Nuclear physics by suman sadhukhan

1.Bulk properties of nuclei
Nuclear mass, charge, size, binding energy, Spin and magnetic moment. Isobars, isotopes and Isotones; Mass spectrometer (Bainbridge)

what are the cause of failure of proton-electron model of nucleus?

1. Problem on the spin and statistics.
2. Problem concerning the uncertainty relation.

The observed spin of $^{14}_{7}N$ is ħ. Show that it cannot explained on the basis of proton electron model of nucleus.

If we consider proton electron model that nucleus $^{14}_{7}\boldsymbol{N}$ Consist of 14 proton and 7 electrons, the total number is 21 particles. As proton as well as electron are Fermions having a spin ħ. As 21 is an odd number then the nucleus of $^{14}_{7}\boldsymbol{N}$ Should have half integral spin according to proton electron model of nucleus.

Justify that electron cannot be a constituent particle of nucleus.

The uncertainty in the position of the electron in the nucleus is $\Delta x = 2 \times 10^{-14}$m
The uncertainty in the momentum is $\Delta P = \frac{6.63\times10^{-34}}{2\times3.14\times2\times10^{-14}} = 5.278 \times 10^{-21}$ Kg.m.s^{-1}
hence if electron exists in the nucleus than its minimum momentum is 5.278×10^{-21} Kg.m.s^{-1}
hence the minimum energy of the electron when it is inside nucleus is $E_{min} = \sqrt{P_{min}^2 c^2 + m_0^2 c^4}$

$$= \sqrt{(5.278 \times 10^{-21})^2 (3 \times 10^8)^2 + (9.11 \times 10^{-31})^2 (3 \times 10^8)^4} \approx 10 \text{ MeV}$$

but experiment shows that the maximum energy of β −Particles emitted from the nucleus ≈ 4 MeV.
Therefore, electron can-not exists inside nucleus

Justify that proton can be a constituent particle of nucleus.

The uncertainty in the position of the proton in the nucleus is $\Delta x = 2 \times 10^{-14}$m
The uncertainty in the momentum is $\Delta P = \frac{6.63\times10^{-34}}{2\times3.14\times2\times10^{-14}} = 5.278 \times 10^{-21}$ Kg.m.s^{-1}
hence if proton exists in the nucleus than its minimum momentum is Kg.m.s^{-1}
hence the minimum energy of the proton when it is inside nucleus is $E_{min} = \sqrt{P_{min}^2 c^2 + m_0^2 c^4}$

$$= \sqrt{(5.278 \times 10^{-21})^2 (3 \times 10^8)^2 + (1.67 \times 10^{-27})^2 (3 \times 10^8)^4} \approx 52 \text{ KeV}$$

but experiment shows that the energy carried out by a proton emitted from the nucleus is greater than 52 KeV.
Therefore, proton can-not exists inside nucleus

What is proton- neutron hypothesis? Give reasons for acceptance of this hypothesis of the nucleus.

According to this hypothesis, the nucleus of mass number A and atomic number Z contains Z protons and (A-Z) neutrons. An atom is electrically neutral, therefore number of peripheral electrons must be equal to Z.
Acceptance of proton and neutron theory –

1. This theory removes the difficulty of having an electron in the nucleus.
2. The presence of protons and neutrons is in accordance with Heisenberg uncertainty principle.

Discuss the point of success of proton neutron model of nucleus.

1. It can explain the phenomena of the radioactivity.
2. The isotopic mass can be easily explained on the basis of proton-neutron model.
3. Nuclear spin can be explained on the basis of proton neutron model.
4. Nuclear magnetic moment associated with nucleus can be explained on the basis of proton-neutron model.

- **MASS OF A NUCLEUS**

Unit of atomic mass John Dalton first suggested a means of expressing relative atomic mass in 1803. He proposed the use of hydrogen-1 (Protium). Wilhelm Ostwald suggested that relative atomic mass would be better if expressed in terms of 1/16th the mass of oxygen. When the existence of isotopes was discovered in 1912 and isotopic oxygen in 1929, the definition based on oxygen became confusing. Some scientists used an AMU based on the natural abundance of oxygen, while others used an AMU based on the oxygen-16 isotope.
So, in 1961 the decision was made to use carbon-12 as the basis for the unit (to avoid any confusion with an oxygen-defined unit). The new unit was given the symbol u to replace amu, plus some scientists called the new unit a Dalton. However, u and Da were not universally adopted. Many scientists kept using the amu, just recognizing it was now based on carbon rather than oxygen. At present, values expressed in u, AMU, amu, and Da all describe the exact same measure.

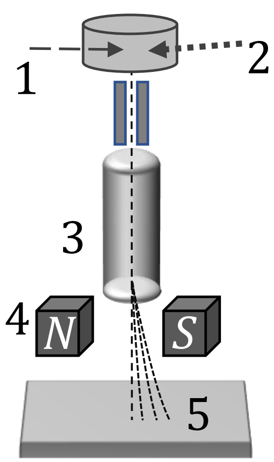
Dalton or unified atomic mass unit The Dalton or unified atomic mass unit (symbols: Da or u) is a unit of mass widely used in physics and chemistry. It is defined precisely as 1/12 of the mass of an unbound neutral atom of carbon-12 in its nuclear and electronic ground state and at rest. The atomic mass constant, denoted m_u, is defined identically, giving $m_u = m(^{12}C)/12 = 1$ Da.

1 Da (or, u) =1.66 Yoctograms=$\mathbf{1.66\times 10^{-27}}$ Kg;
1 Da (or, u)=1822.8 × mass of an electron= $\mathbf{1822.8}m_e$
The mass energy of 1 Da (or, u) corresponds energy value **931.49 MeV.**

Mass of electron	$\mathbf{5.4\times 10^{-4}}$ **u**
Mass of proton	**1.0072 u**
Mass of neutron	**1.0086 u**

What does a mass spectrometer do? A mass spectrometer produces charged particles (ions) from the chemical substances that are to be analyzed. The mass spectrometer then uses electric and magnetic fields to measure the mass ("weight") of the charged particles.

How does a mass spectrometer work? There are numerous different kinds of mass spectrometers, all working in slightly different ways, but the basic process involves broadly the same stages.



1. You place the substance you want to study in a vacuum chamber inside the machine.
2. The substance is bombarded with a beam of electrons so the atoms or molecules it contains are turned into ions. This process is called **ionization**.
3. The ions shoot out from the vacuum chamber into a powerful electric field (the region that develops between two metal plates charged to high voltages), which makes them accelerate. Ions of different atoms have different amounts of electric charge, and the more highly charged ones are accelerated most, so the ions separate out according to the amount of charge they have. (This stage is a bit like the way electrons are accelerated inside an old-style, cathode-ray television.)
4. The ion beam shoots into a magnetic field (the invisible, magnetically active region between the poles of a magnet). When moving particles with an electric charge enter a magnetic field, they *bend* into an arc, with lighter particles (and more positively charged ones) bending more than heavier ones (and more negatively charged ones). The ions split into a spectrum, with each different type of ion bent a different amount according to its mass and its electrical charge.
5. A computerized, electrical detector records a spectrum pattern showing how many ions arrive for each mass/charge. This can be used to identify the atoms or molecules in the original sample. In early spectrometers, photographic detectors were used instead, producing a chart of peaked lines called a **mass spectrograph**. In modern spectrometers, you slowly vary the magnetic field so each separate ion beam hits the detector in turn.

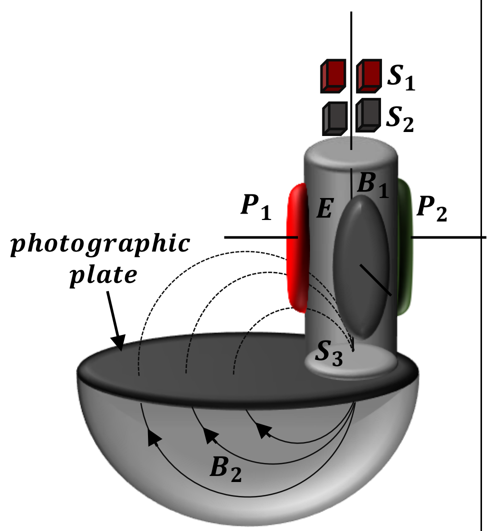
That means a basic mass spectrometer consists of three parts:
1. A source in which ions are produced from the chemical substances to be analyzed.
2. An analyzer in which ions are separated according to mass.
3. A detector which produces a signal from the separated ions.

Bainbridge Mass Spectrometer (spectrograph)

A high resolution and high precision mass spectrograph was built by Bainbridge using new principle called velocity selector.
The filter makes the stream of positive ions perfectly homogeneous before it entry into magnetic field, no matter what the initial velocities of ions are.
A plan view of this spectrometer is shown in figure. A beam of positive ions in a discharge tube is collimated into a narrow beam by two slits S_1& S_2. After emerging from the slit S_2 the positive ions enters into a velocity selector

velocity selector.

It consist of two plates P_1& P_2 between which a steady electric field $\vec{E}$ is maintained in a direction at right angles to the ions beam. And a magnetic field $\overrightarrow{B_1}$ is produced by an electromagnet. The magnetic field is at right and to the electric field as well as the ion beam.
The electric and magnetic field of the velocity selector are so adjusted that the deflection produced by the one is exactly equal to and opposite to the deflection produced by the other. For the net deflection of ions having a particular velocity $\vec{v}$ is zero.
If E & B_1electric field intensity and magnetic induction then,
$qE = qvB_1$ or, $v = \frac{E}{B_1}$ (where q is the charge of the ion)



Working principle-

The positive ions which entered is that chamber D are subjected to a strong magnetic field of induction $\overrightarrow{B_2}$ Perpendicular to the path of radius R. Given by $qvB_2 = \frac{mv^2}{R}$ or, $R = \frac{mv}{qB_2}$

Hence $\frac{q}{m} = \frac{v}{RB_2} = \frac{E}{B_1 B_2 R}$ That means $m \propto R$

After describing the semicircular paths ions strikes the photographic plate.
The ions of different mass will traverse path of different radii and strike the photographic plate at different points. There was a giving a typical mass spectrum. The method is fairly accurate as mass scale is linear

- **CHARGE AND SIZE OF A NUCLEUS.**

Charge	If Z if atomic number or charge number of nucleus, then charge of the nucleus is Ze where e = charge of electron=1.6×10^{-19} C
Radius	As the nucleus is approximately spherical, its volume is proportional to the total number of nucleons that is, $A = \frac{4}{3}\pi r^3$. Hence $r = r_0 A^{1/3}$ where $r_0 = (1.1 - 1.5) \times 10^{-15}$ m
Density	Nuclear density $=\frac{nuclear\ mass}{nuclear\ volume} \approx \frac{Am_N}{\frac{4}{3}\pi {r_0}^3 A} = \frac{1.67\times10^{-27}}{\frac{4}{3}\times3.14\times\left((1.1-1.5)\times10^{-15}\right)^3} \approx 2.3 \times 10^{17}$ Kg.m^{-3} (on average) this shows that the nuclear matter is in a high compressed state and is independent of A, its value is almost same for all nucleus. In the other words nuclear mass is proportional to its volume.

