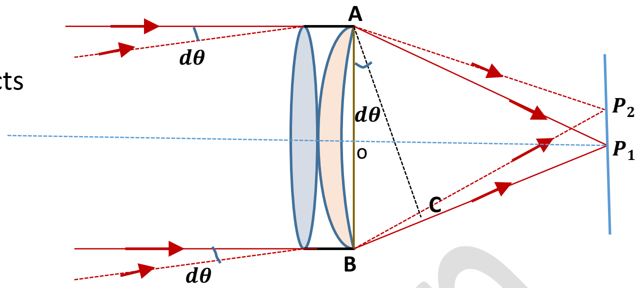
The **Rayleigh criterion** for the diffraction limit to resolution states that *two images are just resolvable when the centre of the diffraction pattern of one is directly over the first minimum of the diffraction pattern of the other.*

As it can be shown that, for a circular aperture of diameter 'd 'the first minimum in the diffraction pattern occurs at $\boldsymbol{\theta = \frac{x}{f} = \frac{1.22\,\lambda}{D}}$ (providing the aperture is large compared with the wavelength of light) so that two point objects are just resolvable if they are separated by the angle $\boldsymbol{\theta = \frac{1.22\,\lambda}{D}}$

where λ is the wavelength of light (or other electromagnetic radiation), lens, mirror, etc., with which the two objects are observed. In this expression θ has units of radians.

The resolving power of a telescope

The resolving power of a telescope is defined as the **reciprocal of the smallest angle** subtended at its objective by two distant close point-objects that can be just recognised as separated ones by telescope.



Let AB be the objective of the telescope of diameter D and O_1& O_2 (not shown in figure) be two distant close point-objects From which parallel rays shown an angular separation $d\theta$ at the objective. The objective (With supporting ring) acts as a circular aperture and forms diffraction pattern as images of O_1& O_2. Let P_1& P_2 are the positions of central maxima of the images.

Path difference $\Delta = BP_2 - AP_2 = BC = AB \ \sin d\theta \ \approx AB \ d\theta = Dd\theta$

If $\Delta = 1.\lambda$, then P_2 will be first minima of the first image. So for just resolved condition $\Delta = Dd\theta = \lambda$

Or $d\theta = \frac{\lambda}{D}$ this condition holds for a rectangular aperture.

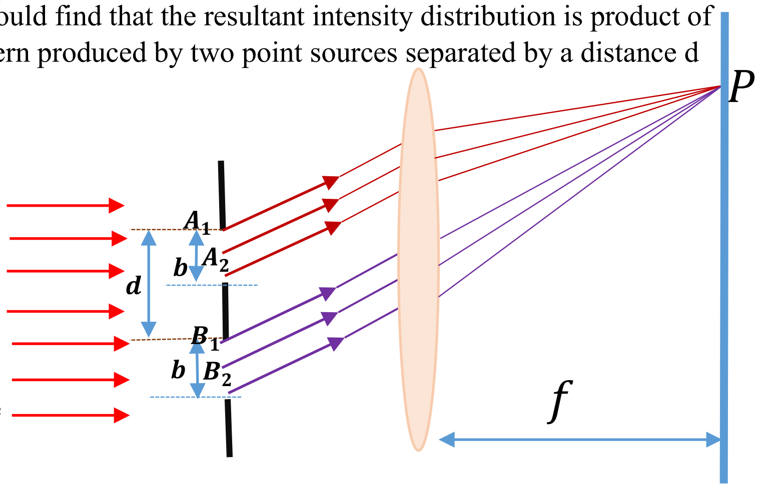
For a circular aperture, Airy modify this condition as $d\boldsymbol{\theta} = \frac{\mathbf{1.22}\,\boldsymbol{\lambda}}{\boldsymbol{D}}$

As $d\boldsymbol{\theta}$ is the minimum resolving angle between two distant close point-objects. So $d\boldsymbol{\theta}$ is limit of resolution.

