P. R. Thakur Govt. College
Department of Physics
B.Sc. Semester-VI Internal Examination, 2024
Paper - **PHSADSE04T**
Advanced Mathematical Physics - II

Time: 1 Hour **Full Marks: 20**

1. Answer any *five* questions from the following: 2 x 5=10

   (a) Prove that every subgroup of an abelian group is normal.

   (b) Construct the multiplication table for the group formed by the set of positive integers less than 5 and relatively prime to 5 under multiplication modulo 5.

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   (c) Show that the equation $\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0$ is a hyperbolic differential equation.

   (d) Show that the centre $Z(G)$ of a group G forms a subgroup.

   (e) Let t be an arbitrary but fixed element of a group G. Then prove that the map $\phi_t : G \to G$ defined by $\phi_t(g) = tgt^{-1}$ for $g \in G$ is a homomorphism.

   (f) Write down a 2-dimensional representation of the group $SU(2)$.

   (g) A dice is rolled five times. Find the probability that exactly three times 1 is obtained.

   (h) A random variable X is distributed as $f(x) = Ae^{-x}$ for $0 < x < \infty$ and 0 elsewhere. Calculate the probability that x lies between 1 and 2.

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2. Answer any *two* questions from the following: 5 x 2=10

   (a) Define a character map on a group. Prove that the conjugate elements of a group have the same character. 2+3

   (b) Prove that a group of prime order is Abelian.

   (c) State and prove Schur's second lemma.

   (d) Find the most general form of a 2 x 2 special orthogonal matrix.

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Paper - **PHSADSE04T**

Advanced Mathematical Physics - II

Time: 1 Hour **Full Marks: 20**

1. Answer any *five* questions from the following: 2 x 5=10

(a) What is an equivalance relation on a set?

(b) A book having 1000 pages contains 10 typographical errors. What is the probability that a single page chosen at random contains at least one error?

(c) A dice is rolled six times. Find the probability that at least one of the faces appears more than once.

(d) State the condition when the binomial distribution reduces to the Poisson distribution.

(e) Show that the set of matrice $A_\alpha = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$, where α is a real number, forms a group under matrix multiplication.

(f) The cumulative distribution for the standard Gaussian is given by $\Phi(z) = Pr(Z < z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{u^2}{2}} du$. Prove that $\Phi(-z) = 1 - \Phi(z)$.

(g) If two events A and B are independent, prove that $\bar{A}$ and $\bar{B}$ are also independent.

(h) Let x be a Poisson-distributed random variable. If $P(x = 1) = P(x = 2)$, find the average value of x.

2. Answer any *two* questions from the following: 5 x 2=10

(a) Define subgroup of a group. Write down the necessary and sufficient condition on a subset H of a group G to be a subgroup of G. Hence prove that $SL_n(\mathbb{R})$ is a subgroup of $GL_n(\mathbb{R})$. 1+2+2

(b) i. If A and B are two events such that $P(A) = \frac{3}{8}$, $P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$. Find $P(A|B)$ and $P(B|A)$. Are A and B independent? 2+1

ii. Prove that for a random variable x, the mean value of x^k is given by $\langle x^k \rangle = \left[\frac{d^k}{dt^k}\langle e^{tx}\rangle\right]_{t=0}$. 2

(c) Define order of an element of a group. State and prove rearrangement theorem in connection to a group. Write down the multiplication table for the group formed by the cube roots of unity. 1+2+2

(d) Solve Poisson's equation in 3-dimensions $\nabla^2 f(\vec{r}) = g(\vec{r})$ using Green function and assuming natural boundary condition $f(\vec{r}) \to 0$ as $|\vec{r}| \to \infty$.

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Department of Physics

B.Sc. Semester-VI Internal Examination, 2022

Paper - **PHSADSE04T**

Advanced Mathematical Physics - II

Time: 1 Hour **Full Marks: 20**

1. Answer any *five* questions from the following: 2 x 5=10

(a) What is an equivalance relation on a set?

(b) Let t be an arbitrary but fixed element of a group G. Then prove that the map $\phi_t : G \to G$ defined by $\phi_t(g) = tgt^{-1}$ for $g \in G$ is a homomorphism.

(c) Derive the multiplication table for the group formed by the set of positive integers less than 5 and relatvely prime to 5 under multiplication modulo 5.

(d) Show that the map $\phi : GL_n(\mathbb{R}) \to \mathbb{R}\backslash\{0\}$ defined by $\phi(M) = det(M)$ is a homomorphism.

(e) Given a group G of non-singular $n \times n$ matrices ($n > 1$), construct a 1-dimensional representation of G.

(f) Let $\phi : G \to G^*$ be a homomorphism. Prove that if H is a subgroup of G, then its image $\phi(H)$ is a subgroup of G^*.

(g) Define centre of a group G. Prove that it is a normal subgroup of G.

(h) Prove that all irreducible representations of an Abelian group are 1-dimensional.

2. Answer any *two* questions from the following: 5 x 2=10

(a) Derive a 2-dimensional representation of D_3, the symmetry group of equilateral triangle, by acting its elements (3 reflections & 3 rotations) on the standard basis $\{e_1, e_2\}$ of the vector space $\mathbb{R}^2$.

(b) What is a cyclic group? Let G be a group and let g be a fixed element of G. Prove that $gG = Gg = G$. 2+3

(c) State and prove Schur's first lemma.

(d) Define a character map on a group. Prove that the conjugate elements of a group have the same character. 2+3

P. R. Thakur Govt. College

Department of Physics

B.Sc. Semester-VI Internal Examination, 2021

Paper - PHSADSE04T

Advanced Mathematical Physics - II

Time: 1 Hour **Full Marks: 20**

1. Answer any *five* questions from the following: 2 x 5=10

   (a) What is an equivalance relation on a set?
   (b) Derive the multiplication table for the group formed by the solutions of the equation $z^3 = 1$ under ordinary multiplication where z is a complex number.
   (c) Prove that the sufficient condition for a subset H of a group G to be a subgroup of G is that along with each pair of elements x and y in H, the product xy^{-1} is in H.
   (d) Let $\psi : G \to G'$ be a homomorphism of a group G into the group G'. Show that $Ker(\psi)$, the kernel of the map ψ is a normal subgroup of G.
   (e) Given a group G of non-singular $n \times n$ matrices $(n > 1)$, construct a 1-dimensional representation of G.
   (f) Define inner automorphism on a group.
   (g) Define centre of a group G. Prove that it is a normal subgroup of G.
   (h) Prove that all irreducible representations of an Abelian group are 1-dimensional.

2. Answer any *two* questions from the following: 5 x 2=10

   (a) Derive a 2-dimensional representation of D_3, the symmetry group of equilateral triangle, by acting its elements (3 reflections & 3 rotations) on the standard basis $\{e_1, e_2\}$ of the vector space $\mathbf{R}^2$.
   (b) Prove that a group of prime order is Abelian.
   (c) State and prove Schur's first lemma.
   (d) State the character orthogonality theorem for a group G. Prove that # of irreps of $G \leq$ # of equivalance classes of G. 2+3