P. R. Thakur Govt. College

Department of Physics

B.Sc. Semester-V Internal Examination, 2023-24

Paper - **PHSADSE01T**

Advanced Mathematical Physics - I

Time: 1 Hour **Full Marks: 20**

1. Answer any *five* questions from the following: 2 x 5=10

   (a) If $\tilde{f}(s) = \mathcal{L}[f(t)]$, prove that $\mathcal{L}\left[\int_0^t f(u)du\right] = \tilde{f}(s)/s$.

   (b) Prove that $\mathcal{L}[\sinh(at)] = \frac{a}{s^2-a^2}$ for $s > |a|$.

   (c) State the convolution theorem in connection to Laplace transform.

   (d) Define basis and dimension of a vector space.

   (e) Prove that if a linear transformation is non-singular, it is injective.

   (f) Show that the set $S = \{(1,0,0), (1,1,1), (0,1,0)\}$ spans the vector space $\mathbb{R}^3$.

   (g) Check whether the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is linear or not: $T(x_1, x_2) = (1 + x_1, x_2)$.

   (h) For $t_0 > 0$, find $\mathcal{L}[f(t)]$ where

Department of Physics
P.R. Thakur Govt. College

$$f(t) = \begin{cases} 1, & \text{if } t > t_0 \\ 0, & \text{otherwise.} \end{cases}$$

Answer any *one* question from the following:

2. (a) Is Laplace transform linear? Solve the following differential equation by Laplace transform method:

$$Y''(t) + 9Y(t) = 18t,$$

$Y'(0)=0$ where $Y(0) = 0$ and $\cancel{Y(\frac{\pi}{2}) = 0}$ 1+4

   (b) If $\tilde{f}(s)$ is the Laplace transform of $f(t)$, prove that

$$\mathcal{L}\left[\frac{d^2f}{dt^2}\right] = s^2\tilde{f}(s) - sf(0) - \frac{df}{dt}(0)$$ 2

Department of Physics
P.R. Thakur Govt. College

   (c) Let $\beta = \{\alpha_1, \alpha_2, \alpha_3\}$ be the ordered basis of $\mathbb{R}^3$ with $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 1, 1)$, $\alpha_3 = (1, 0, 0)$. Find the components of the vector (a, b, c) in the basis β. 3

3. (a) Let $T : V \to W$ be a linear transformation and let R_T be the range of T in W. Prove that R_T is a subspace of W. 3

   (b) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(1, -1) = (1, 0)$ and $T(1, 1) = (0, 1)$. Find $T(x, y)$. 3

   (c) Apply Gram-Schmidt process to obtain an orthonormal set starting from the vectors $\alpha = (3, 0, 4)$, $\beta = (-1, 0, 7)$, $\gamma = (2, 9, 11)$ in $\mathbb{R}^3$. 4

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Advanced Mathematical Physics - I

Time: 1 Hour **Full Marks: 20**

1. Answer any *five* questions from the following: 2 x 5=10

(a) If $\tilde{f}(s)$ is the Laplace transform of $f(t)$, prove that for some constant a, $\mathcal{L}[f(at)] = \frac{1}{a}\tilde{f}(\frac{s}{a})$.

(b) If $\tilde{f}(s)$ is the Laplace transform of $f(t)$, then prove that $\mathcal{L}[tf(t)] = -\frac{d}{ds}\tilde{f}(s)$.

(c) State the convolution theorem in connection to Laplace transform.

(d) Prove that $\mathcal{L}[t\cos(at)] = \frac{s^2-a^2}{(s^2+a^2)^2}$.

Department of Physics
P. R. Thakur Govt. College

(e) Define group. Give an example of it.

Wrong (f) Show that the set $S = \{(1,0,0),(1,1,1),(0,1,0)\}$ spans the vector space $\mathbb{R}^3$, but does not form a basis for $\mathbb{R}^3$.

(g) Check whether the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is linear or not: $T(x_1, x_2) = (x_1 - x_2, 0)$.

(h) Show that the vector $(2, -5, 3)$ is not in the span of the set S consisting of vectors $\alpha = (1, -3, 2)$, $\beta = (2, -4, -1)$, $\gamma = (1, -5, 7)$.

2. Answer any *two* questions from the following: 5 x 2=10

(a) Is Laplace transform linear? Solve the following differential equation by Laplace transform method:

$$X''(t) + 4X(t) = 9t,$$

where $X(0) = 0$ and $X'(0) = 7$. 1+4

(b) If $f(t+T) = f(t)$ for all $t \geq 0$, show that

Department of Physics
P. R. Thakur Govt. College

$$\mathcal{L}[f(t)] = \frac{\int_0^T e^{-st} f(t)dt}{1 - e^{-sT}}.$$

Evaluate $\mathcal{L}^{-1}\left[\frac{1}{s-2} + \frac{2}{s+5} + \frac{6}{s^4}\right]$. 2+3

(c) If a vector in $\mathbb{R}^3$ is represented in the standard basis by $(3, 2, 1)$, find its components in a new basis consisting of vectors $(1, 1, 1)$, $(1, 1, 0)$, $(1, 0, 0)$.

Let $T : V \to W$ be a linear transformation and suppose N is defined by the set $\{\alpha \in V \text{ such that } T\alpha = 0\}$. Prove that N is a subspace of V. 3+2

(d) Describe a linear transformation from $\mathbb{R}^3$ into $\mathbb{R}^3$ which has its range spanned by the vectors $\{(1, 0, 1), (1, 2, 2)\}$.