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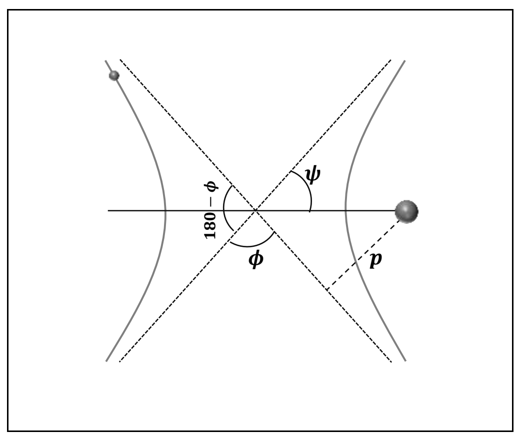
Assumptions of Rutherford.	1. That contents in nucleus of charge Ze where Z is the atomic number of the atom. 2. The nucleus can be treated as point particle. 3. As the nucleus is sufficiently massive, is recoil of nucleus may be neglected. 4. That the laws of classical mechanics and electromagnetism can be applied and that no other forces are at present (strong force is not considered). 5. The collision is elastic.
Impact parameter	The impact parameter is defined as the minimum distance to which the alpha particle would approach the nucleus in there were no between them.
Distance of closest approach	Distance of closest approach is the minimum distance between α −particle and Centre of nucleus just before the alpha particle reflects back by 180°
Relation between distance of closest approach and impact parameter Distance of closest approach is used to determine the approximate size of the nucleus size of nucleus is smaller than the distance of closest approach. The relation between the impact parameter(p) and the distance of closest approach(d) is $p = \frac{d}{2}\tan\frac{\phi}{2}$ Where ϕ is the scattering angle.	
Find expression for closest approach If the collision between the incident particle whose kinetic energy is $\frac{1}{2}mv_0^2$ and a nucleus of electric charge is Ze, where Z is the atomic number of the nucleus as target. Considering head-on collision between nucleus and the particle if d be the distance of closest approach we have $\frac{1}{2}mv_0^2 = \frac{2Ze^2}{4\pi\varepsilon_0 d}$ then $d = \frac{2Ze^2}{4\pi\varepsilon_0 \frac{1}{2}mv_0^2} = \frac{Ze^2}{\pi\varepsilon_0\, mv_0^2}$	

Show that the path traverse by an alpha particle in Rutherford experiment is hyperbola:

According to Coulombs law the force of repulsion between alpha particle and the nucleus is $F = \frac{2Ze^2}{4\pi\varepsilon_0 r^2}$

[symbols have their usual meaning]

Here (r,θ) is the position coordinate at any instant of alpha particle with respect to the nucleus.



Then $a_r = \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = \frac{F}{m} = \frac{2Ze^2}{4\pi\varepsilon_0 m\, r^2} = \frac{J}{r^2}$ [where $J = \frac{2Ze^2}{4\pi\varepsilon_0 m}$]

And $a_\theta = \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = \frac{1}{r}\frac{d}{dr}\left(r^2\frac{d\theta}{dt}\right) = 0$ or, $r^2\frac{d\theta}{dt} = k\ (constant)$

or, $\boldsymbol{\frac{d\theta}{dt} = \frac{k}{r^2}}$

Now let $u = \frac{1}{r}$ then $\frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} = \frac{dr}{du}\frac{du}{d\theta}\frac{d\theta}{dt} = \left(-\frac{1}{u^2}\right)\frac{du}{d\theta}ku^2 = -k\frac{du}{d\theta}$

$\frac{d^2r}{dt^2} = \frac{d}{dt}\left(\frac{dr}{dt}\right) = \frac{d}{dt}\left(-k\frac{du}{d\theta}\right) = -k\frac{d^2u}{d\theta^2}\frac{d\theta}{dt} = k\frac{d^2u}{d\theta^2}(ku^2) = -k^2u^2\frac{d^2u}{d\theta^2}$

Then we have $-k^2u^2\frac{d^2u}{d\theta^2} - \frac{1}{u}\left(-k\frac{du}{d\theta}\right)^2 = Ju^2$

$\frac{d^2u}{d\theta^2} + u = -\frac{J}{k^2}$ or, $\frac{d^2u}{d\theta^2} + \left(u + \frac{J}{k^2}\right) = 0$ or, $\frac{d^2\left(u+\frac{J}{k^2}\right)}{d\theta^2} + \left(u + \frac{J}{k^2}\right) = 0$ [as $\frac{J}{k^2} = constant.$]

Then $u + \frac{J}{k^2} = A\cos(\theta - \delta)$

By proper choice of the axis δ may be taken as zero and if we take $A = \frac{J\epsilon}{k^2}$

$u = \frac{1}{r} = -\frac{J}{k^2}(1 - \epsilon\cos\theta)$. Which represent a conic section with the eccentricity ϵ

Let the initial velocity of $\alpha -$ particle be v_0 at any instant the velocity is v. Then $\frac{1}{2}m\, {v_0}^2 = \frac{1}{2}m\, v^2 + \frac{2Ze^2}{4\pi\varepsilon_0 r}$

[from conservation of energy]

Then $ {v_0}^2 = v^2 + \frac{2Ze^2}{2m\pi\varepsilon_0 r} = v^2 + \frac{2J}{r}$

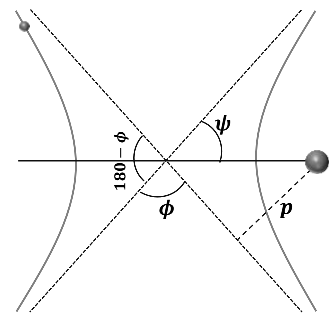
Now $v^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2 = k^2\left(\frac{du}{d\theta}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2 = \left(\frac{du}{d\theta}\right)^2 + r^2\frac{k^2}{r^4} = \frac{k^2J^2\epsilon^2}{k^4}\sin^2\theta + \frac{k^2J^2}{k^4}(1-\epsilon\cos\theta)^2$

$= \frac{J^2}{k^2}(1+\epsilon^2 - 2\epsilon\ \cos\theta)$

Then ${v_0}^2 = \frac{J^2}{k^2}(1+\epsilon^2-2\epsilon\ \cos\theta) - \frac{2\,J^2}{k^2}(1-\epsilon\cos\theta) = \frac{J^2}{k^2}(\epsilon^2-1)$ Then $\epsilon = \sqrt{1+\left(\frac{kv_0}{J}\right)^2}$

Again from conservation law of angular momentum $mr^2\frac{d\theta}{dt} = mk = mv_0 p$ where p =Impact parameter. Then $k = v_0 p$

Hence $\epsilon = \sqrt{1+\left(\frac{kv_0}{J}\right)^2} = \epsilon = \sqrt{1+\left(\frac{4\pi\varepsilon_0 m{v_0}^2 p}{2Ze^2}\right)^2} > 1.$ Hence the conic is hyperbola.



derive relation between impact parameter p, closest approach (d) and scattering angle ϕ

Equation of hyperbola in rectangular co-ordinate system as $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Hence equation of asymptotes are $y = \pm\frac{b}{a}x$

The slope of the asymptote is $\frac{b}{a} = \tan\psi = \tan\frac{180-\phi}{2} = \cot\frac{\phi}{2}$

from figure where ϕ= scattering angle.

Again for hyperbola $\frac{b^2}{a^2} = \epsilon^2 - 1$. Hence $\cot^2\frac{\phi}{2} = \left(\frac{4\pi\varepsilon_0 m{v_0}^2 p}{2Ze^2}\right)^2$ or, $\cot\frac{\phi}{2} = \frac{4\pi\varepsilon_0 m{v_0}^2 p}{2Ze^2} = \frac{p}{\frac{d}{2}}$. Hence $\boldsymbol{p = \frac{d}{2}\cot\frac{\phi}{2}}$

Derive differential cross-section $\frac{d\sigma}{d\phi}$ and probability of deviation per unit solid angle that is a Rutherford scattering formula

Let F be the number of incident particles are arriving by the unit area per unit second at the target, that means flux of particles.

The number of particles $dN(p)$ with impact parameter in between p & $p+dp$ is $dN(p) = F(2\pi\, p\, dp)$

Now as $p = \frac{d}{2}\cot\frac{\phi}{2}$ then $dp = -\frac{d}{4\sin^2\frac{\phi}{2}}\, d\phi$.

Then out of F particles the particles scattered through an angle between ϕ & $\phi + d\phi$ is $dN(p) = F\pi\frac{d^2}{4}\frac{\cos\frac{\phi}{2}}{\sin^3\frac{\phi}{2}}\, d\phi$

Differential cross-section= number of scattered particles between per unit flux, per unit range of scattering angle.

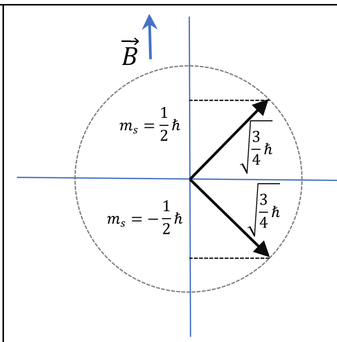
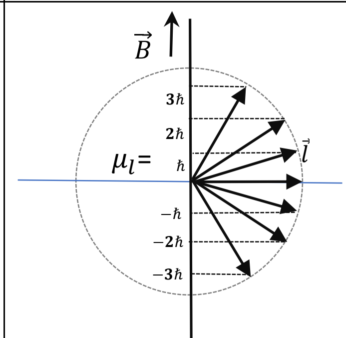
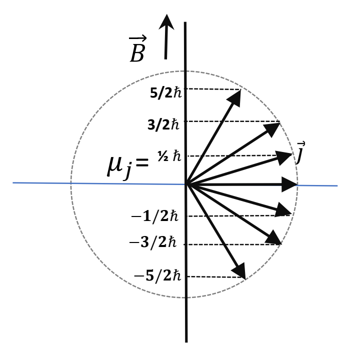
That means $\frac{d\sigma}{d\phi} = \frac{dN(p)}{F\, d\phi} = \pi\frac{d^2}{4}\frac{\cos\frac{\phi}{2}}{\sin^3\frac{\phi}{2}}$

Again probability of deviation per unit solid angle $W(\phi) = \frac{dN(p)}{F\, d\Omega} = \frac{d\sigma}{d\Omega} = F\pi\frac{d^2}{4}\frac{\cos\frac{\phi}{2}}{\sin^3\frac{\phi}{2}}\frac{d\phi}{(2\,\pi\sin\phi\ d\phi)} = \frac{d^2}{16}\frac{1}{\sin^4\frac{\phi}{2}} = \left(\frac{Ze^2}{\pi\varepsilon_0\, mv_0^2}\right)^2\frac{1}{16\sin^4\frac{\phi}{2}}$

OR, $\boldsymbol{\frac{d\sigma}{d\Omega}} \propto \begin{cases} \boldsymbol{Z^2} \\ \boldsymbol{\frac{1}{v_0^4} \propto \frac{1}{T^2}} \\ \frac{1}{\sin^4\frac{\phi}{2}} \end{cases}$ Note that, T= K.E & scattering cross-section= πp^2

- **ANGULAR MOMENTUM OF NUCLEUS**

Angular momentum of nucleons

1. Spin angular momentum:	Each nucleons has its intrinsic angular momentum which may be pictured as being caused by particles spinning motion adult axis through its centre of mass. This is known as spin angular momentum and the magnitude is given by $s = \lvert\vec{s}\rvert = \sqrt{s(s+1)}\,\hbar$ where $s = \frac{1}{2}$ = spin angular momentum quantum number. The ***magnetic spin quantum*** number m_s is the component of $\vec{s}$ in a specified quantization direction (applied magnetic field direction, usually taken as Z axis). With $s = \frac{1}{2}$ the values of m_s are $+\frac{1}{2}$ & $-\frac{1}{2}$ corresponding to parallel and antiparallel alignment w.r.t Z axis.	
2. Orbital angular momentum:	Each individual nucleon may be pictured as having an angular momentum associated with its orbital motion within the nucleus this is known as orbital angular momentum the magnitude is given by $l = \lvert\vec{l}\rvert = \sqrt{l(l+1)}\,\hbar$ where l = orbital angular momentum quantum number $l = 0,1,2,3 \ldots\ldots$ Integer number The ***orbital magnetic quantum*** number m_l is the component of $\vec{l}$ in a specified quantization direction (applied magnetic field direction, usually taken as Z axis). m_l can take $(2l+1)$ possible values as $l, (l-1), (l-2), \ldots\ldots -(l-2), -(l-1), -l$.	
3. Total angular momentum:	The total angular momentum $\vec{j}$ of a nucleon is the vector sum of orbital and spin angular momentum $\vec{j} = \vec{l} + \vec{s}$ The magnitude of the total angular momentum of a nucleon is given by $j = \lvert\vec{j}\rvert = \sqrt{j(j+1)}\,\hbar$ where $j =$ total angular momentum quantum number $j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \ldots\ldots$ half integral value (as both proton and neutron are Fermions then $s = \frac{1}{2}$, and as already told that l =integer, then j can take half integral values.) The ***total magnetic quantum*** number m_j is the component of $\vec{l}$ in a specified quantization direction (applied magnetic field direction, usually taken as Z axis). m_l can take $(2j+1)$ possible values as $j, (j-1), (j-2), \ldots\ldots -(j-2), -(j-1), -j$.	

