WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2023-24

PHSADSE01T-PHYSICS (DSE1/2)

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* questions from the rest

1. Answer any ***ten*** questions from the following: $2\times 10=20$

(a) Prove that if $L[f(x)]=\bar{f}(p)$, then $L[e^{ax}f(x)]=\bar{f}(p-a)$.

(b) Prove that δ^r_q is a mixed tensor of the second rank.

(c) State the quotient law of tensor.

(d) Define unit step function and find its Laplace transformation.

(e) Find $L[x\sinh(ax)]$.

(f) For which value of R, the vector $\vec{u}=(1,-2,R)$ in $\mathcal{R}^3$, will be a linear combination of the vectors $\vec{v}=(3,0,2)$ and $\vec{w}=(2,-1,5)$.

(g) Define the subspace of a vector space.

(h) Evaluate $L\{t^n\}$ if n is positive integer.

(i) If $\Phi=a_{jk}A^jA^k$, show that it is always possible to write $\Phi=b_{jk}A^jA^k$, where b_{jk} is a symmetric tensor.

(j) Evaluate: $L^{-1}\left[\dfrac{2}{(s-4)^2+4}\right]$

(k) Show that the operator $|\phi\rangle\langle\phi|$ is a projection operator only when $|\phi\rangle$ is normalized.

(l) If $L[f(t)]=\tilde{f}(s)$, then show that $L[f(at)]=\frac{1}{a}\tilde{f}(\frac{s}{a})$, where a is a constant.

(m) Define inner-product of a pair of vectors. What is norm of a vector?

(n) If a vector $|\alpha\rangle\in\mathbb{R}^3$ in standard basis is $|\alpha\rangle=(3,2,-1)$, find its components in the new basis $\{(1,1,1),(1,1,0),(1,0,0)\}$.

2. (a) Find the eigen-values and eigen-vectors (Normalized) of the matrix $A=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$. Check whether the eigen-vectors are orthogonal or not. 2+3+1

(b) Given, $\psi_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\psi_2 = \begin{bmatrix} +1 \\ 0 \\ 1 \end{bmatrix}$, $\psi_3 = \begin{bmatrix} +1 \\ -2 \\ -1 \end{bmatrix}$; construct an orthonormal set using Gram-Schmidt process. 4

3. (a) Find the Laplace transform of a periodic function. 5

$$f(t) = \begin{cases} \sin \omega t & , \quad 0 < t < \pi/\omega \\ 0 & , \quad \pi/\omega < t < 2\pi/\omega \end{cases}$$

where $f(t + 2\pi/\omega) = f(t)$.

(b) Use Convolution theorem to evaluate $L^{-1}\left[\dfrac{s}{(s^2 + a^2)^2}\right]$. 5

4. (a) Prove that $L[x^n f(x)] = (-1)^n \dfrac{d^n}{ds^n}[\tilde{f}(s)]$, where $L[f(x)] = \tilde{f}(s)$. 3

(b) If A^j and B^j are two non-null vectors such that $g_{ij}U^iU^j = g_{ij}V^iV^j$, where $U^i = A^i + B^i$ and $V^i = A^i - B^i$, show that A^j and B^j are orthogonal. 3

(c) Using tensor notation establish the vector identity: 4

$$\vec{\nabla} \cdot (\vec{A} \cdot \vec{B}) = \vec{A} \times (\vec{\nabla} \times \vec{B}) + \vec{B} \times (\vec{\nabla} \times \vec{A}) + (\vec{A} \cdot \vec{\nabla})\vec{B} + (\vec{B} \cdot \vec{\nabla})\vec{A}$$

5. (a) Prove that $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$ using tensor notation for an arbitrary vector $\vec{A}$. 2

(b) Distinguish between the covariant and contravariant component of a vector using their transformation rule. 2

(c) Show that velocity is a contravariant vector while gradient of a scalar is a covariant vector. 3

(d) Show that the contraction of the outer product of tensors A_j and B_k results in an invariant. 3

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2022-23

PHSADSE01T-PHYSICS (DSE1/2)

ADVANCED MATHEMATICAL PHYSICS-I

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any *ten* questions from the following: $2\times 10=20$

(a) Define isomorphism of vector space.

(b) Consider the following two kets

$$|\psi\rangle=\begin{pmatrix}-3i\\ 2+i\\ 4\end{pmatrix} \text{ and } |\phi\rangle=\begin{pmatrix}2\\ -i\\ -3i\end{pmatrix}$$

Evaluate the scalar product $\langle\phi|\psi\rangle$.