Apply Gram-Schmidt process to obtain an orthonormal set starting from the vectors $\alpha = (-1, 0, 1)$, $\beta = (1, -1, 0)$ in $\mathbb{R}^3$.

2+3

P. R. Thakur Govt. College
Department of Physics
B.Sc. Semester-V Internal Examination, 2022

Paper - **PHSADSE01T**

Advanced Mathematical Physics - I

Time: 1 Hour **Full Marks: 20**

1. Answer any *five* questions from the following: 2 x 5=10

(a) Let $\mathcal{L}[f(t)] = \tilde{f}(s)$ and $g(t) = \begin{cases} f(t-a) & t > a \\ 0 & t < a, \end{cases}$

then prove that $\mathcal{L}[g(t)] = e^{-as}\tilde{f}(s)$.

(b) If $\mathcal{L}[f(t)] = \tilde{f}(s)$, then show that $\mathcal{L}[\int_0^t f(u)du] = \frac{\tilde{f}(s)}{s}$.

(c) Prove that $\mathcal{L}[\sinh(at)] = \frac{a}{s^2-a^2}$ for $s > |a|$.

(d) Construct two orthogonal vectors starting from the linearly independent set $\{\alpha_1, \alpha_2\}$, where $\alpha_1 = (3,0,4)$, $\alpha_2 = (-1,0,7)$.

(e) Check whether the transformation $T : \mathbf{R}^2 \to \mathbf{R}^2$ defined by $T(x_1, x_2) = (1 + x_1, x_2)$ is linear or not.

(f) Show that Kronecker delta is a mixed tensor of order two.

(g) Show that velocity of a fluid at any pount is a contravarient tensor of rank one

(h) Compute tha Laplace transform of the function $f(t) = \delta(t-1)$ $t \geq 0$.

Department of Physics
P.R. Thakur Govt. College

2. Answer any *two* questions from the following: 5 x 2=10

(a) i. Let $\beta = \{\alpha_1, \alpha_2, \alpha_3\}$ be the ordered basis for $\mathbf{R}^3$ where $\alpha_1 = (1,0,-1)$, $\alpha_2 = (1,1,1)$, $\alpha_3 = (1,0,0)$. Compute the coordinates of the vector (a,b,c) in the ordered basis β. $2\frac{1}{2}$

ii. Let $T : V \to W$ be a linear transformation and let R be the range of T. Prove that R is a subspace of W. $2\frac{1}{2}$

(b) Let T be a linear operator on $\mathbf{R}^3$ defined by
$T(x_1, x_2, x_3) = (3x_1 + x_3, -2x_1 + x_2, -x_1 + 2x_2 + 4x_3)$.
What is the matrix of T in the standard basis for $\mathbf{R}^3$? Prove that T is invertible and find the rule for T^{-1}. 2+3

(c) i. State and prove the convolution theorem for Laplace transform. 3

ii. Let $f(t)$ be a periodic function with period T. Prove that its Laplace transform $\tilde{f}(s)$ is given by

Department of Physics
P.R. Thakur Govt. College

$$\tilde{f}(s) = \frac{\int_0^T e^{-st} f(t)dt}{1 - e^{-sT}}.$$

2

(d) A covarient tensor has components $xy, zy - z^2, xz$ in rectangular co-ordinates. find its components in spherical components 5

P. R. Thakur Govt. College

Department of Physics

B.Sc. Semester-V Internal Examination, 2021

Paper - PHSADSE01T

Advanced Mathematical Physics I

1. Answer any five (5× 2 = 10)

a) Show that the vectors $\alpha_1 = (1,0,-1), \alpha_2 = (1,2,1)$ & $\alpha_3 = (0,-3,2)$ form a basis of $\mathbb{R}^3$.
b) Show that the gradient of a scalar field is a covariant tensor of rank one.
c) Write down the transformation rule of a mixed tensor of contravariant rank one and covariant rank two.
d) In Cartesian coordinate system, the contravariant and the covariant parts of a tensor are same – why?
e) Find the Laplace transformation of $e^{t/2}$.
f) Define function of class A.

Department of Physics
P.R. Thakur Govt. College

g) Show that F(t) $=e^{t^2}$ is not exponential order as t→ ∞.
h) Write down the condition under which two tensors A and B are conjugate.

2. Answer any two. (2× 5 = 10)

a) Let T be a linear transformation of $\mathbb{R}^3$ into $\mathbb{R}^3$ defined by

$$T(X_1, X_2, X_3) = (3X_1, X_1 - X_2, 2X_1 + X_2 + X_3)$$

Is T invertible? If so find a rule for T^{-1} like the one which define T. 5

b) (i) Describe a linear transformation of $\mathbb{R}^3$ into $\mathbb{R}^3$ which has its range the subspace spanned by $(1,0,-1)$ & $(1,2,2)$.

Department of Physics
P.R. Thakur Govt. College

(ii) Let $T: V \rightarrow W$ be a linear transformation and suppose N be subset of V defined by $\{\alpha \in V\ such that\ T\alpha = 0\}$. Prove that N is a subspace of V. (2.5 +2.5)

c) Find the Laplace's transformation of $e^{-2t}\sin^2 t$. 5

d) A covariant tensor has components xy, yz, xz In rectangular co-ordinate system. Find covariant components in spherical polar system. 5