WEST BENGAL STATE UNIVERSITY
B.Sc. Honours 6th Semester Examination, 2024

PHSADSE04T-PHYSICS (DSE3/4)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No.1 and any *two* questions from the rest

1. Answer any ***fifteen*** questions from the following: $2\times15=30$

(a) Show that for $a, b \in G$, each of the equations $a \circ x = b$ and $y \circ a = b$ has a unique solution.

(b) Prove that the identity of a sub-group is the same as that of the group.

(c) Prove that a group of order 3 is always a cyclic group.

(d) If N is the set of all positive integers, then show that division and subtraction are not binary operations in N.

(e) Show that $SU(2)$ is not an abelian group in general.

(f) Does the set of all orthogonal 3×3 matrices with determinant $=-1$ forms a group? If so, what is the unit element?

(g) $x+y=10$ is defined in natural number N. Show that this relation is not transitive.

(h) Show that the two cosets are identical if a single common element occurs.

(i) If H and K be subgroups of G and $x, y \in G$ with $Hx = Ky$. Then show that $H = K$.

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(j) Show that the centre of a group, $\mathbb{Z}(G, \ )$ forms a subgroup.

(k) X and Y stand in a line at random with 10 other people. What is the probability that there are 3 people between X and Y?

(l) Let A and B be independent events with $P(A)=0.3$ and $P(B)=0.4$. Find $P(A\cap B)$ and $P(A\cup B)$.

(m) Find the probability of at most 5 defective fuse to be found in a box of 200 fuses if experience shows that 2% of such fuses are defective.

(n) Arithmetic mean and standard derivation of a binomial distribution are respectively 4 and $\sqrt{8/3}$. Find the probability of success in each trial.

(o) A random variable x follows Poisson distribution with parameter 3. Find the probability that the variable assumes the value less than 3. Given $e^{-3}=0.0498$.

(p) Show that Laplace equation is an elliptic partial differential equation.

(q) Write down the limitations of separation of variables technique of solving partial differential equation.

(r) Find the particular integral of the differential equation

$$\frac{\partial^2 u}{\partial x^2}-\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}-2\frac{\partial^2 u}{\partial y^2}=(y-1)e^x$$

(s) Find partial differential equations satisfied by the function $u(x, y) = f(x^2 - y^2)$ for all arbitrary functions f.

(t) Show that the equation $\left(a^2 \frac{\partial^2}{\partial x^2} - b^2 \frac{\partial^2}{\partial y^2}\right)\phi(x, y) = 0$ can be expressed as the product of two linear partial differential equations with real coefficients.

2. (a) Prove that the order of a subgroup of a finite group must be an integral divisor of the order of a group. 3+4+(2+1)

(b) (i) Construct the group multiplication table for the following six functions.

$$f_1 = x \ , \ f_2 = \frac{1}{1-x} \ , \ f_3 = \frac{x-1}{x} \ , \ f_4 = \frac{1}{x} \ , \ f_5 = 1-x \ , \ f_6 = \frac{x}{x-1}$$

(ii) Find out its classes. Name it's proper subgroups.

3. (a) If the mapping f is a homomorphism from a group G to G' and if e and e' be their respective identities then prove that $f(e) = e'$. 3+3+2+2

(b) Find the permutation group isomorphic to the multiplicative group $G = \{1, -1, i, -i\}$.

(c) Express the permutation

$$P = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 6 & 4 & 1 & 2 & 5 \end{bmatrix}$$ as the product of disjoint cycles.

(d) If $(R, \circ)$ be a multiplicative group $x \in R$, then find homomorphism and their kernels in the mapping $x \rightarrow |x|$.

4. (a) Use the method of separation of variables to solve 4+(2+2)+2

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u \quad , \quad \text{given } u(0, t) = 4e^{-3t}$$

(b) Show that the mean and standard deviation of a binomial distribution are given by np and $\sqrt{npq}$ respectively. The symbols have their usual significance.

(c) State the condition when a binomial distribution can be approximated to normal distribution.

5. (a) Using the method of separation of variables solve the one dimensional heat flow equation 5+4+1

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{K}\frac{\partial u}{\partial t},$$ under the following boundary conditions

(i) $u(0, t) = u(l, t) = 0 \quad , \quad t > 0$

(ii) $u(x, 0) = \dfrac{x(l-x)}{l^2} \quad , \quad 0 < x < l$.

(b) Consider two successive events A and B. Let $P(A)$ be the probability that A occurs and $P(AB)$ be the probability that both A and B; and $P(B/A)$ be the conditional probability that B will occur if we know that A has occured. Establish the Bayes' Formula, i.e., the relation among the above probabilities.