(c) Write in full the following

$$g_{ij}\,dx^i\,dx^j \quad (i,\,j=1,\,2,\,3)$$

(d) Show that $f_1(x)=Ae^{mx}$ and $f_2(x)=Be^{nx}$ $(m\neq n)$ are linearly independent.

(e) Show that the operator $|\phi\rangle\langle\phi|$ is a projection operator only when $|\phi\rangle$ is normalized.

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(f) Write down the Minkowski metric in the mixed tensor form showing the components.

(g) Prove that every vector in a finite dimensional vector space V over the field F can be uniquely expressed as a linear combination of the basis vectors.

(h) Define Laplace transform of a function. What conditions are to be satisfied for the transform to exist?

(i) Evaluate $L[e^{-2x}(2\cos 5x-3\sin 5x)]$

(j) Find the metric for the three-dimensional Euclidean space in terms of spherical polar coordinates.

(k) Find $L^{-1}\left[\dfrac{1}{\sqrt{s+2}}\right]$.

(l) State the convolution theorem of Laplace transform.

(m) Show that for Laplace inverse transform

$$L^{-1}\{c_1 f_1(p)+c_2 f_2(p)\}=L^{-1}\{c_1 f_1(p)\}+L^{-1}\{c_2 f_2(p)\}$$

(n) Any arbitrary second rank tensor M_{ij} can be written as a sum of a symmetric and an antisymmetric tensor.

2. (a) Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 3 & i \\ -i & 3 \end{pmatrix}$. 2+2+2

Check whether the eigenvalues are real and the eigenvectors are orthogonal. Find the normalized eigenvectors of the same.

(b) Prove that $\nabla^2\phi=\dfrac{1}{\sqrt{g}}\dfrac{\partial}{\partial x^k}\left(\sqrt{g}\, g^{kr}\dfrac{\partial\phi}{\partial x^r}\right)$ 4

3. (a) Solve $\dfrac{dy}{dx}+2y=\cos x$, given $y(0)=1$, using Laplace transform method. 4

(b) Find the Laplace transform of the Heaviside function: 2

$$H(t-a)=\begin{cases}0 & \text{for } t<a, \\ 1 & \text{for } t\geq 0,\ a\geq 0\end{cases}.$$

(c) Prove that $L[f(t)]=\dfrac{1}{1-e^{-pt}}\displaystyle\int_0^T e^{-pt}f(t)dt$ where, $f(t+T)=f(t)$. 4

4. (a) Find a unit vector orthogonal to 2

$$\alpha_1=\begin{bmatrix}0\\2\\1\end{bmatrix} \text{ and } \alpha_2=\begin{bmatrix}3\\1\\2\end{bmatrix}$$

(b) Prove that $\delta^i_j a^{jk}=a^{ik}$. 2

(c) Show that $\varepsilon_{ijk}\varepsilon_{ijk}=6$. 3

(d) Consider the states $|\psi\rangle=3i|\phi_1\rangle-7i|\phi_2\rangle$ and $|\chi\rangle=-|\phi_1\rangle+2i|\phi_2\rangle$, where $|\phi_1\rangle$ and $|\phi_2\rangle$ are orthonormal. Show that the states $|\psi\rangle$ and $|\chi\rangle$ satisfy the Schwarz inequality and triangle inequality. 3

5. (a) Expand $\phi=a_{ij}u^i v^j$, where $i, j=1,2,3$ assuming Einstein summation convention. 2

(b) Show that $\delta^i_i=n$ where, n is the dimension. 2

(c) Prove that every basis of a finite dimensional vector space has the same number of vectors. 4

(d) Show that $\dfrac{\partial A_p}{\partial x^q}$ is not a tensor even though A_p is a covariant tensor of rank one. 2

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2021-22

PHSADSE01T-PHYSICS (DSE1/2)

ADVANCED MATHEMATICAL PHYSICS-I

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any ***ten*** questions from the following: $2\times10=20$

(a) Prove that the contraction of tensors A_q^p is an invariant.

(b) If $\varphi=a_{jk}A^jA^k$ then show that one can write $\varphi=b_{jk}A^jA^k$, where b_{jk} is symmetric.

(c) Show that the Kronecker delta is a mixed tensor of order two.

(d) For an orthogonal basis prove that norm of a nonzero vector is positive definite.

(e) Why is Laplace transformation a linear operation?