Hence resolving power of a telescope is $\boxed{\frac{\mathbf{1}}{d\boldsymbol{\theta}} = \frac{\boldsymbol{D}}{\mathbf{1.22}\,\boldsymbol{\lambda}}}$

Fraunhofer Diffraction double slit.

Let two slits (of width b) separated by a distance d. we would find that the resultant intensity distribution is product of the single slit diffraction pattern and the interference pattern produced by two point sources separated by a distance d



We assume that each slit consists of a large number of equally spaced point sources and that each point on the slit is a source of Huygens secondary wavelets which interfere with the wavelets emanating from other points. Let the point sources be at A_1, A_2, A_3, A_4, A_5 ….. (in the first slit) Let the point sources be at B_1, B_2, B_3, B_4, B_5 ….. (in the second slit) and let the distance between two consecutive points be Δ. If the diffracted rays make an angle θ with the normal to the plane of the slits, then the path difference between the disturbances reaching the point P from two consecutive point in a slit will be $\Delta \sin\theta$.

The field produced by the first slit at the point P will, $E_1 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta)$, similarly the second slit will produce a field at the point P is $E_2 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta - \varphi_1)$,

[where $\boldsymbol{\varphi_1} = \frac{2\pi}{\lambda}\boldsymbol{d}\sin\boldsymbol{\theta}$ = phase difference between corresponding points $(\boldsymbol{A_1}, \boldsymbol{B_1}), (\boldsymbol{A_2}, \boldsymbol{B_2}), (\boldsymbol{A_3}, \boldsymbol{B_3})$, ……separated by a distance $\boldsymbol{d}$]

$$\boldsymbol{E} = \boldsymbol{E_1} + \boldsymbol{E_2} = A\frac{\sin\beta}{\beta}[\cos(\omega t - \beta) + \cos(\omega t - \beta - \varphi_1)] = A\frac{\sin\beta}{\beta}[2\cos\frac{\varphi_1}{2}\cos(\omega t - \beta - \frac{\varphi_1}{2})]$$

Or, $\boldsymbol{E} = \mathbf{2}\boldsymbol{A}\frac{\sin\boldsymbol{\beta}}{\boldsymbol{\beta}}\cos\boldsymbol{\gamma}\cos(\boldsymbol{\omega t} - \boldsymbol{\beta} - \boldsymbol{\gamma})$ [where $\frac{\varphi_1}{2} = \boldsymbol{\gamma} = \frac{\pi}{\lambda}\boldsymbol{d}\sin\boldsymbol{\theta}$ note that $\boldsymbol{\beta} = \frac{\pi}{\lambda}\boldsymbol{b}\sin\boldsymbol{\theta}$]

$E = E_0\cos(\boldsymbol{\omega t} - \boldsymbol{\beta} - \boldsymbol{\gamma})$ **[where $\boldsymbol{E_0} = \mathbf{2}\boldsymbol{A}\frac{\sin\boldsymbol{\beta}}{\boldsymbol{\beta}}\cos\boldsymbol{\gamma}$]**

Then intensity distribution will be in form of $I = E_0 E^*{}_0 = 4A^2 \frac{\sin^2\beta}{\beta^2}\cos^2\gamma = 4I_0 \frac{\sin^2\beta}{\beta^2}\cos^2\gamma$