The total angular momentum of nucleus (nuclear spin): The total angular momentum of nucleus for the particular nuclear state is the result of individual total angular momentum of all nucleons. Its magnitude is $I = \lvert\vec{I}\rvert = \sqrt{I(I+1)}\,\hbar$

where $I =$ total angular momentum quantum number

Even A	*Z=even , A= even, i.e., N=even*	*I in ground state is always zero (0)*
	Z=odd , A= even, i.e., N=odd	$I = n\hbar$, *where* $n = 1,2,3,4 \ldots\ldots\ldots\ldots\ldots$
Odd A	*Z=even , A= odd, i.e., N=odd*	$I = half\ integral = \frac{n}{2}\hbar$, *where* $n = 1,2,3,4 \ldots\ldots\ldots\ldots\ldots$
	Z=odd , A= odd, i.e., N= even	*Note that I lying between* $\frac{\hbar}{2}$ *to* $\frac{9\hbar}{2}$

Why the nuclei do not have any electric dipole moment

the expectation value of dipole moment operator for states of definite parity is zero. This is because the dipole moment operator is odd operator. That's why the nuclei don't have an electric dipole moment.

What is electrical quadrupole moment of the nucleus? What is its role in nuclear physics?

The nuclear Electric quadrupole moment is a parameter which describe the effective shape of the ellipsoid of nuclear charge distribution. A nonzero quadrupole moment Q indicates that the charge distribution is not spherically symmetric. By convention the value of Q is taken to be positive if the ellipsoid is Prolate and negative if it is a Oblate.

This intrinsic quadrupole moment of a nucleus is given by $Q_0 = \frac{1}{e}\iiint_V (3z'^2 - r'^2)\rho(r')d\tau'$

[Where symbols have the usual meanings]

The integration is over **the** whole volume of the nucleus and r' is measured from the center of mass of the nucleus, which has the symmetry axis along Z.

For spherically symmetric charge distribution

$$\iiint_V \rho(r')x'^2 d\tau' = \iiint_V \rho(r')y'^2 d\tau' = \iiint_V \rho(r')z'^2 d\tau' = \frac{1}{3}\iiint_V \rho(r')r'^2 d\tau'$$

This gives $x'^2 = y'^2 = z'^2 = \frac{1}{3}r'^2$

then for spherically symmetric nucleus $Q_0 = \frac{1}{e}\iiint_V (3z'^2 - r'^2)\rho(r')d\tau' = \frac{1}{e}\iiint_V \left(3.\frac{1}{3}r'^2 - r'^2\right)\rho(r')d\tau' = 0$

If $z'^2 > \frac{1}{3}r'^2$ then $3z'^2 - r'^2 > 0$ then $Q_0 > 0$ Hence the shape is prolate.

If $z'^2 < \frac{1}{3}r'^2$ then $3z'^2 - r'^2 < 0$ then $Q_0 < 0$ Hence the shape is oblate.

The smallest value of I (nuclear spin) for $Q \neq 0$

For non-spherical nuclei that intrinsic quadrupole moment Q_0 is measured in a reference frame fixed to the nucleus, where as the observed quadrupole moment $\boldsymbol{Q}$ is measured in a reference frame fixed in the laboratory $\boldsymbol{Q}$& Q_0 are related by the question

$Q = \frac{I(2I-1)}{(I+1)(2I+3)}Q_0$ [where I is the nucleus being]. Now for $I = 0\ or, I = \frac{1}{2}$ we get $\boldsymbol{Q = 0}$ **even** $Q_0 \neq 0$.

As smallest possible value of I (nuclear spin) for $Q \neq 0$ is $I = 1$.

Note that for $I \gg 1$ then $Q \to Q_0$

Note that the dimension of quadrupole moment is that of area in nuclear physics area is measured in burn ($1\ burn = 10^{-28}\ m^2$). Quadrupole is also measured in $fm^2$and the value mostly lies in the range 1.2 burn for $^{123}_{51}Sb$ to 8 burn for $^{176}_{71}Lu$. Note that for deuteron has the value $Q = 2.74 \times 10^{-31}m^2$.

Nuclear magnetic dipole moment

For proton: - The spin angular momentum and orbital angular momentum of protons within the nuclei produce magnetic fields which can be described in terms of resultant magnetic dipole moment looked at the Centre of the nucleus. A spinning positive charge produce a magnetic field whose N-pole direction is parallel to the direction of spin. The magnetic moment in such a case is said to be positive.

If a proton having a charge e circulated about a force center with the frequency ν than the equivalent current $I = e\nu$.

Then orbital magnetic dipole moment $\vec{\mu_l} = I\vec{A} = e\nu\pi r^2\hat{n} = \frac{e\nu\pi r^2\hat{n}}{2\pi r} = \frac{e}{2m}\ m v r\hat{n} = \frac{e}{2m}\ \vec{l}$

Or we may write $\vec{\mu_l} = g_l\frac{e}{2m}\ \vec{l}$

where g_l =orbital gyromagnetic ratio= 1, $\vec{s}$ = spin angular momentum. M= mass of a proton

similarly the spin of proton will produce a spin magnetic dipole moment of an amount $\vec{\mu_s} = g_s\frac{e}{2m}\ \vec{s}$

where g_s =spin gyromagnetic ratio $\vec{s}$ = spin angular momentum.

For the neutron: - A neutron being and uncharged particle can only have a spin magnetic moment of amount $\vec{\mu_s} = g_s\frac{e}{2m}\ \vec{s}$ where g_s =spin gyromagnetic ratio $\vec{s}$ = spin angular momentum. Clearly we take m as mass of neutron exactly same as that was for proton.

Hence total magnetic dipole moment of a proton $\vec{\mu_p} = \vec{\mu_l} + \vec{\mu_s}$ At for neutron $\vec{\mu_n} = \vec{\mu_s}$

Then total magnetic moment of the nucleus $\vec{\mu_N} = \frac{e}{2m}\{\sum_{k=1}^{Z} g_l \vec{l}_k + \sum_{k=1}^{Z} g_s \vec{s}_k + \sum_{k=1}^{N} g_s \vec{s}_k \}$

$$= \frac{e}{2m}\{\sum_{k=1}^{Z} g_l \vec{l}_k + \sum_{k=1}^{A} g_s \vec{s}_k \} = \frac{e}{2m} g \vec{I}$$

Then $\vec{\mu_N} = \frac{e\hbar}{2m} g \frac{\vec{I}}{\hbar} = \left(g \frac{\vec{I}}{\hbar}\right)\beta_N$ where $\beta_N = Bohr\ magneton = \frac{e\hbar}{2m}$

What is nuclear magneton or both magneton: nuclear magneton is a physical constant of magnetic moment represented as $\beta_N = \frac{e\hbar}{2m}$ where e= elementary charge and m= Mass of nucleon.

The nuclear magneton is then natural unit of expressing dipole moment of heavy particles such as nucleons and atomic nuclei.

Even A	*Z=even , A= even, i.e., N=even*	*I in ground state is always zero (0).* $\vec{\mu_N} = \vec{0}$
	Z=odd , A= even, i.e., N=odd	***$\vec{\mu_N}$ is determined by the superposition of magnetic moment of unpaired neutron and proton***
Odd A	*Z=even , A= odd, i.e., N=odd*	***$\vec{\mu_N}$ is determined by the magnetic moment of unpaired neutron or proton***
	Z=odd , A= odd, i.e., N= even	For unpaired proton $\vec{\mu_p} = g_p \frac{1}{2}\beta_N = (\frac{5.58}{2})\beta_N = 2.792\beta_N$ For unpaired proton $\vec{\mu_n} = g_n \frac{1}{2}\beta_N = (\frac{-3.82}{2})\beta_N = -1.913\,\beta_N$

Wave mechanical properties of nucleus

The nucleus has the following wave mechanical properties 1. Statistics 2. Parity 3. Isospin

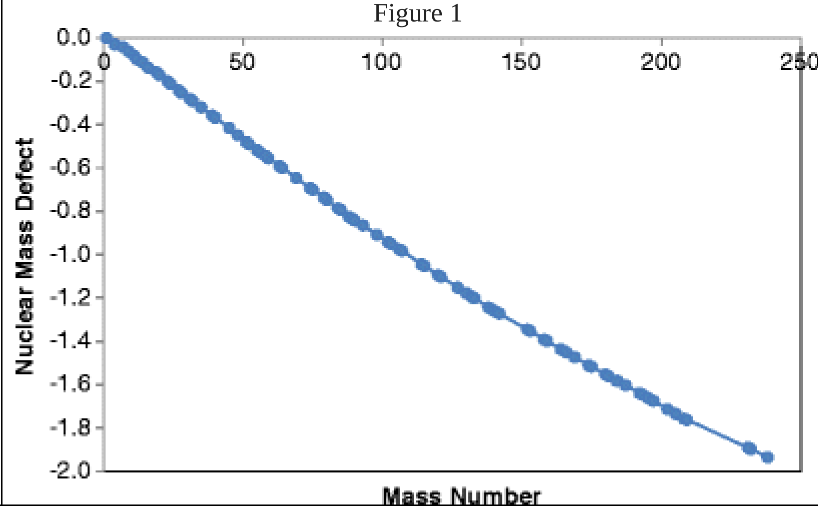
1. **Statistics:** All particles (nucleus with even A) with integral spin (in unit of ħ) or zero spin obey B.E statistics and called Bosons. The wave function of the system obeys B.E statistics is symmetric.
   Then, $\psi(x_1, x_2, \dots\dots\dots.. x_i, x_j, \dots\dots\dots.. x_N) = \psi(x_1, x_2, \dots\dots\dots.. x_j, x_i, \dots\dots\dots.. x_N)$
   All particles with half integral spin (in unit of ħ) obey M. B. statistics and called Fermions. The wave function of the system obeys M.B statistics is anti-symmetric.
   Then, $\psi(x_1, x_2, \dots\dots\dots.. x_i, x_j, \dots\dots\dots.. x_N) = -\psi(x_1, x_2, \dots\dots\dots.. x_j, x_i, \dots\dots\dots.. x_N)$
2. **Parity:** the parity of the system refers to behavior of the function ψ under the inversion of coordinate the origin
   $\psi(x_1, x_2, \dots\dots\dots.. x_i, x_j, \dots\dots\dots.. x_N) = \psi(-x_1, -x_2, \dots\dots\dots.. -x_i, -x_j, \dots\dots\dots.. -x_N)$ *EVEN PARITY*
   $\psi(x_1, x_2, \dots\dots\dots.. x_i, x_j, \dots\dots\dots.. x_N) = -\psi(-x_1, -x_2, \dots\dots\dots.. -x_i, -x_j, \dots\dots\dots.. -x_N)$ *ODD PARITY*
   the intrinsic parity of protons, neutrons are even. And the parity of whole system is product of parities of individual particles.
   Parity is purely quantum mechanical concept. Nuclear states are characterized by a definite parity which may different for different states of same nucleus. In nuclear reaction parity is conserved.
3. **Isospin:** neutrons and protons are similar in all respect except charge. On this basis these particles are considered to be different manifestation of the same inherent particles called nucleon. To describe the quantum states of nucleon quantum number used is known as isotropic spin quantum number of Isospin or, T-spin.
   Isospin is conserved in nuclear transformation. A nucleon is assigned isospin $\frac{1}{2}$ and it is an electromagnetic field two charge states with Isospin component $+\frac{1}{2}$ & $-\frac{1}{2}$ can be distinguished as proton and neutron.