(f) Show that the Laplace transform of the integral of $f(x)$, i.e. $L\left[\int_0^x f(x)dx\right]=\frac{1}{p}\bar{f}(p)$, where $L[f(x)]=\bar{f}(p)$.

(g) Prove that every vector in a finite-dimensional vector space V over the field F can be uniquely expressed as a linear combination of the vectors of its basis.

(h) Show that the vectors $(2,-5,3)$ cannot be expressed as a linear combination of the vectors $\alpha_1=(1,-3,2)$, $\alpha_2=(2,-4,-1)$ and $\alpha_3=(1,-5,7)$.

(i) Show that the four matrices $E=\begin{bmatrix}1&0\\0&1\end{bmatrix}$, $A=\begin{bmatrix}0&1\\-1&0\end{bmatrix}$, $B=\begin{bmatrix}-1&0\\0&-1\end{bmatrix}$ and $C=\begin{bmatrix}0&-1\\1&2\end{bmatrix}$ from a group under matrix multiplication.

(j) Find Laplace transform of $f(x)$, where $f(x)=\begin{cases}0\ , & x<a\\ 1\ , & x>a\end{cases}$.

(k) Find Laplace transform of $e^{4x}\sin(2x)\cos(x)$.

(l) Examine if the following operator is linear:

$$Af(x)=x^2f(x)$$

(m) Evaluate $L^{-1}\left[\frac{1}{s-2}+\frac{2}{s+5}+\frac{6}{s^4}\right]$.

2. (a) Show that the velocity at any point of a fluid $\frac{dx^k}{dt} = v^k$ is a tensor but acceleration $\frac{dv^k}{dt}$ is not a tensor. 2+1

(b) Using Levi-Civita symbol establish the relation $\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{B}) - \vec{C}(\vec{A}\cdot\vec{B})$. 4

(c) What are stress and strain tensors? Write down the tensorial form of Hooke's law in Elasticity. 3

3. (a) Distinguish between isomorphism and homomorphism in connection with two groups. 2

(b) Define basis and dimension of a vector space. 2

(c) Define orthogonal and orthonormal set of vectors. 2

(d) Show that the sets $S\{(1,\ 0,\ 0),\ (1,\ 1,\ 1),\ (0,\ 1,\ 0)\}$ spans a vector space R^3 but is not a basis set. 4

4. (a) For the set of basis vectors, $(0,\ 2,\ 0,\ 0)$, $(3,\ -4,\ 0,\ 0)$ and $(1,\ 2,\ 3,\ 4)$ use Gram-Schmidt process to construct an orthonormal set. 4

(b) A linear transformation T is defined as $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 - x_3 \end{pmatrix}$ that transforms a vector a 3-D real space to 2-D real space. Show that the transformation matrix is $T = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$. 2

(c) Use the convolution theorem to find the inverse Laplace transform of $\frac{1}{(s^2+4)^2}$. 4

5. (a) If $LT[f(x)] = \bar{f}(s)$, then prove that $LT[f''(x)] = s^2\bar{f}(s) - sf(0) - f'(0)$. 3

(b) Find inverse Laplace transform of $\frac{s+2}{s^2(s+1)(s-2)}$. 4

(c) Using Laplace transform solve the differential equation 3

$$2\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 2y = e^{-2x} \quad \text{with } y(0) = 1,\ \frac{dy}{dx}(0) = 1$$

N.B.: *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2020, held in 2021

PHSADSE01T-PHYSICS (DSE1/2)

ADVANCED MATHEMATICAL PHYSICS I

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
Answer mu be precise and to the point to earn credit
All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any ***ten*** questions from the following: $2\times10 = 20$

(a) Show that Laplace transformation is a linear transformation.

(b) Find the Laplace transform $\mathcal{L}\{f(t)\}$ of the function $f(t)=\begin{cases}1 \text{ for } t\geq t_0 \\ 0 \text{ for } t\leq t_0\end{cases}$, t_0 being a constant.

(c) If $\tilde{f}(s)$ is the Laplace transform of $f(t)$, show that multiplying $f(t)$ by e^{at} moves the origin of s by and amount a.

(d) Find the Laplace transform of $f'(t)$ in terms of that of $f(t)$.

(e) From the definition of Laplace transform, show that $\mathcal{L}\{f(\omega t)\}=\frac{1}{\omega}\tilde{f}\left(\frac{s}{\omega}\right)$, for $\omega>0$, where $\tilde{f}(s)$ is the Laplace transform of $f(t)$.