(c) What is the form of the above relation when A and B are independent events?

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Answer Question No.1 and any *two* questions from the rest

1. Answer any ***fifteen*** questions from the following: $2\times15=30$

(a) Show that the identity element in a group is unique.

(b) Prove that the order of an element 'a' of a group is same as that of its inverse i.e., 'a^{-1}'.

(c) If 'a' is conjugate to 'b', 'b' is conjugate to 'c' then show that 'a' is conjugate to 'c'.

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(d) If $A=\{1, 2, 3\}$, then write down the non-empty subsets of A.

(e) Show that the set S of all integers does not form a group under the operation defined by

$$a*b=a-b \quad \forall\, a, b\in S .$$

(f) Let $f:N\rightarrow N: f(x)=2x$ for all x in N. Check whether f is a bijection when N is the set of all positive integers.

(g) Show that the set $U(n)$ of all unitary matrices of order n, where n is a fixed finite positive integer forms group under matrix multiplication.

(h) Show that a set of nonzero rational numbers is an ablian group under multiplication.

(i) When can a binary relation be called an equivalence relation?

(j) State whether the permutation $p=\begin{bmatrix}1 & 2 & 3 & 4\\ 2 & 3 & 4 & 1\end{bmatrix}$ is odd or even permutation.

(k) A committee is formed from two men and two women. What is the probability that the committee will have no man?

(l) A problem in mechanics is given to three students A, B and C whose chance of solving it are $1/2$, $1/3$ and $1/4$ respectively. What is the probability that the problem will be solved?

(m) A biased six-sided dice has probabilities $\frac{p}{2}, p, p, p, p, 2p$ of getting 1, 2, 3, 4, 5, 6 respectively. Calculate p.

(n) The probability density function $\psi(x)$ of a continuous random variable x is defined by the relation

$$\psi(x)=\begin{cases} A/x^3 & \text{for } 5\le x\le 10 \\ 0 & \text{elsewhere}\end{cases}$$

Evaluate A.

(o) If $P(A)=0.3$, $P(A\cup B)=0.6$, where A and B are independent then find $P(B)$.

(p) The mean and standard deviation of a binomial distribution are 10 and 2 respectively. Find the probability of success in each trial.

(q) A random variable x has a probability density function $f(x)=e^{-x}$ in the interval $0<x<\infty$, and zero elsewhere. Find the value of probability that x lies in the interval $1\le x\le 2$.

(r) Show that the equation $\frac{\partial^2 u}{\partial t^2}-c^2\frac{\partial^2 u}{\partial x^2}=0$ is a hyperbolic differential equation.

(s) Find the solution of the equation $\frac{\partial u}{\partial t}+3\frac{\partial u}{\partial x}=4x$ with $u(x,0)=6e^{-3t}$.

(t) Show that the function, $U(x,y)=f(x+iy)+g(x-iy)+x^3+y^3$ satisfies the partial differential equation $\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}=6(x+y)$ where f and g are arbitrary functions and $i=\sqrt{-1}$.

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2. (a) Show that the set of four 2×2 matrices given by 4+1

$$A=\begin{pmatrix}1&0\\0&1\end{pmatrix}, B=\begin{pmatrix}0&1\\-1&0\end{pmatrix}, C=\begin{pmatrix}-1&0\\0&-1\end{pmatrix}, D=\begin{pmatrix}0&-1\\1&0\end{pmatrix}$$ forms a group of order four under multiplication and the group is abelian.

(b) Prove that the elements common to two intersecting subgroups H_1 and H_2 form a subgroup of the group G. 3

(c) Prove that a group where order is a prime number must be a cyclic group. 2

3. (a) Show that a non-empty subset H of a group G is a subgroup of G if $a\in H$ and $b\in H$ implies $ab^{-1}\in H$. 3

(b) Show that the mapping $f:(Z,+)\to(2Z,+)$ such that $f(x)=2x$, $\forall x\in Z$ is an isomorphism from Z to $2Z$, where Z is the set of integers. 3

(c) A density function is defined by 2+2

$$f(x)=\begin{cases} e^{-x}, & x\ge 0\\ 0, & x<0\end{cases}.$$

(i) Determine the probability that the variate having this density will fall in the interval (1, 2).

(ii) Find the cumulative probability function $F(2)$.

4. (a) What are Dirichlet and Neumann boundary conditions? 2

(b) What are characteristic curves or characteristics of partial differential equations? 2

(c) An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of a scooter, car and a truck meeting an accident are 0.01, 0.03, 0.15 respectively. If one of the insured persons meets with an accident, find the probability that he is a scooter driver. 4

(d) Show that every subgroup of an abelian group is a normal subgroup. 2

5. (a) Solve $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x\partial y}-6\frac{\partial^2 z}{\partial y^2}=y\cos x$ 5

(b) The Gaussian distribution with mean μ and standard deviation σ is given by 3

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Find $<x>$ and $<x^2>$.