Mass defect Mass of the nucleus is slightly less than the added masses of its constituent protons and neutrons and this mass difference is called nuclear mass defect.

Mass defect $\Delta m = M(A,Z) - \{Zm_p + (A-Z)m_n\}$ [symbols have the usual meanings.]. Mass defect exists universally in bound systems of all sizes in which the components are bound together by force. It is applicable to small systems such as the nucleus of an atom as well as large systems such as the solar system.

The nuclear mass defect is a fundamental property of a nucleus and is a fixed value corresponding to a certain amount of binding energy for that nucleus. Mass defect and binding energy are important factors in the energy involved in nuclear reactions. Looking at how mass defect and binding energy change from one element to another will make the relationship between mass defect, binding energy, and nuclear energy more apparent.

Figure 1



A plot of nuclear mass defect versus mass number for different elements is shown in Figure 1. The nuclear mass defect changes with mass number from **zero for hydrogen** to a value close to –2 for uranium.

According to this definition Nuclear mass defect **is always negative** and has the same sign for all elements and therefore binding energies as the energy that keeps the nucleus together will all have the same sign as expected.

Binding energy:- The binding energy of a nucleus may be defined as the energy equivalent to the mass defect of the nucleus.

Binding energy

$E_B = [\{Zm_p + (A-Z)m_n\} - M(A,Z)]c^2$

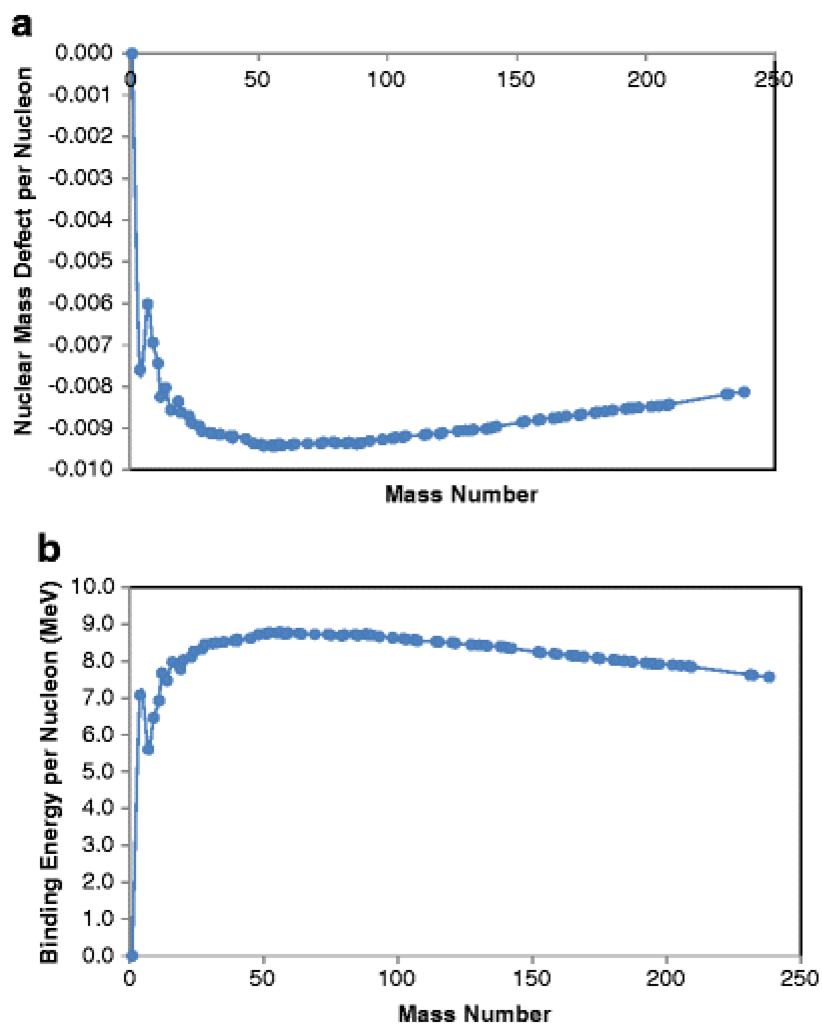
According to this definition binding energy **is always positive.**

nuclear mass defect per nucleon:- $\frac{\Delta m}{A}$

Binding energy per nucleon:- $f_B = \frac{E_B}{A}$

Where A= mass number.

The nuclear mass defect per nucleon, which is mass defect divided by mass number, is a more useful value. It provides a more meaningful way of comparison between different elements and is plotted against mass number in Figure 2a. The corresponding energy, the binding energy per nucleon, is the amount of energy that is released per nucleon upon the formation of a nucleus and is an indication of its stability Figure 2b



WE CAN DEFINE MASS DEFECT IN AN ONOTHER WAY: -

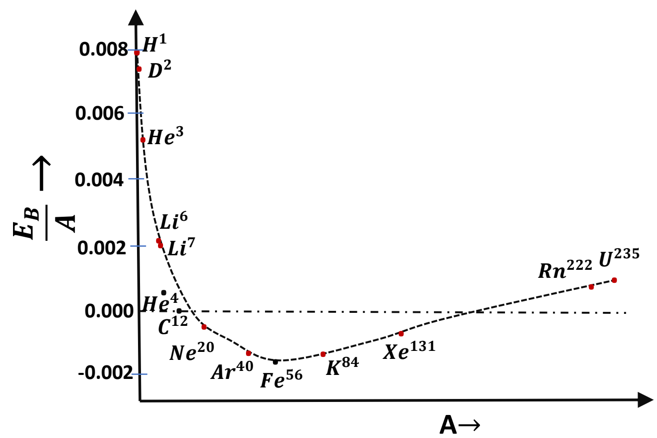
The mass excess or mass defect of a nuclide is the difference between its **actual mass (**in atomic mass units u**)** and **its mass number**. It is one of the predominant methods for tabulating nuclear mass. The mass of an atomic nucleus is well approximated (less than 0.1% difference for most nuclides) by its mass number, which indicates that most of the mass of a nucleus arises from mass of its constituent protons and neutrons. Thus, the mass excess is an expression of the nuclear binding energy, **relative to the binding energy per nucleon of carbon-12** (which defines the atomic mass unit). If the mass excess is negative, the nucleus has more binding energy than $^{12}_{6}C$, and vice versa. If a nucleus has a large excess of mass compared to a nearby nuclear species, it can radioactively decay, releasing energy.
Mass excess or defect $\Delta m' = M(A,Z) - A$

The mass defect is positive for very light and for very heavy atoms and is negative for atoms in having intermediate mass number.

Packing fraction. According to Aston, packing fraction of an atomic nucleus is the mass defect per nucleon of the atom. If be the mass defect $\Delta m'$ and A the mass number (i.e., the number of nucleons), then the packing fraction is given by $f = \frac{M(A,Z)-A}{A}$ or, $M(A,Z) = A(1+f)$

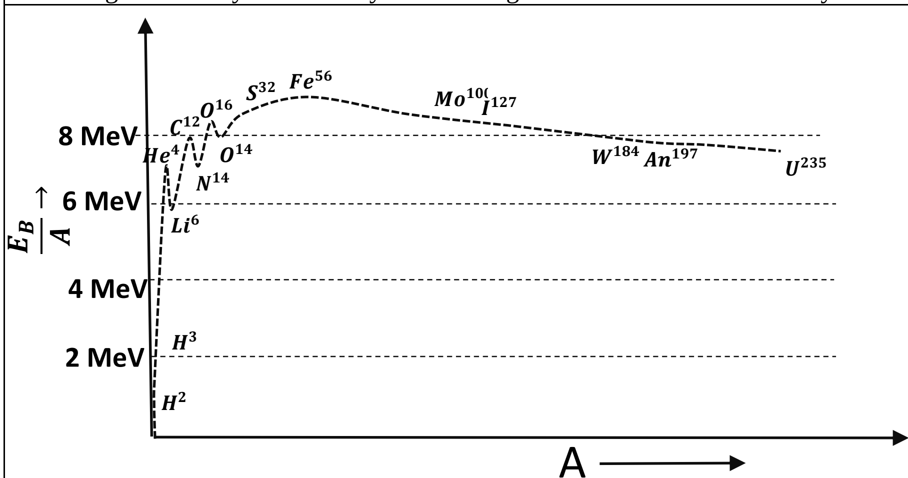
The plot of packing fraction against the mass number (A) (called the packing fraction curve) is shown in Fig. 1.1. The curve shows that,

1. f is positive for very light nuclei,
2. as A increases f decreases becoming negative for A > 20,
3. f attains a minimum value, which is negative at A~ 60 after which f starts increasing slowly,
4. for A ~160, f becomes positive again.



PACKING FRACTION CURVE

NOTE THAT: - All nuclei have a tendency to move from a region of higher packing fraction to lower packing fraction to attain higher stability. This is why fusion of light nuclei and fission of heavy nuclei take place with a release of energy.



A number of interesting conclusions can be drawn by critically noting the features of $\boldsymbol{f_B}$ against A curve.
1. From Fig it is evident that $\boldsymbol{f_B}$ is small for very light nuclei and goes on increasing with (curve AB) the increasing of A reaching a value~ 8 MeV for A ~20. The rise of the curve after A ~20, is much slower assuming a value of 8.7 MeV for A =56 . Thereafter the curve decreases gradually
2. In the range 20 < A < 180 variation $\boldsymbol{f_B}$ is very slight and hence, it may be considered as constant having a mean value ~ 8.5 MeV.
3. For heavy nuclei (A > 180) $\boldsymbol{f_B}$ decreases monotonically with the increase of A and becomes 7.5 MeV for the heaviest nuclei,

the curve shows a rapid fluctuation of $\boldsymbol{f_B}$ with some peaks for very light even-even nuclei, viz., ^{4}He, ^{8}Be, ^{12}C, ^{16}0, etc. with mass number $A = 4n, n = 1,2,3$
The curve shows also peaks for nuclei with Z or N equal to 20, 28, 50, 82, 160. These nuclei are called magic number nuclei. The EB valves of these nuclei are large representing their high stability. The high stability of the nuclei is reflected in high abundance of isotopes in nature. The phenomenon of nuclear fission, fusion as well as a-decay involving high energy release can be qualitatively explained with the help of $\boldsymbol{f_B}$ vs. A curve. It is evident that nuclear fusion is favourable for very light nuclei, whereas heavy nuclei are involved in the process of nuclear fission.

Complementarity of binding energy per nucleon and packing fraction curves.