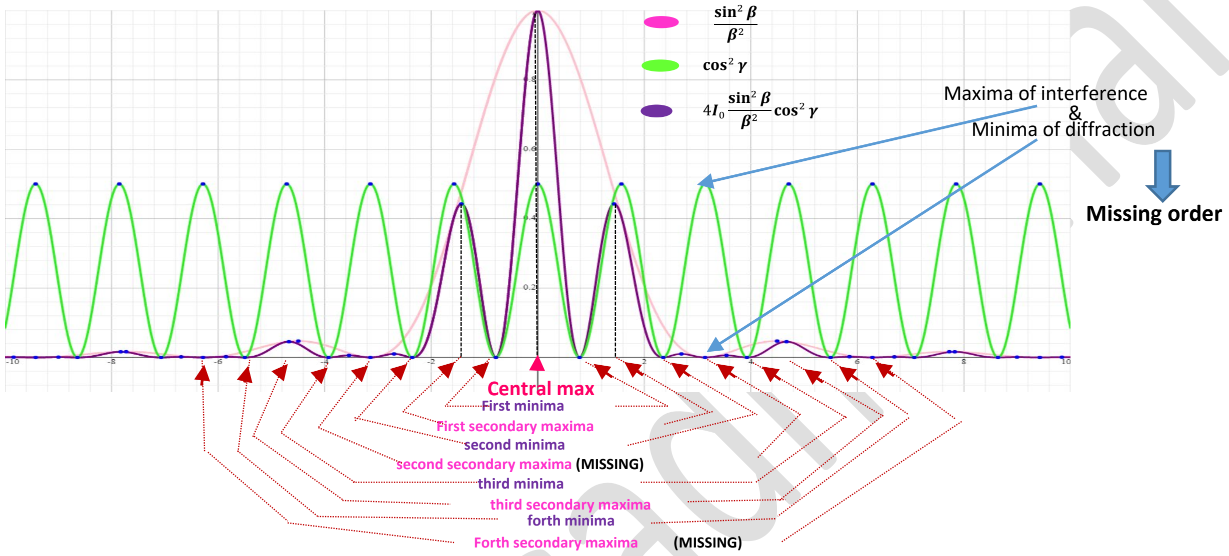
$$I = 4I_0 \frac{\sin^2\beta}{\beta^2}\cos^2\gamma$$

$I_0 \frac{\sin^2\beta}{\beta^2}$ **represent the intensity distribution of diffraction pattern produced by one single slit of width** b

$4\cos^2\gamma$ **represent the intensity distribution of interference pattern produced by two point sources separated by a distance** d.

Note that when b→ 0 then for $\beta \to 0$ **as** $\lim_{\beta\to 0}\frac{\sin\beta}{\beta} \to 1$ **then** $I = 4I_0\cos^2\gamma$

Simply we obtain the Young's experiment pattern.



Position of maximum and minimum

For diffraction pattern: The **central maxima** for **β**=0 that means for **θ**=0

The **secondary maxima** occur at the values of β approaching $\pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}$ ....

The **minima** due to diffraction pattern occur at the values of β=$\pm\pi, \pm 2\pi, \pm 3\pi$......

Hence the condition for **minimum(diffraction)** is $\frac{\pi}{\lambda} b\sin\theta = m\pi$ or, $\boxed{\boldsymbol{b\sin\theta = m\lambda} \quad \text{where } m = \pm1, \pm2, \pm3 \ldots}$

For interference pattern: The maxima occur when $\cos^2\gamma$**=1** that means $\gamma = s\pi$ where $s = \mathbf{0}, \pm1, \pm2, \pm3 \ldots$

Hence the condition for **maxima (interference)** is $\frac{\pi}{\lambda} d\sin\theta = s\pi$ or, $\boxed{\boldsymbol{d\sin\theta = s\lambda} \quad \text{where } s = 0, \pm1, \pm2, \pm3 \ldots}$

The minima occur when $\cos^2\gamma$**=0** that means $\gamma = \frac{(2s+1)}{2}\pi$ where $s = \mathbf{0}, \pm1, \pm2, \pm3 \ldots$

The condition for **minima (interference)** is $\frac{\pi}{\lambda} d\sin\theta = \frac{(2s+1)}{2}\pi$ or, $\boxed{\boldsymbol{d\sin\theta = \frac{(2s+1)}{2}\lambda} \quad \text{where } s = 0, \pm1, \pm2, \pm3 \ldots}$