(c) Show that the function $u=f\left(\frac{x}{y}\right)$ satisfies the equation $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$. 2

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Answer Question No.1 and any *two* questions from the rest

1. Answer any ***fifteen*** questions from the following: $2\times 15 = 30$

(a) $x+y=10$ is defined in natural number N. Show that this relation is not *transitive*.

(b) Show that the centre of a group, $Z(G)$, forms a subgroup.

(c) Show that Inverse of an element in a subgroup is the inverse of the element of the group.

(d) Show that two left cosets of a subgroup H either coincide completely, or else have no elements in common at all.

(e) Show three cube roots of unity form an abelian group under multiplication.

(f) Give irreducible representation of SU(2) group.

(g) Prove that the group of order two is always cyclic.

(h) A random number $\tilde{x}$ is distributed uniformly between [0, 1]. Find its variance.

(i) A book of 1000 pages contains 20 typographical errors. What is the probability that there is at least one error in a single page?

(j) In a one dimensional random walk, the probability of moving one step in the right is p for each step. Find the probability of reaching the starting point by a random walker after taking $2n$ number of steps.

(k) Find the standard deviation of the uniform distribution $f(x)=\frac{1}{n}$; $(x=0, 1, 2, \cdots, n)$.

(l) Let x be distributed in the Poisson form. If $P(x=1)=P(x=2)$; Find the expectation value.

(m) Four coins are tossed simultaneously. Find the probability of obtaining 2 heads and 2 tails.

(n) Show that total area under a normal curve is unity.

(o) State the condition when a binomial distribution can be approximated to normal distribution.

(p) Let a homomorphism $f:(G,\bullet)\rightarrow(H,\bullet)$. Show that $f(e_G)=e_H$, where e_I is the identity element of group I.

(q) State the nature of the equations:

(i) $4\dfrac{\partial^2 U}{\partial x^2}+\dfrac{\partial^2 U}{\partial y^2}=0$ (ii) $\dfrac{\partial^2 U}{\partial x^2}-2\dfrac{\partial^2 U}{\partial x\partial y}+\dfrac{\partial^2 U}{\partial y^2}=0$

(r) Find the solution of one dimensional Laplace's equation, where at the boundaries, the solution $\psi(x=0)=\psi(x=l)=0$.

(s) Determine the condition under which the following differential equation can be solved by the method of separation of variables:

$$C_1\frac{\partial\phi(x,t)}{\partial t}+C_2\nabla^2\phi(x,t)+V(x,t)\phi(x,t)=0,$$ where C_1 and C_2 are constants.

(t) Show that the equation $\left[a^2\dfrac{\partial^2}{\partial x^2}-b^2\dfrac{\partial^2}{\partial y^2}\right]\phi(x,y)=0$ can be expressed as the product of two linear partial differential equations with real coefficients.

2. (a) A multiplication table of a Group consists 6 elements is given below 3+2+2

e	a_1	a_2	a_3	a_4	a_5
a_1	e	a_4	a_5	a_2	a_3
a_2	a_5	e	a_4	a_3	a_1
a_3	a_4	a_5	e	a_1	a_2
a_4	a_3	a_1	a_2	a_5	e
a_5	a_2	a_3	a_1	e	a_4

(i) Identify the two elements and 3 elements subgroups

(ii) Find the left cosets of any one 2 elements subgroup.

(iii) Show that three elements subgroup is the invariant subgroup.

(b) Show that the set $\{1,-1,i,-i\}$ forms a cyclic group for multiplication. Find its generator. 3

3. (a) Using the method of least squares, fit a straight line to the four points: 4

x	1	2	3	4
y	1.7	1.8	2.3	3.2

(b) Let f be a homomorphism from $G\rightarrow G'$. Denote by K the set of all elements of G which are mapped to the identity element of G', Then K forms an invariant subgroup of G. Prove it. 3

(c) Statistical average of some function $f(x)$ is defined as 3

$$\langle f(x)\rangle=\sum_i f(x_i)P_i.$$ Show that $$\left(\frac{d^k}{dt^k}\langle e^{tx}\rangle\right)_{t=0}=\langle x^k\rangle$$

4. (a) Prove that Poisson distribution may be obtained as a limiting case of Binomial distribution. 3

(b) In a bolt factory, machines A_1, A_2 and A_3 manufacture respectively 25, 35 and 40% of the total. Of their output 5, 4 and 2% are defective bolts. A bolt is drawn at random and found it defective. What is the probabilities that it was manufactured by the machine A_1, A_2 or A_3? 4