It can be easily shown that the binding energy per nucleon vs. mass number curve is complementary to the packing fraction vs. mass number curve. Binding energy $E_B = \left[\left\{Zm_p + (A-Z)m_n\right\} - M(A,Z)\right]$ in energy unit.

$$= Z(1+f_p) + (A-Z)(1+f_n) - A(1+f) = Zf_p + Nf_n - Af$$

Hence $\boldsymbol{f_B} = \frac{E_B}{A} = \frac{Zf_p+Nf_n}{A} - f$

The term $\frac{Zf_p+Nf_n}{A}$ is approximately constant for lower A-values, when Z≈ N ≈A/2. So, $\boldsymbol{f_B}$ increases or decreases as f decreases or increases, i.e., the $\boldsymbol{f_B}$vs. A curve and f *vs.* A curve is complementary to each other. The minimum of $(f — A)$ curve and the maximum of ($\boldsymbol{f_B}$—A) occures at the same value of A. Also for lower A-values the region of negative slope of $(f — A)$ curve corresponds to the region of positive slope in ($\boldsymbol{f_B}$ —A) curve. For higher A-values the region of positive slope in $(f — A)$ curve corresponds to the region of negative slope in the ($\boldsymbol{f_B}$ —A) curve. Thus the complementary of binding energy per nucleon and packing curves are explained.

Why we get maxima and minima peaks in B.E curve?

Maxima and minima peaks in B. E curve. The five peaks in B.E curve correspond to He^4, Be_4^8 , C^{12}, 0^{16} and Ne_{10}^{20} nuclei. These nuclei have even number of protons and even number of neutrons, i.e. these do not have any unpaired nucleon. These nuclei are, therefore, more stable than their immediate neighbours and have a higher value of binding energy giving rise to a maxima.

The binding energy on either side of these nuclei falls sharply due to the unpaired nucleon and hence give rise to minima at H^I,Li^7, B^9 etc.

Why is binding energy curve so significant?

The B.E curve is highly significant. The fact that binding energy exists at all means that nuclei more complex than single proton of hydrogen can be stable. Such stability, in turn, accounts for the existence of various elements and hence explains the reason for the existence of different forms of matter. The B.E curve rises slowly as mass number A increases, has a peak value in the middle at Fe_{26}^{56} and then slowly slopes downward.

As the binding energy per nucleon increases with mass number A, it explains the cause of release of energy in the fusion of light nuclei into heavier ones. The energy which comes from the fusion of protons and neutrons to form heavier nuclei explains how sun and stars get their energy i. e. indirectly the curve explains the evolution of the universe.

On the other hand, breaking of heavier nuclei into lighter ones (fission) also releases energy which is being used for production of electric energy in nuclear reactors for peaceful purposes and also in weapons of war thereby bringing about a change in modern civilisation.

What will happen if $\boldsymbol{f_B}$ versus A curve has only positive slope.

If $\boldsymbol{f_B}$ versus A curve has only positive slope. If the B/A (binding energy per nucleon) versus A (mass number) curve has only a positive slope it would mean that binding energy per nucleon goes on increasing with mass number. In such a case energy would be released only by fusion i.e. combining of lighter nuclei into heavier one and not by fission i.e. breaking of heavier nuclei into lighter ones.

Classification of elements. This theory also leads to more refined classification of elements as follows:

(a) **Isotopes.** These are nuclei having the same atomic (proton) number Z but different atomic masses (A) e.g.O_8^{16} & O_8^{17}.
The isotopes of an element contain the same number of protons but different number neutrons. All the isotopes of an element have identical chemical properties and differ physically only in mass.

(b) **Isotones.** These are nuclei having the same neutron number N e.g., C_6^{13} & N_7^{14}. Both have $(13 — 6) = 7$ or $(14 — 7) = 7$ neutrons.

(c) **Isobars.** These are nuclei having the same mass number A (total number of nucleons i.e., (protons plus neutrons) but different atomic number Z, e.g., C_6^{14} & N_7^{14}. Both have the same mass number A =14 but Z = 6 for C^{14} and Z = 7 for N^{14}.

(d) **Isomers.** These are atoms which have the same atomic mass A and same atomic number Z but differ from one another in their nuclear energy states. They also exhibit differences in their internal structure and have different life times. They are also known as isomeric nuclei.

(e) **Mirror nuclei.** Nuclei with same mass number A but with proton and neutron number interchanged i.e., the number of protons in one is equal to the number of neutrons in the other, are called mirror nuclei e.g., B_4^7 $(Z = 4, N = 3)$ and Li_3^7 $(Z = 3, N = 4)$.

2.NUCLEAR STRUCTURE

Nature of forces between nucleons, nuclear stability and nuclear bindings. The liquid drop model (descriptive) and the Bethe –Wiezacker semi-empirical mass formula, application of stability consideration, extreme single particle shell model (qualitative discussion with emphasis on phenomenology with examples)

Need for nuclear models

There is in number of features of the nucleus that need to be explained in terms of nuclear model

1. Why the binding energy per nucleon is almost constant?
2. The shape of the binding energy curve needs an explanation.
3. Why the magic number nuclei are stable, why double magic number nuclei are more stable?
4. Why even-even nuclei are very stable? Why there is hardly any isotope with odd values of Z & N?
5. Certain nuclei emits all α −particle, β −particles though these particles do not exists within the nucleus- needs an explanation
6. why do nuclei have spins that is measured for them?
7. Although in nucleus in ground state is more stable, what is case of excited state of the nucleus and nuclear energy level.

Name of various models of nucleus which have been suggested are

1. alpha particle model. 2. Degenerate gas model. 3. Liquid drop model. 4. Shell model & 5. Collective model.

Give the main assumptions of liquid drop model of the nucleus

this model was proposed by Neals Bohr in 1937 uses the concept of potential barrier developed by Gamow. According to this model, the nuclei of all elements behave like a liquid drop of electrically charged, incompressible liquid of constant density but varying mass.

Assumptions:

1. The nucleus is supposed to be spherical in shape in its stable state just as a liquid drop due to symmetrical force of surface tension.
2. The volume as well as the mass of nucleus being proportional to the atomic mass number A, the density of the nucleus is constant, independent of volume. Similarly, the density of a liquid drop is constant independent of its volume.
3. The nucleons are supposed to move about within a spherical enclosure represented by the nuclear potential barrier just as the molecules move about within a spherical drop of the liquid, the shape of which is determined by properties of surface tension.
4. Just as the latent heat of vaporization is constant for liquid, the binding energy per nucleon is also constant.
5. The molecule evaporates from a liquid drop on rising the temperature of the liquid due to their increased energy of thermal agitation. Similarly, when energy is given to a nucleus by bombarding it with the nuclei projectile, a compound nucleus is formed which emits nuclear radiation almost immediately.
6. When a small drop of liquid is allowed to oscillate it breaks up into two similarly, a nucleus when oscillate breaks up into two smaller nuclei.

Justify the name liquid drop model

from the Above of assumptions, we find that the properties of a nucleus is very similar to the properties of liquid. Hence it is justified to call the model of nucleus as liquid drop model.

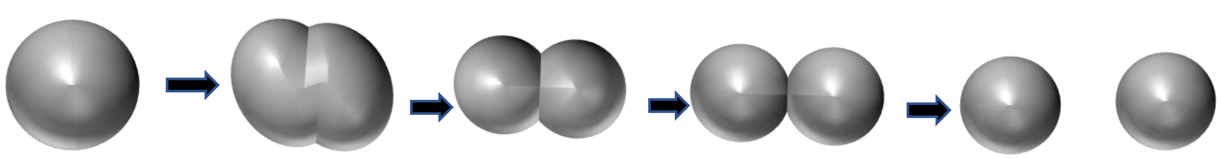
Achievements (utility) of liquid drop model

1. **Stable nucleus:** We can explain the stability of the nucleus on the basis of liquid drop model. The stability of the liquid drop is due to the force of cohesion between molecules, similarly the stability of the nucleus is due to binding energy of each nucleon just as to remove a molecule from a liquid drop energy has to be supplied to it in the form of liquid drop energy has to be supplied to it in form of heat energy to the nucleon equal or greater than the binding energy to remove it.

2. **Radioactive nucleus: -** In a liquid, with the increase of temperature thermal agitation of its molecule becomes more and more rapid and a particle of stage evaporation may take place. A molecule, in a liquid drop, evaporates by gaining energy from its neighboring molecules during the process of coalitions. Similarly, a nucleon or a group of nucleons may leave the nucleus by gaining energy from the neighboring nucleons during the process of collision thus exhibiting the phenomenon of radioactivity.

3. **Artificial radioactivity: -** The liquid drop model explains the phenomena of artificial radioactivity. It is supposed that point in nucleus is bombarded by fast-moving particles, an incoming particles enter the target nucleus forming a compound nucleus. It quickly sheers its energy with the nucleon already present for that no single particle has sufficient energy for escape. The decay of disintegration of the compound nucleus occurs when the energy is again accidentally concentrated on someone particle which escapes giving rise to the phenomenon of artificial radioactivity or energy may be lost by emission of gamma rays.

4. **Fission:-** nucleus is normally maintained in equilibrium under the combined action of short-range nuclear force of attraction among the nucleons in it which try to maintain the shape of the nucleus as such and long-range Coulomb force of repulsion among the protons in it which try to destroy the shape. When some energy is imparted to the drop, she threw the capture of a neutron, oscillations are set up in drop which aimed to destroy the spherical shape of the nucleus, while the surface tension force try to restore it. When the excitation energy is sufficiently large that, compound nucleus formed in an excited state and is sufficiently distorted in shape like dumbbell exceeds the nuclear force holding the nucleons, the nucleus breaks of into two fragments and fission is said to be take place.



5. Binding energy (Bethe –Wiezacker semi-empirical mass formula).

The semi-empirical formula for the nuclear masses or nuclear binding energy gives a connection between the theory of nuclear matter with experimental in formation and based on the liquid drop model of the nucleus.

The binding energy $\boldsymbol{E_B}$ **can be expressed** as the sum of a number of terms given below.

1. <u>Volume energy term:-</u>	The first term $B_V = a_V A$, if the volume effect representing the volume energy of all nucleons. The larger the total number of nucleons A, the more difficult it is to remove an individual nucleon from the nucleus. Since the nuclear density is nearly constant, the nucleus of mass is proportional to the nuclear volume which again is proportional for spherical nucleus to R^3 where R is the radius of the nucleus. We know that $R \propto A^{\frac{1}{3}}$. Then we can say the volume energy term $B_V \propto A$ hence $B_V = a_V A$ where a_V is a constant called volume coefficient= 14 MeV.
<u>Note that: - a.</u> the basis for volume energy term in the strong nuclear force. The strong force affects both protons and neutrons and as expected this term is independent of Z because the number of pairs that can be taken from A number of particles is $\frac{A(A-1)}{2}$one might expect a term proportional to A^2 however the strong force has a very limited range, and a giving number of nucleon may only interact strongly with its nearest neighbours and next nearest neighbours. Therefore, the number of pairs of particles that actually interact is roughly proportional to A. **<u>b.</u>** The coefficient a_V is smaller than the binding energy of nucleons to the neighbours $\boldsymbol{E_b}$, which is order of 40 MeV. This is because large the number nucleons in the nucleus means larger kinetic energy, due to Pauli exclusion principle. If one treats the nucleus as a 'Fermi Ball' of A nucleons, the total kinetic energy is equal to $\frac{3}{5} A\, \epsilon_F$. Where ϵ_F =Fermi energy=28 MeV. That the expected value of a_V in this model= $\frac{3}{5}\epsilon_F \approx 17$ MeV which is not far from measured values.	
2. **<u>Surface energy term:-</u>**	the nucleus on the surface of the nucleus are not completely surrounded by other nucleons, which lead to the reduction of total binding energy due to nucleons on the surface. As is the volume term suggests that each nucleon interacts with the constant number of nucleons independent of A but for the nucleons on the surface of the nucleus have fewer nearest neighbours, justifying this correction.
So, the correction due to surface energy B_s is proportional to the surface area of the nucleus and hence it is proportional to R^2that means it is proportional to $\boldsymbol{A^{\frac{2}{3}}}$**. Then we can write** $B_S = a_S \boldsymbol{A^{\frac{2}{3}}}$ where a_S is a constant called surface coefficient $= 13\ MeV$. B_S is analogues to surface tension of the liquid.	
3. **<u>Coulomb energy term</u>**	The Coulomb repulsion tends to disrupt the nucleus and, therefore its effect appears as in the negative term in the binding energy expression. Since it charge particle repulse all other charged particles, this term is proportional to the possible number of combinations for a giving proton number Z, which is $\frac{Z(Z-1)}{2}$.
The energy of interaction between the protons being inversely proportional to the distance of separation R that the energy associated Coulomb repulsion is $B_c = k\frac{Z(Z-1)}{2R} = k\frac{Z(Z-1)}{2\boldsymbol{R_0 A^{\frac{1}{3}}}} = a_C \boldsymbol{Z(Z-1)A^{-\frac{1}{3}}}$ where a_S is a constant called Coulomb coefficient $= 0.60\ MeV$.	