Condition for absent spectra

The condition for **maxima (interference)** is $\frac{\pi}{\lambda} d\sin\theta = s\pi$ or, $\boldsymbol{d\sin\theta = s\lambda}$ where $s = 0, \pm1, \pm2, \pm3 \ldots$

And the for m^th **minimum(diffraction)** is $\frac{\pi}{\lambda} b\sin\theta = m\pi$ or, $\boldsymbol{b\sin\theta = m\lambda}$ where $m = \pm1, \pm2, \pm3 \ldots$

If these two condition satisfy simultaneously then **s^th secondary maxima** will be absent from the resulting spectra.

Then we get the condition for absent of **s^th principal maxima is** $\boxed{\frac{d}{b} = \frac{s}{m}}$

Fraunhofer Diffraction N-slit:

We next consider the diffraction pattern produced by N parallel slits, each of width b; the distance between two consecutive slit be d

As before, we assume that each slit consists of n equally spaced point with spacing Δ thus fieil of any arbitrary point P will essentially be a sum of N terms:

$$\boldsymbol{E} = A\frac{\sin\beta}{\beta}[\cos(\omega t-\beta)+\cos(\omega t-\beta-\varphi_1)+\cdots\ldots\ldots+\cos(\omega t-\beta-(N-1)\varphi_1)]$$

$$= \boldsymbol{A}\ \frac{\sin\boldsymbol{\beta}}{\boldsymbol{\beta}}\frac{\sin\frac{N\varphi_1}{2}}{\sin\frac{\varphi_1}{2}}\cos\left(\boldsymbol{\omega t}-\boldsymbol{\beta}-\frac{N-1}{2}\varphi_1\right)$$

$$= \boldsymbol{A}\ \frac{\sin\boldsymbol{\beta}}{\boldsymbol{\beta}}\frac{\sin N\boldsymbol{\gamma}}{\sin\boldsymbol{\gamma}}\cos\left(\boldsymbol{\omega t}-\boldsymbol{\beta}-\frac{N-1}{2}\boldsymbol{\gamma}\right) \qquad \left[\text{where } \frac{\varphi_1}{2}=\boldsymbol{\gamma}=\frac{\pi}{\lambda}\boldsymbol{d}\sin\boldsymbol{\theta} \quad \text{note that } \boldsymbol{\beta}=\frac{\pi}{\lambda}\boldsymbol{b}\sin\boldsymbol{\theta}\ \right]$$

$$E = E_0\cos(\boldsymbol{\omega t}-\boldsymbol{\beta}-\boldsymbol{\gamma}) \qquad \left[\textbf{ where } \boldsymbol{E_0} = \boldsymbol{A}\ \frac{\sin\boldsymbol{\beta}}{\boldsymbol{\beta}}\frac{\sin N\boldsymbol{\gamma}}{\sin\boldsymbol{\gamma}}\right]$$

Then intensity distribution will be in form of $\boldsymbol{I} = E_0E^*{}_0=A^2\frac{\sin^2\boldsymbol{\beta}}{\boldsymbol{\beta}^2}\frac{\sin^2 N\boldsymbol{\gamma}}{\sin^2\boldsymbol{\gamma}} = \boldsymbol{I_0}\frac{\sin^2\boldsymbol{\beta}}{\boldsymbol{\beta}^2}\frac{\sin^2 N\boldsymbol{\gamma}}{\sin^2\boldsymbol{\gamma}}$

$$\boldsymbol{I} = \boldsymbol{I_0}\frac{\sin^2\boldsymbol{\beta}}{\boldsymbol{\beta}^2}\frac{\sin^2 N\boldsymbol{\gamma}}{\sin^2\boldsymbol{\gamma}}$$

$\boldsymbol{I_0}\frac{\sin^2\boldsymbol{\beta}}{\boldsymbol{\beta}^2}$ **represent the intensity distribution of diffraction pattern produced by one single slit of width** $\boldsymbol{b}$

