P. R. Thakur Govt. College
Department of Physics
B.Sc. Semester-V Internal Examination, 2024-25
Paper - **PHSADSE02T**
Advanced Dynamics

Time: 1 Hour **Full Marks: 20**

1. Answer any *five* questions from the following: 2 x 5=10

   (a) Show that the conjugate momentum, corresponding to a cyclic coordinate in the Lagrangian, is conserved.

   (b) If all the generalised coordinates $q_1, q_2, \ldots, q_n$ of a system become cyclic, what will be the time dependence of those coordinates?

   (c) If $f(q,p)$ is a dynamical variable in phase space, then show that $\frac{df}{dt} = \{f, H\}$, where H is the Hamiltonian.

   (d) Show that the transformation $P = \frac{1}{2}(p^2 + q^2)$ and $Q = \tan^{-1}(\frac{q}{p})$ is canonical.

   (e) Define Euler's angles for the orientation of a rigid body.

   (f) Derive the equation of continuity for an incompressible fluid.

   (g) Define Reynold number. How does the estimation of Reynold number help determine whether a motion is turbulent or streamline?

   (h) Write down the dimensions of coefficient of viscosity and surface tension.

Answer any *two* questions from the following: 5 x 2=10

2. Find the canonical transformation starting from the generating function $F_1(q,Q) = \mu q^2 \cot Q$, where μ is a constant. If the Hamiltonian in (q,p) representation is uuu

$$H(q,p) = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2,$$

   find the Hamiltonian in (Q,P) representation. Determine μ so that this Hamiltonian independent of Q. 2+2+1

3. Show that Poisson bracket of two phase space variables remains invariant under canonical transformation.

4. (a) What do you mean by normal modes of vibration? Explain the meaning of normal coordinates and normal frequencies. 1+1+1

   (b) Potential energy of a particle moving along the x-axis is given by $U = \frac{1}{3}x^3 + \frac{9}{2}x^2 + 20x$. Find the position of stable equilibrium state. 2

5. (a) A rigid body with one point fixed rotates with angular velocity $\vec{\omega}$ and has an angular momentum $\vec{L}$. Show that its kinetic energy is $\frac{1}{2}\vec{L}\cdot\vec{\omega}$.

   (b) Derive Euler's equations of motion for a rigid body with a fixed point. 2+3