4. <u>Asymmetry energy term:-</u>

The nuclei are more stable if the number of protons be equal to the number of neutrons. With the increase of value of A, the number of neutrons increases. This transition from symmetric to asymmetric configuration reduced the stability of nuclei that means binding energy decreases.

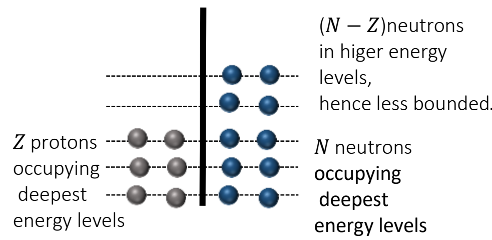
The asymmetric energy is directly proportional to

a. the neutron excess (N-Z) present in the asymmetric nuclei.
b. The fraction of nuclear volume in which the excess neutrons are present. The nuclear volume being proportional to A, the fractional volume of the nucleus in which excess neutrons are at present will be proportional to $\frac{N-Z}{A}$

Hence $B_N \propto \frac{(N-Z)^2}{A}$ **Then** $B_N = a_N \frac{(A-2Z)^2}{A}$ where a_N is a constant called asymmetric coefficient $= 19\ MeV$.

<u>The asymmetry gives a disruptive term to the nuclear binding energy</u>. This can be understood as follows.

Let Z protons and N neutrons are put into a nucleus. Then the lowest Z energy levels are field of first and the excess (N-Z) neutrons by Pauli's exclusion principle, must go to higher unoccupied quantum states as the first Z quantum states are already filled up with protons and neutrons as shown in figure. Consequently, the excess neutrons are less tightly bound than the first 2Z nucleons occupying the deepest energy levels. Thus, the asymmetry leads to a disruptive term to nuclear binding energy.



Pairing energy term:- so far all the energy terms have involved a smooth variation of binding energy, when-ever the proton number Z or the neutron number N are changed. But binding energy per nucleon versus A curve shows number of kinks and there is evidence of favored pairing.

Some nucleus with magic numbers Z (or N) or double magic number Z and N have large binding energies. **That means magic number cannot be explained on the basis of liquid drop model again in this model that intrinsic spin of the nucleons and the shell effect are disregarded.** Note that the omission has been partly corrected by adding is pairing energy term B_P to the nuclear binding energy.

The data indicate that the nuclei with even Z and even N are the most stable and most abundant. Odd nuclei are least stable. The odd-even nuclei have intermediate stability. The correction term $B_P = \pm\frac{\delta}{A^{\frac{3}{4}}}$ **where** δ is a constant

Even A	*Z=even , A= even, i.e., N=even*	$\delta = -33.5$ MeV	$B_P = -\frac{\delta}{A^{\frac{3}{4}}}$
	Z=odd , A= even, i.e., N=odd	$\delta = +33.5$ MeV	*0*
Odd A	*Z=even , A= odd, i.e., N=odd*	$\delta = 0$	*0*
	Z=odd , A= odd, i.e., N= even	$\delta = 0$	$B_P = \pm\frac{\delta}{A^{\frac{3}{4}}}$

$$\boldsymbol{E_B} = a_V \boldsymbol{A} - a_S \boldsymbol{A}^{\frac{2}{3}} - a_C \boldsymbol{Z}(\boldsymbol{Z}-\boldsymbol{1})\boldsymbol{A}^{-\frac{1}{3}} - a_N \frac{(\boldsymbol{A}-\boldsymbol{2Z})^2}{\boldsymbol{A}} \pm \frac{\delta}{\boldsymbol{A}^{\frac{3}{4}}}$$

The single particle shell model effectively reduces the many nucleon problems into a single particle problem. The model is the simplest nuclear model of obtained by applying quantum mechanics.
Assumptions on single particle shell model
The Shell model is based on the assumption that

1. each nucleon stays in a well-defined quantum state. In contrast to the situation in atom the nucleus has no massive central body acting as a fixed force centre of charge.
2. It is assumed that each nucleon experiences a central attractive force, which can be described to the average effect of remaining (A-1) nucleons acting as a core in the nucleus.
3. The nucleon is confined to its own orbit completing several revolutions before being disturbed by collision with the others. That is, the mean free path between collisions among the nucleons is much larger than nuclear diameter.
4. The Shell model thus assumes the interaction among the nucleons.

Note that the weak interaction paradox was solved by Weisskopf by invoking Pauli's exclusion principle. The nucleons being fermions cannot occupy identical quantum states. Hence experimentally expected strong interaction among the nucleons in the nucleus cannot show up itself because all low-lying quantum states into which the nucleon might get scattered out of the collision are already occupied by other nucleons.

3. UNSTABLE NUCLEI

(a) alpha decay: α −particle spectra, velocity and energy of α −particle. Geiger's-Nuttal Law.

What do you meant by α −decay?

α −decay is a nuclear process in which an α −particle is spontaneously ejected from a nucleus. In this process the parent loss an aggregate of two protons and two neutrons. The process may be expressed as ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$ where is ${}^{A}_{Z}X$ parent nucleus and ${}^{A-4}_{Z-2}Y$ is daughter nucleus.

The process is energetically possible if sum of the binding energies of the last two protons and two neutrons in the nucleus is less than the α −binding energy of the value 28.3 MeV.

Disintegration energy of spontaneous emission of α −particle

Let us consider single α −decay process represented by ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He + Q_\alpha$

Hence disintegration energy-$Q_\alpha = (M_X - M_Y - M_\alpha)c^2$ where M_X, M_Y & M_α are the mass of parent, daughter and α −particle.

From conservation of momentum, $0 = M_\alpha v_\alpha - M_Y v_Y$ OR, $v_Y = \frac{M_\alpha v_\alpha}{M_Y}$ [assuming parent nucleus be at rest during decay].

Then, $Q_\alpha = \frac{1}{2}M_\alpha v_\alpha^2 + \frac{1}{2}M_Y v_Y^2 = \frac{1}{2}M_\alpha v_\alpha^2 + \frac{1}{2}M_Y\left(\frac{M_\alpha v_\alpha}{M_Y}\right)^2 = \frac{1}{2}M_\alpha v_\alpha^2\left(1 + \frac{M_\alpha}{M_Y}\right) = T_\alpha\left(1 + \frac{M_\alpha}{M_Y}\right) \approx T_\alpha\left(1 + \frac{4}{A-4}\right)$

$Q_\alpha = T_\alpha \frac{A}{A-4}$ For A large $Q_\alpha \approx T_\alpha$

This means for large nuclei, most of disintegration energy is carried away by α −particle.

Why is the energy of α −particles are discrete in alpha decay?

The formation of an α −particle within the nucleus makes available the kinetic energy of the particle must have to escape from the nucleus. The kinetic energy known as disintegration energy Q_α. Disintegration energy released in the process of emission of an α −particle is given by $Q_\alpha = (M_P - M_D - M_\alpha)c^2$. Where M_P, M_D & M_α are the mass of parent, daughter and α −particle.
Thus if the daughter nucleus does not carry in the energy of the alpha particle has a discrete value of energy as the all other factors are constant. Even if the daughter nucleus carries some energy the alpha particle energy will be less by the same amount but it will still have a discrete value.

Show that ${}^{236}_{94}Pu$ will be spontaneously decay by α − emission. Given $M_{Pu} = 236.046$ u , $M_U = 232.037$ u , $M_{He} = 4.0020$ u,

- The decay mode is ${}^{236}_{94}Pu \rightarrow {}^{232}_{92}U + {}^{4}_{2}He + Q_\alpha$

$\frac{Q_\alpha}{c^2} = (M_{Pu} - M_U - M_\alpha) = (236.046 - 232.037 - 4.0020)\text{u} = 0.007 \text{ u} > 0$

Then spontaneous decay is possible.

The K.E of the α −particle. Is nearly equal to $\frac{232}{236}(0.007 \times 931.5)$ MeV= 6.40 MeV

Polonium 212 emits an α −particle of 8.776 MeV energy. Calculate the disintegration energy.

- $Q_\alpha = T_\alpha \frac{A}{A-4} = 8.776\left(1 + \frac{4}{208}\right) = 8.945$

Calculate the K.E of the α −particle emitted by ${}^{222}_{84}Rn$. Given that $M_{Rn} = 222.017531$ u M_{Po} 218.0089 u $M_{He} = 4.0020$ u,

Here $\frac{Q_\alpha}{c^2} = (M_{Rn} - M_{Po} - M_\alpha) = (222.017531 - 218.0089 - 4.0020)\text{u} = 0.016278 \text{ u} > 0$

$Q_\alpha = 0.016278 \times 931.5 \text{ u} = 15.256$ MeV

$T_\alpha = Q_\alpha \frac{218}{222} = 14.98$ MeV

Explain why proton emission (spontaneous) is not possible from $^{232}_{92}U$

Let consider the imaginary decay process $^{232}_{92}U \rightarrow {}^{231}_{91}Pa + {}^{1}_{1}H + Q_p$

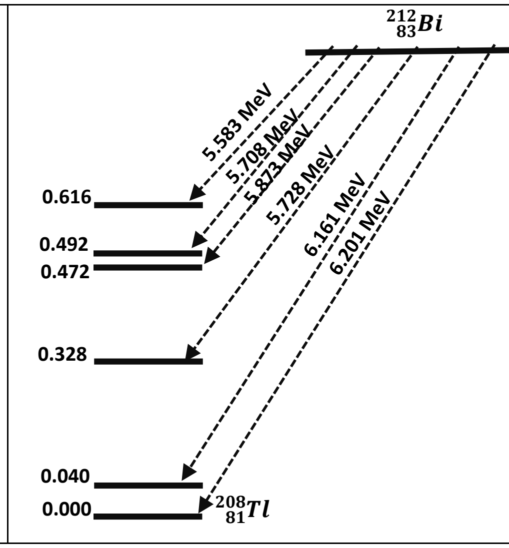
$\frac{Q_\alpha}{c^2} = (M_U - M_{Pa} - M_p) = (232.0371562 - 231.03588 - 1.007276)\text{u} = -5.99 \times 10^{-3}\ \text{u} < 0$

$Q_\alpha = -5.99 \times 10^{-3} \times 931.5\ \text{u} = -5.58\ \text{MeV}$

Here disintegration energy is negative hence emission is not possible spontaneously.