$\frac{\sin^2 N\boldsymbol{\gamma}}{\sin^2\boldsymbol{\gamma}}$ **represent the intensity distribution of interference pattern produced by N equally spaced point sources separated by a distance** $\boldsymbol{d}$**.**

<u>Condition for maximum and minima</u>

The variation of $\frac{\sin^2\beta}{\beta^2}$ is small and the maxima will be determined by entirely by the factor $\frac{\sin^2 N\gamma}{\sin^2\gamma}$

Then $I = I_0\frac{\sin^2\beta}{\beta^2}\frac{\sin^2 N\gamma}{\sin^2\gamma} = I_1\frac{\sin^2 N\gamma}{\sin^2\gamma}$ [as variation of $I_1 = I_0\frac{\sin^2\beta}{\beta^2} \approx$ constant]

Or, $\frac{\mathrm{dI}}{\mathrm{d\gamma}} = I_1\left[\frac{\sin^2\gamma\,2(\sin N\gamma)N(\cos N\gamma)-\sin^2 N\gamma\,2(\sin\gamma)(\cos\gamma)}{\sin^4\gamma}\right] = 2I_1\left[\frac{N(\sin N\gamma)(\cos N\gamma)}{\sin^2\gamma}-\frac{\sin^2 N\gamma(\cos\gamma)}{\sin^3\gamma}\right]$

$$= 2I_1\left(\frac{\sin^2 N\gamma}{\sin^2\gamma}\right)(N\cot N\gamma-\cot\gamma)$$

Hence for maxima or minima either $\frac{\sin^2 N\gamma}{\sin^2\gamma} = 0$ or, $(N\cot N\gamma-\cot\gamma) = 0$

Again when $(N\cot N\gamma-\cot\gamma) = 0$ and $\sin\gamma = 0$ we get $\frac{\mathrm{d^2I}}{\mathrm{d\gamma^2}} < 0$

Then $\boldsymbol{\sin\gamma = 0}$ and $\boldsymbol{(N\cot N\gamma-\cot\gamma) = 0}$ gives the condition of **maximum.**

Now when $(N\cot N\gamma-\cot\gamma) \neq 0$, $\sin\gamma \neq 0$, but $\sin N\gamma = 0$ we get $\frac{\mathrm{d^2I}}{\mathrm{d\gamma^2}} > 0$

Then $\boldsymbol{\sin N\gamma = 0}$ gives the condition of **minimum.**

MAXIMUM POSITIONS :

Principal maxima:

when $\sin\gamma = 0$ then $\gamma = s\pi$ where $s = 0, \pm1, \pm2, \pm3 \ldots$

Hence the condition for **principal maxima** is $\frac{\pi}{\lambda} d\sin\theta = s\pi$ or, $\boxed{\boldsymbol{d\sin\theta = s\lambda} \quad \text{where } s = 0, \pm1, \pm2, \pm3 \ldots}$

The significance of $\pm$ is there are two principal maxima for each order on either side of the zeroth order maxima.

Now when $\gamma = s\pi$ then $\lim_{\gamma\to s\pi} \frac{\sin N\gamma}{\sin\gamma} = \lim_{\gamma\to s\pi} \frac{N\cos N\gamma}{\cos N\gamma} = \frac{N\cos N(s\pi)}{\cos N(s\pi)} = N$ then Intensity $I_P = I_0 \frac{\sin^2\beta}{\beta^2} N^2 = I_1 N^2$

Similarly we get zero order principal maxima for $s = 0$ $\gamma \to 0$ then $\lim_{\gamma\to 0} \frac{\sin N\gamma}{\sin\gamma} = \lim_{\gamma\to 0} \frac{N\cos N\gamma}{\cos N\gamma} = N$

then $I = I_0 \frac{\sin^2\beta}{\beta^2} N^2$

Note that

(i) Hence intensity of principal maxima increases as number of slits N increases.