(c) Deduce the expression for mean of a binomial distribution. 3

5. (a) Solve: $\dfrac{\partial^2 T}{\partial x^2}+\dfrac{\partial^2 T}{\partial y^2}=0$ given 4

$$T(0, y)=0\,;\; T(x, \infty)=0$$

$$T(a, y)=0\,;\; T(x, 0)=\sin\left(\frac{\pi x}{a}\right)$$

(b) Let U and V are two solutions of Laplace's equation. If both of them satisfy either Dirichlet or Neumann boundary condition, then show that they are at most differ by a constant otherwise identical. 3

(c) Show that following transformation forms a group under multiplication, 3

$$\begin{pmatrix} ct' \\ x' \end{pmatrix}=\begin{pmatrix} \cosh\Psi & \sinh\Psi \\ \sinh\Psi & \cosh\Psi \end{pmatrix}\begin{pmatrix} ct \\ x \end{pmatrix}$$

Ignore the fact that range of Ψ, $(-\infty, \infty)$ is not finite.

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Answer Question No.1 and any *two* questions from the rest

1. Answer any ***fifteen*** questions from the following: $2\times 15 = 30$

(a) For a Poisson distribution; $P(x=0)=P(x=1)$ find the $P(x>0)$.

(b) What do you mean by proper subset? Explain with an example.

(c) Distinguish between Onto and Into mapping.

(d) If N is the set of all positive integers, then show that division and subtraction are not binary operations in N.

(e) When can a binary relation be called an equivalence relation?

(f) For what value of a will be the function $f(x)=ax$; $x=1,2,3,\ldots n$ be the probability mass function of a discrete random variable x?

(g) Show that the set of matrices $A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, where α is real forms a group under multiplication.

(h) Prove that if event A and B are independent, $\overline{A}$ and $\overline{B}$ are also independent.

(i) Show that the identity of a subgroup of a group is the same as that of the group.

(j) Show that the intersection of any two normal subgroups of a groups is a normal subgroup.

(k) If $(R,\cdot)$ be a multiplicative group and $x\in R$, then find homomorphisms and their kernels in the mapping $x\rightarrow |x|$.

(l) What do you mean by faithful representation? Give an example.

(m) If in an experiment with random outcome the number of outcomes favouring an event A is less than the number of outcomes favouring event B in a large number of trials N, show that $P(A)<P(B)$. The symbols have their usual meaning.

(n) A dice is rolled six times. Find the probability that each of the six faces appears only once.

(o) If $f(x)=e^{-x}$, $0\le x\le\infty$ show that it is a probability density function and find it's mean.

(p) Write down the limitations of separation of variable technique of solving partial differential equation.

(q) Explain with example, what are homogeneous and non-homogeneous partial differential equations.

(r) Show that if there exists a relation among the three variables x, y, z like $f(x, y, z) = 0$, then show that $\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$.

(s) For what value of x and y the equation

$$(y+1)\frac{\partial^2 u}{\partial x^2} + 2x\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = x + y$$

is parabolic?

(t) Write down the Laplace's equation in spherical polar coordinates.

2. (a) Define order of an element of a group. Prove that the order of an element a of a group is the same as that of its inverse a^{-1}. 3

(b) Show that the group formed by the set $(1, \omega, \omega^2)$, ω being cube root of unity, is a cyclic group of order 3 with respect to multiplication. 2

(c) Write down orthogonality theorem for the Irreducible representation of a group and prove it. 2+3

3. (a) Find the regular permutation group isomorphic to the group $G = (a, b, c, d)$ with the composition table. 3

(b) If a matrix commutes with all the matrices of an irreducible representation, then show that it is a multiple of unit matrix. 4

(c) For any two events A and B, the probability that either A or B or both occur is given by $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. 3

4. (a) Consider the Gaussian distribution $\rho(x) = Ae^{-\lambda(x-a)^2}$, where A, a, λ are constants. Determine σ^2. 4

(b) If A and B are two events such that $P(A) = \frac{3}{8}$, $P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$, find $P\left(\frac{A}{B}\right)$ and $P\left(\frac{B}{A}\right)$. Are A and B independent? 3

(c) If X denotes a random variable having binomial distribution with mean 6 and variance 3. Obtain $P(X \geq 1)$. 3

5. (a) Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ where $u = 0$ for $t = \infty$ and for $x = 0$ or 1. 4

(b) Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2} = 0$ for $0 < x < \pi$, $0 < y < \pi$ subject to the conditions $u(0, y) = u(\pi, y) = u(x, \pi) = 0$ and $u(x, 0) = \sin^2 x$. 6

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