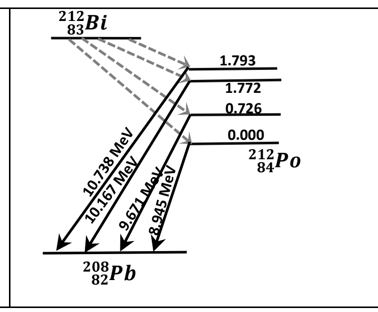
What is meant what is meant by the structure α −ray spectrum? How because it leads to the existence of energy levels in nuclei?

Accurate Energy measurements of α −particle emitted by the radioactive substance indicate that the number of α −particle emitter show a group of discrete α −particle energies rather than a single energy. This is known as fine structure in α −ray energy spectrum.
As an example consider the decay of $^{212}_{83}Bi$ by α −emission to Thallium $^{208}_{81}Tl$. The most energetic α −ray corresponds to a disintegration energy Q_α = 6.201 MeV. In addition to this, there are α-rays corresponding to disintegration energies of 6.161, 5.873, 5.728, 5.708 and 5.583 MeV giving rise to a fine structure.
The fine structure in the α −ray spectrum can easily be explained in terms of nuclear energy levels. The nuclear-energy level diagram for $^{212}_{83}Bi$ decaying to $^{208}_{81}Tl$ is shown in Fig1. According to the explanation, when a parent element decays by α-emission, the daughter element may go over to any one of the number of allowed energy states. The state corresponding to the lowest energy is known as the ground state and those corresponding to higher energies are called excited states.



The most energetic α-particle corresponds to decay to the ground state of the daughter nucleus and other α-particles correspond to decay to the excited state of the daughter nucleus. The difference Fig. 24.10. between pairs of disintegration energies gives directly the energy difference between various levels of the daughter nucleus. Thus, the first excited level of $^{208}_{81}Tl$ has an energy $(6.201 - 6.161) = 0.040$ MeV and the second excited level has an energy (6.201 - 5.873)= 0.328 MeV and so on. Thus, the fine structure in the α-ray spectrum not only gives an indication of the existence of energy states in the nucleus but also provides a method of measuring their energies. A daughter nucleus formed in an excited state sooner or later passes to its ground state successive emission of one or more γ— rays. The energies of these coincide γ— rays within experimental error with the energy level difference found from α-particle fine structure.

Long range a-particles. Decay of $^{212}_{83}Bi$ by α −emission to Thallium $^{208}_{81}Tl$ happens in 34% cases. In the remaining 66% of the cases, $^{212}_{83}Bi$ decays to $^{212}_{84}Po$ by the emission of a ß-particle. In has been observed that $^{212}_{84}Po$ emits α-particles a few of which have energies much greater than that of the main group. These are known as long range a-particles. For example, the main group of α-particles from $^{212}_{84}Po$ − corresponds to a disintegration energy of 8.945 MeV whereas the long range α-particles energies of 9.67 MeV, 10.167 MeV and 10.738 MeV. The explanation is again based on energy levels. When $^{212}_{83}Bi$decays by the of a ß-particle, $^{212}_{84}Po$ which is formed, may exist in the ground state or in one of the excited states.

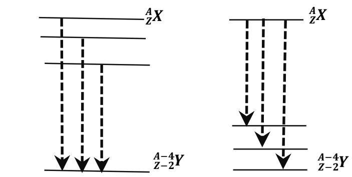


As $^{212}_{84}Po$ is very short lived in the ground state, the excited states are even more short lived. Normally these excited nuclei pass to the lower level within 10^{-12}sec. or so by γ −emission. But this γ-emission is a relatively slow process on a nuclear time scale and sometimes the excited nucleus emits an α-particle in less than 10^{-12}sec. i.e., before the nucleus can de-excite itself by γ −emission. This α-particle carries more energy than the normal i. e., the energy from the ground state of the parent to the ground state of the daughter plus the energy from the excited state of the parent to the ground state of the parent. Thus, the study of the long range α-particles gives us information about the energy levels of the parent α-emitter itself, whereas the study of fine structure α-ray spectrum gives us information about the energy levels of daughter nucleus.

Discuss the origin of fine structure of α-ray spectra in short.

α −emission by the radioactive substance are usually mono-energetic. This shows that they are emitted from a definite energy state of parent nucleus to a definite energy state of a daughter nucleus (product nucleus).

In case of some nuclei more than one group of α −particles, each of a definite energy are found to be emitted by some radioactive substance. This can happen due to transmission from different discrete states (ground or excited) of the parent nucleus to the different discrete states (ground or excited) of the daughter nucleus.



Origin of long range alpha particle   Origin of fine structure of alpha particle

Transition may be grouped into two classes-

1. Transition from different excited state of parent nucleus to the ground state of the product nucleus (long-range)
2. Transition from ground state of parent nucleus to the different excited states of product or daughter nucleus (fine structure).

Note that: if the product nucleus is formed in the excited state, then it subsequently emits γ − rays to transit to the ground state.

What is meant by a range of an α −particle? What the factors that determine the range of the α −particle?

The range of an α −particle in any medium is defined as the distance traversed by a α −particle before stopping to ionized it. In other words, the range is sharply defined ionization path length in a medium.

The range of α −particle in a gas depends on the following factors:

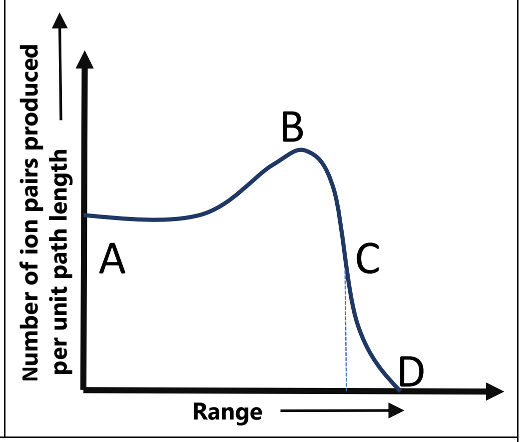
1. the initial energy of the α −particle.
2. The Irish and potential of the gas or the medium.
3. The chance of collision between the α −particle and the atoms and molecules of the medium.
4. On the nature, temperature and pressure of the medium.

Note that: the range of α −particle is highest in the gases, less in liquids and least in solids due to more and more packing of the particles in the medium.

Unit of range:	Range of an ionizing particle is measured either in unit of length or unit of mass of the material. In unit of length, the range of a few MeV α −particle in solids and liquids is$\sim 10^{-3} mm$, which is very small.
In unit of mass the range is defined as the mass per unit area (kg.m^{-2}). It is actually the mass of matter of the cylinder of unit cross section and length equal to the range R of the α −particle. If R be the range and ρ density of the substance, then amount of mass in aforesaid cylinder is $R\rho$. In solid liquid or gas the order of magnitude of range is same when it is expressed in unit of mass.	
Geiger's empirical relation:-	Geiger experimentally found that for more energetic α −particles moving with velocity v, the range in standard air is proportional to v^3 that is $R = a\, v^3$ where a is a constant having value 9.416×10^{-24}. This is known as Geiger's law.
Which is valid only in a limited velocity range. Again since energy $E = \frac{1}{2} m\, v^2$then $R \propto E^{3/2}$ Or $R = bE^{3/2}$ Where $b = 3.25 \times 10^{-3}$is a constant. If E is expressed in MeV, then R is in meter. Note that actually at lower range of velocity R is approximately proportional to $v^{\frac{3}{2}}$ & $E^{\frac{3}{4}}$ And higher range of velocity to v^4 & E^2.	
Specific ionization:-	The specific ionization is defined as the number of ion pairs produced per unit path length of the α −particle and is denoted by I. Since $E \propto R^{2/3}$ then $I = \frac{dE}{dR} \propto R^{-\frac{1}{3}}$ Or $\frac{dE}{dR} \propto v^{-1}$.
Thus the ionization produced by an α −particle at any point in its path is inversely proportional to its velocity at that point.	
Validate the of Geiger's law:-	Geiger's law is valid for a limited medium velocity range for any kind of charge particles in any medium. It does not hold for a new value of v rather it is valid for $v \ll c$ where c is velocity of light in free space.

Struggling of range:-	The range of the α −particles, having same initial kinetic energy are more or less same. The small spread of the values of the range about mean range of the α −particles, having same initial kinetic energy is known as struggling of the range of α −particle.

The plot of number of ion pairs produced along the path of an of an α −particle against the distance from the source is shown in figure. It is observed that towards the end of the curve the number of ion pairs produced at a maximum and then steeply drops down to zero the curve is called the Braggs curve. And the maximum B, is called the Braggs humps. So the maximum number of ion produced immediately before the α −particles stop ionising. The reason behind it is the α −particles move slowly in the region and as such find more time for interaction with the atoms and the molecules. At the point the ion density sharply reduced to zero giving the range of the α −particles. The Braggs curve shows that the ionization is fairly constant in the initial path and tries to the humps towards the end when the energy of the α −particles falls below the ionisation energy of the gas, a sharp fall in the curve occurs but the curve does not meet the X axis abruptly instead it tails off called the struggling.



The curve BCD, through a steep, has a finite slope. This would be abruptly vertical downward if the α −particles with the same kinetic energy would make equal number of collisions. The curve CD is due to struggling of the α −particles

Factors causing struggling:-

Struggling in the range of α −particles arise due to

1. statistical fluctuations in the number of collisions
2. statistical fluctuations about a mean value in the energy loss per collision.
3. Apart from the Apple factors multiple scattering during collisions, inhomogeneity in the density of the observer and electron capture also contribute to the process of struggling.

Derivation of Geiger's law:-

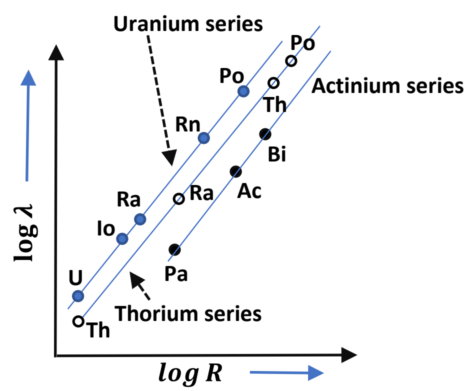
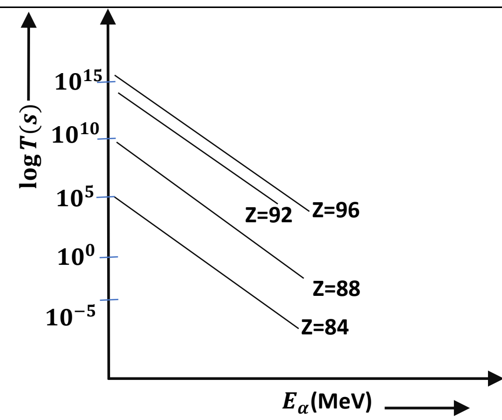
While passing through medium an α −particle loses more energy per unit length near the end of the range where they move more slowly and interact with atoms for more time.