(ii) The intensity of principal maxima **decreases** with **increase** in **order number** due to the presence of $\frac{\sin^2\beta}{\beta^2}$ Whose value **decreases with increase of angle of diffraction θ**

Secondary maxima:

When $(N\cot N\gamma - \cot\gamma) = 0$ then $N^2\cot^2 N\gamma - \cot^2\gamma = 0$ or $\cot^2 N\gamma = \frac{\cot^2\gamma}{N^2}$

Then intensity of maxima(secondary) with condition $(N\cot N\gamma - \cot\gamma) = 0$ is

$$I_s = I_1 \frac{\sin^2 N\gamma}{\sin^2\gamma} = I_1 \frac{1}{\sin^2\gamma\csc^2 N\gamma} = I_1 \frac{1}{\sin^2\gamma(1+\cot^2 N\gamma)} = I_1 \frac{1}{\sin^2\gamma(1+\frac{\cot^2\gamma}{N^2})} \quad \text{as } \cot^2 N\gamma = \frac{\cot^2\gamma}{N^2}$$

$$= I_1 \frac{N^2}{\sin^2\gamma(N^2+\cot^2\gamma)} = I_1 \frac{N^2}{\sin^2\gamma N^2 + \cos^2\gamma} = \frac{I_1 N^2}{\sin^2\gamma(N^2-1)+1} = \frac{I_P}{1+(N^2-1)\sin^2\gamma}$$

Then $\boxed{\frac{I_s}{I_P} = \frac{1}{1+(N^2-1)\sin^2\gamma}}$ then for large value of N $\frac{I_s}{I_P} \to 0$ hence **they are too weak to be generally visible in N-slit.**

Hence we don't need their positions.

MAXIMUM POSITIONS :

minima:

when $\boldsymbol{\sin N\gamma = 0}$ **then** $N\gamma = s\pi$

or, $\gamma = \frac{s\pi}{N}$ where $s = \pm1, \pm2, \pm3 \ldots\ldots\ldots\ldots\ldots$

But $s \neq 0, \pm1N, \pm2N, \pm3N \ldots$ Because then condition of principal maxima is satisfied.

Hence the condition for **minima** is $\frac{\pi}{\lambda} d\sin\theta = \frac{s\pi}{N}$ or, $\boldsymbol{d\sin\theta = \frac{s\lambda}{N}}$ where $s = \pm1, \pm2, \pm3 \ldots$

But $s \neq 0, \pm1N, \pm2N, \pm3N \ldots$

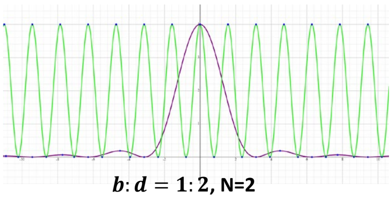
Condition for absent spectra

The condition for s^{th} **principal maxima** is $\frac{\pi}{\lambda} d\sin\theta = s\pi$ or, $\boldsymbol{d\sin\theta = s\lambda}$ where $s = 0, \pm1, \pm2, \pm3 \ldots$

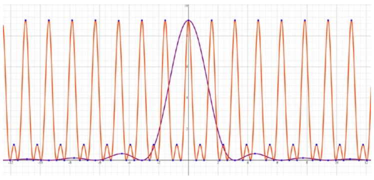
And the for $\boldsymbol{m^{th}}$ **minimum(diffraction)** is $\frac{\pi}{\lambda} b\sin\theta = m\pi$ or, $\boldsymbol{b\sin\theta = m\lambda}$ where $m = \pm1, \pm2, \pm3 \ldots$

If these two condition satisfy simultaneously then s^{th} **principal maxima** will be absent from the resulting spectra.