(f) Show that the vectors $|a\rangle=(2, 1, 1)$; $|b\rangle=(-2, 1, 2)$; $|c\rangle=(0, 0, 1)$ are linearly independent.

(g) What is the significance of a non-singular transformation in a vector space?

(h) If a vector $|a\rangle\in\mathbb{R}^3$ in standard basis is $|a\rangle=(3, 2, -1)$, find its components in the new basis (1, 1, 1), (1, 1, 0), (1, 0, 0).

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(i) When are two finite dimensional vector spaces V and V' isomorphic?

(j) Show that members of an orthogonal set of vectors are linearly independent of each other.

(k) In the relation $L=I\omega$, show using quotient law that I is a tensor of rank two if L and ω are tensors of rank one.

(l) Prove the vector identity $\vec{A}\times(\vec{B}\times\vec{C})=(\vec{A}.\vec{C})\vec{B}-(\vec{A}.\vec{B})\vec{C}$ using properties of Levi-Civita symbol ε_{ijk}.

(m) Show that in three dimensions any anti-symmetric second rank tensor A_{ij} can be expressed in terms of a dual vector.

(n) If v_i are the components of a first rank tensor, show that $\frac{\partial v_i}{\partial x_i}$ is a tensor of rank zero.

2. (a) Find the Laplace transform of the periodic saw-tooth function with period T defined by 4+3+3

$$V(t) = V_0\frac{t}{T}, \text{ for } 0 \le t \le T.$$

(b) In a two-dimensional complex linear vector space, $\{|a\rangle, |b\rangle\}$ from a non-orthogonal normalized basis where inner product of $|a\rangle$ and $|b\rangle$ is given by k. Form another orthonormal basis where $|c\rangle = \alpha|a\rangle + \beta|b\rangle$ is a basis element.

(c) From the defining properties of Krönecker delta, show that it is an invariant second-rank Cartesian tensor.

3. (a) Using tensorial symbols, establish the following vector identity: 4+4+2

$$\vec{\nabla}\times(\vec{A}\times\vec{B}) = (\vec{B}.\nabla)\vec{A} - (\vec{A}.\vec{\nabla})\vec{B} + A(\vec{\nabla}.\vec{B}) - B(\vec{\nabla}.\vec{A}).$$

(b) For a 3-dimensional vector space with an orthonormal basis formed by $|1\rangle$, $|2\rangle$ and $|3\rangle$, An operator $\hat{A}$ is defined by the relations: $\hat{A}|1\rangle = |2\rangle$, $\hat{A}|2\rangle = |1\rangle$ and $\hat{A}|3\rangle = |3\rangle$.

Find the matrix representation of A, using this basis. Is this operator unitary?

(c) Prove that the negative gradient of a scalar field is a first rank tensor.

4. (a) Show that $\mathcal{L}\left\{\frac{d^2 f}{dt^2}\right\} = s^2\tilde{f}(s) - sf(0+) - \frac{df}{dt}(0+)$, where $\tilde{f}(s) = \mathcal{L}\{f(t)\}$. 3+3+(3+1)

(b) Using techniques of linear transformation, decouple the following coupled first order differential equations: $\frac{dx}{dt} = -ay$, $\frac{dy}{dt} = ax$.

(c) Consider a collection of rigidly connected N particles, rotating about an axis through the origin with angular velocity $\vec{\omega}$. If the particles, positioned at $\vec{r}_\alpha$, have mass m_α (where $\alpha = 0, 1, 2, \ldots, N$), find the component I_{ij} of the inertia tensor. Find the number of independent components of this tensor.

5. (a) From Newton's law for impulsive force acting on a particle of mass m, initially resting at the origin, the governing equation of its displacement $x(t)$ is written as $m\frac{d^2x}{dt^2} = P\delta(t)$, where P is a constant denoting the strength of the impulsive force. Show that the Laplace transform of $x(t)$ may be written as $\tilde{x}(s) = \frac{P}{ms^2}$. 4+4+2

(b) Apply Gram-Schmidt process to obtain an orthonormal set for the given vectors

$$\vec{A} = (-1, 0, 1);\ \vec{B} = (1, -1, 0);\ \vec{C} = (0, 0, 1) \text{ in } \mathbb{R}^3.$$

(c) Find the value of $\varepsilon_{ijk}\,\varepsilon_{ijk}$, where ε_{ijk} is the antisymmetric Cartesian tensor in three dimensions.

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