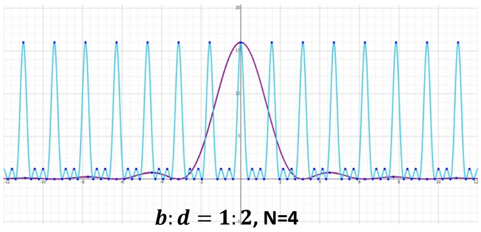
Then we get the condition for absent of s^{th} **principal maxima is** $\boxed{\frac{d}{b} = \frac{s}{m}}$



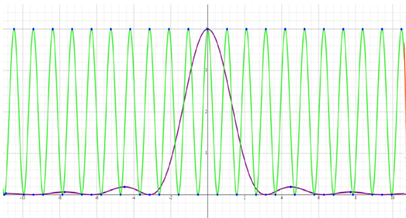
$b:d = 1:2$, N=2



$b:d = 1:2$, N=3



$b:d = 1:2$, N=4



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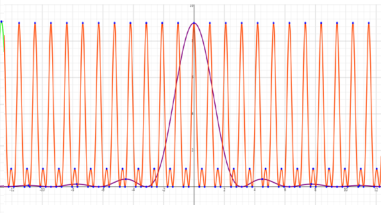

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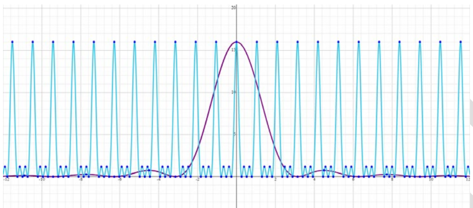

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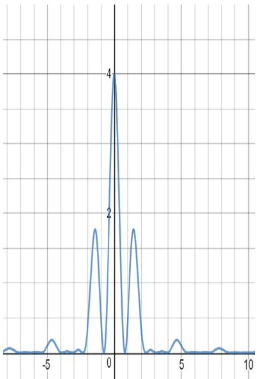
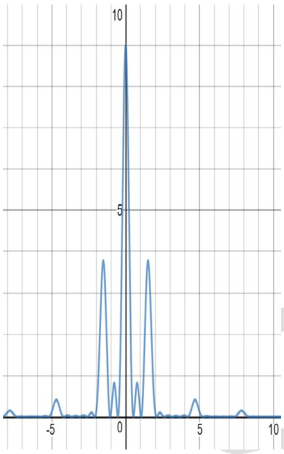
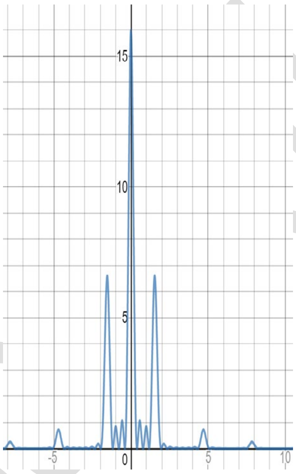


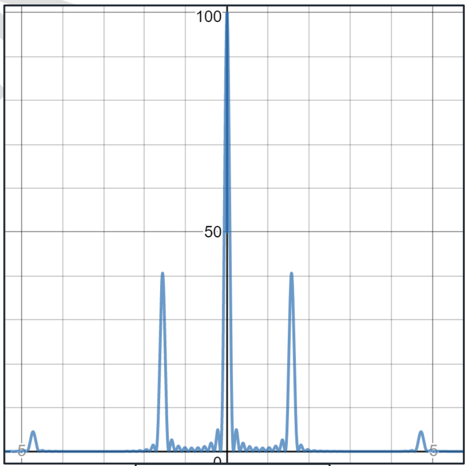
$b:d = 1:2$, N=2



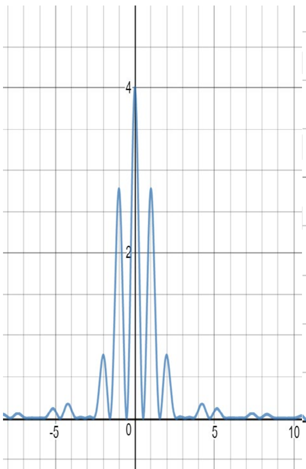
$b:d = 1:2$, N=3



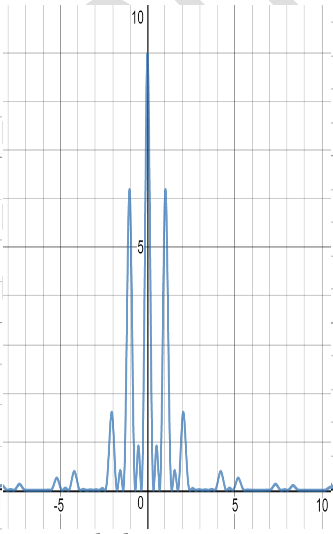
$b:d = 1:2$, N=4



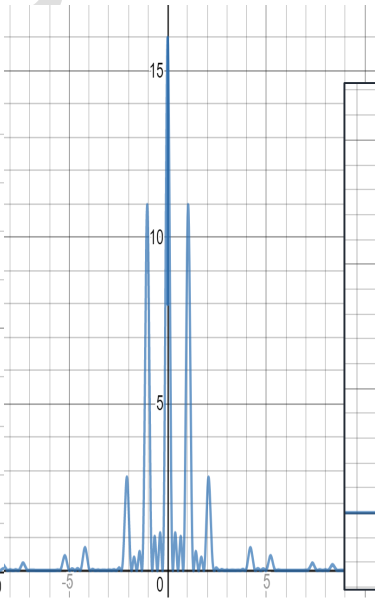
$b:d = 1:2$, N=10



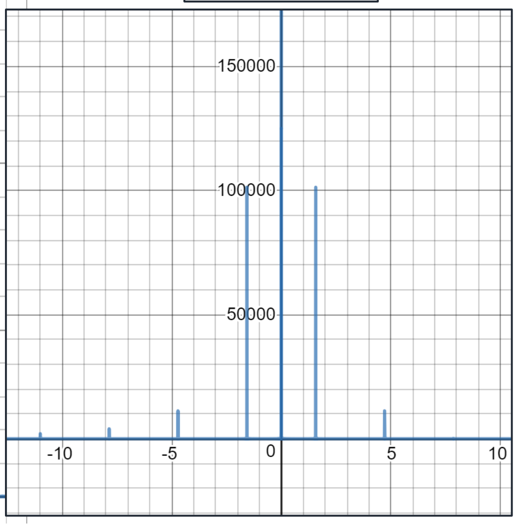
$b:d = 1:3$, N=2



$b:d = 1:3$, N=3



$b:d = 1:3$, N=4



$b:d = 1:2$, N=500

Diffraction grating is an arrangement consisting of a large number of parallel slits of same width and separated by equal opaque space. Grating may be two types:

(1) Transmission grating.
(2) Reflection grating.

A plane transmission grating is a well-polished thin uniform optically plane transparent glass plate on which equidistance, extremely close lines (ruling) are drawn with a sharp diamond point.

When the rulings are made on a polished metal surface, it is called Reflection grating.

Note that **d** is called **grating element or grating constant and $\frac{1}{d}$ is number of lines per unit length of grating. Some author take $d = b + a$ where b is slit width and a is opaque space width.**

Errors in grating and ghosts line:

Practically no grating is perfect. The imperfectness in ruling of any diffraction grating causes some errors. the error may **periodic** or **random** or **progressive** in nature.

Ghosts line are defined as **false spectral lines** arising from **periodic errors** in in ruling of any diffraction grating.

If the **error is random** the grating gives a continuous background illumination.

If the **error is progressive** in nature the spectral lines becomes sharper in planes which are different from the focal plane of the optical system