Assuming energy loss per unit length of the medium inversely proportional to the velocity we have $-\frac{dE}{dx} = \frac{k}{v}$

now non-relativistically kinetic energy of an α −particle is $E = \frac{1}{2} m\, v^2$ then $dE = mvdv$ hence $-\frac{mvdv}{dx} = \frac{k}{v}$

hence $\int_0^R k\, dx = -\int_v^0 m\, v^2\, dv$ or, $kR = \frac{1}{3}\, m\, v^3$ then we can write $\boldsymbol{R = a\, v^3}$ where a is a constant$= \frac{m}{3k}$

stopping power:-	It is the amount of energy loss of an α −particle per unit length in an absorber.
Equivalent stopping power	It is the thickness of the absorber that produces the same energy loss of the alpha particle as does unit thickness of standard air.
The relative stopping power	It is the ratio of the energy loss of the α −particle in travelling a unit distance in the absorber to that in the travelling in unit distance in standard air.
Mass stopping power	It is defined as stopping power divided by the density of the absorber. Mass stopping power$= -\boldsymbol{\frac{1}{\rho}\frac{dE}{dx}}$
atomic stopping power	It is defined as mass stopping power divided by number of atoms per volume of the material. Atomic stopping power $= -\boldsymbol{\frac{1}{n\,\rho}\frac{dE}{dx}}$

<table>
<tr><td><u>Geiger’s and Nuttall law</u></td><td>Geiger’s and Nuttall in 1911, experimentally obtained an important relation between the range of the α −particle and decay constant of the emitting nuclei in form $\boldsymbol{\log_{10}\lambda = A + B\log_{10} R}$, known as Geiger’s and Nuttall law.</td></tr>
<tr><td colspan="2">Here A and B are constants having values different for different radioactive series. According to this law α −emitter having large λ having large range and vice versa. [Note that half life $T = \frac{0.693}{\lambda}$]</td></tr>
<tr><td><u>The importance of Geiger’s and Nuttall law</u></td><td>The importance of Geiger’s and Nuttall law is that it enables us to estimate the half-lives of common alpha emitters that could be determined easily experimentally.</td></tr>
<tr><td colspan="2"><u>Note that:</u>
1. Since $R \propto E^{3/2}$the Geiger’s and Nuttall relation can also be written as $\boldsymbol{\log_{10}\lambda = C + D\log_{10} E}$
2. Since half-lives $T = \frac{0.693}{\lambda}$, Geiger’s and Nuttall law also be expressed by the variation of $\boldsymbol{\log T}$ with $\boldsymbol{\log R}$ OR, $\boldsymbol{\log E}$ the straight lines are with negative slope.
3. Geiger’s and Nuttall law was theoretically derived by Gammow to explain quantum mechanical tunneling.
4. Geiger’s and Nuttall law is very accurate in its modern form $\boldsymbol{\ln\lambda = -a_1\frac{Z}{\sqrt{E}} + a_2}$ where a_1, a_2 are constant.
5. The law works best for nuclei with even Z and even A the trend is still true for even-odd, odd-even and odd-odd nuclei but not as pronounced.</td></tr>
<tr><td>
</td><td>
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<u>In order</u> to understand the mechanism of α-decay it is essential to have a thorough knowledge of transmission of plane wave through a potential barrier, with special reference to tunnel effect.

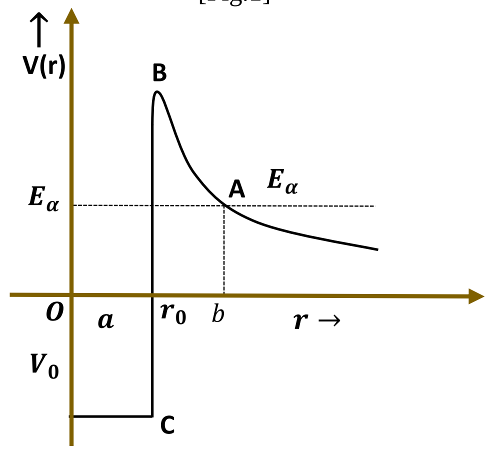
<u>What is tunnel effect?</u> If an α-particle is impinging on the barrier with energy certainly less than the height of potential barrier, it will not necessarily be totally reflected by the barrier but there is always a probability that it may cross the barrier and continue its forward motion. The probability of crossing the barrier is known as tunnel effect.

According to Rutherford's α-scattering experiment, the fastest α-particle available cannot penetrate the nucleus to show departure from Coulomb's law. This means the potential energy up to a distance 2×10^{-2}cm is still given by $V(r) = \frac{2Ze^2}{4\pi\varepsilon_0 r}$ Coulomb's law breaks down at distances smaller than 2×10^{-2}cm. The repulsive potential pre-vents the α −particle from penetrating the nucleus and as such may be considered to form a potential barrier. The plot is V(r) against r is shown in Fig1.

The rise of the curve from A to B is due to the increase of repulsive force as the α −particle approaches the nucleus. The shape of the potential energy curve for region $r < r_0$ inside the nucleus is not known as the nature of the nuclear force in the said region is not exactly known but can be represented by a constant attractive potential V_0. Over the distance r_0, called the effective radius of the nucleus. Potential of this nature is called a potential well of depth V_0 and width r_0.

For the existence of the α-particle inside the nucleus it should have energy (E_α — V), where E_α represents the α −disintegration energy, when the particle is far from the nucleus. To escape from the nucleus, the α-particle should have K.E (E_α) at least equal to the value of the maximum energy (V_m) in the potential energy curve [Fig. 1]. The α-particle approaching the nucleus from outside can penetrate the nucleus provided its K.E is greater than the maximum energy(V_m) shown in the potential energy curve Fig.1

Coulomb potential barrier for α-disintegration [Fig.1]



Thus, for an a-particle to escape from the nucleus or enter into it E_α must be greater than the maximum energy in the curve of Fig.2. But α-particles from ThC' are found to scatter by uranium. This is possible provided the uranium nucleus emits α-particle with energy greater than 9 MeV. But experiments show that the energy of the a-particles emitted by uranium is 4 MeV. It is thus surprising how an α-particle with energy 4 MeV can go over a potential barrier, which is more than twice as high as their total energy.

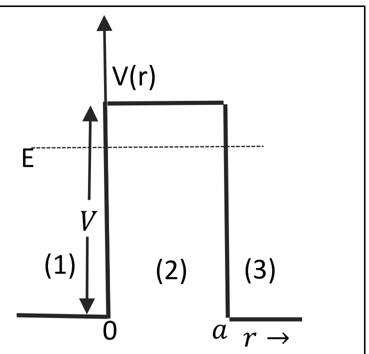
In view of classical mechanics, this paradox cannot be solved. Wave mechanics, however, shows that a certain probability of the α-particle penetrating through the potential barrier does exist. The problem was solved by Gamow, Gurney and Condon and is known as the Gamow's theory a-decay.

Gamow's theory of a-decay

Gamow's theory is regarded as a successful theory of a-decay due to the following reasons:

1. The theory helps to derive an expression for the decay constant.
2. It explains the phenomenon of α-disintegration.
3. A relation between λ and E of the a-particle similar to Geiger and Nuttall can be established with its help.

For simplicity we consider that the one dimensional potential barrier is rectangular in shape of width a and height V (V>E)



from figure for region 1 and 3 we have $\frac{d^2\psi_1}{dr^2} + \frac{2\mu E}{\hbar^2}\psi_1 = 0$ Where $\mu = \frac{M_\alpha M_D}{M_\alpha + M_D}$

and for region 2 we have $\frac{d^2\psi_2}{dr^2} + \frac{2\mu(E-V)}{\hbar^2}\psi_2 = 0$

Let $k_1 = \sqrt{\frac{2\mu E}{\hbar^2}}$ $k_2 = \sqrt{\frac{2\mu(V-E)}{\hbar^2}}$

since region 1 and 2 has both incident and reflected wave but not for region 3.

We may write $\psi_1 = A_1 e^{i k_1 r} + B_1 e^{-i k_1 r}$; $\psi_2 = A_2 e^{k_2 r} + B_2 e^{-k_2 r}$; $\psi_3 = A_3 e^{k_1 r}$

we have the boundary condition $\psi_1 = \psi_2$ & $\frac{d\psi_1}{dr} = \frac{d\psi_2}{dr}$ at $r = 0$ $\psi_3 = \psi_2$ & $\frac{d\psi_3}{dr} = \frac{d\psi_2}{dr}$ at $r = a$

then $A_1 + B_1 = A_2 + B_2$ & $ik_1 A_1 - ik_1 B_1 = k_2 A_2 - k_2 B_2$……………………………(2)

$A_2 e^{k_2 a} + B_2 e^{-k_2 a} = A_3 e^{k_1 a}$ & $k_2 A_2 e^{k_2 a} - k_2 B_2 e^{-k_2 a} = ik_1 A_3 e^{k_1 a}$……………………(3)

$A_2 = \frac{1}{2} A_3 \left(1 + i\frac{k_1}{k_2}\right) e^{(ik_1 - k_2)a}$ ………………(4) & $B_2 = \frac{1}{2} A_3 \left(1 - i\frac{k_1}{k_2}\right) e^{(ik_1 + k_2)a}$………………(5)

From equation 2 $A_1 = \frac{1}{2} A_2 \left(1 + \frac{k_2}{ik_1}\right) + \frac{1}{2} B_2 \left(1 - \frac{k_2}{ik_1}\right)$

from equation 4 and 5 $A_1 = \frac{1}{2}\frac{1}{2} A_3 \left(1 + i\frac{k_1}{k_2}\right) e^{(ik_1 - k_2)a} \left(1 + \frac{k_2}{ik_1}\right) + \frac{1}{2}\frac{1}{2} A_3 \left(1 - i\frac{k_1}{k_2}\right) e^{(ik_1 + k_2)a} \left(1 - \frac{k_2}{ik_1}\right)$

practically $k_2 a \gg 1$ hence 1st term is neglected then $A_1 \approx \frac{1}{4} A_3 \left(1 - i\frac{k_1}{k_2}\right)\left(1 - \frac{k_2}{ik_1}\right) e^{(ik_1 + k_2)a}$

$\frac{A_1}{A_3} \approx \frac{1}{4}\left(1 - i\frac{k_1}{k_2}\right)\left(1 - \frac{k_2}{ik_1}\right) e^{(ik_1 + k_2)a}$ or, $\left|\frac{A_1}{A_3}\right|^2 = \frac{1}{16}\left(1 + \left(\frac{k_1}{k_2}\right)^2\right)\left(1 + \left(\frac{k_2}{k_1}\right)^2\right) e^{2k_2 a} = \frac{1}{16}\frac{\left(k_1^2 + k_2^2\right)^2}{k_1^2 k_2^2} e^{2k_2 a}$

$Transmission\ probability = \frac{transmitted\ flux}{incident\ flux} = \left|\frac{A_3}{A_3}\right|^2 \frac{v}{v}$ [as velocity of α particle is same in region 1 and 3.]

$$T = \frac{16k_1^2 k_2^2}{\left(k_1^2 + k_2^2\right)^2} e^{-2k_2 a} = \frac{16k_1^2 k_2^2}{\left(k_1^2 + k_2^2\right)^2} e^{-2k_2 a} = \frac{16\frac{2\mu E}{\hbar^2}\frac{2\mu(V-E)}{\hbar^2}}{\left(\frac{2\mu E}{\hbar^2} + \frac{2\mu(V-E)}{\hbar^2}\right)^2} e^{-2\sqrt{\frac{2\mu(V-E)}{\hbar^2}}a} = \frac{16E(V-E)}{V^2} e^{-2\sqrt{\frac{2\mu(V-E)}{\hbar^2}}a}$$

Hence probability of transmission decreases exponentially as thickness a increases the dominant term here is the exponential. The energy dependence factor $\frac{16E(V-E)}{V^2}$ is of order of unity (its maximum value is 4)so for the calculation of magnitude, we can write $T = e^{-2\sqrt{\frac{2\mu(V-E)}{\hbar^2}}a} = e^{-2k_2 a}$.

In case where V is not constant then $T = e^{-2\int k_2\, dr}$

If α particle moves inside the potential with a velocity v_0

then collision frequency $\omega = \frac{v_0}{2R_0}$ where R_0 is the radius of nucleus

Then decay constant $\lambda = \omega\, T = \frac{v_0}{2R_0}\, e^{-2\int k_2\, dr}$.

Disintegration energy of spontaneous emission of α −particle

Let us consider single α −decay process represented by $\ ^{A}_{Z}X \rightarrow \ ^{A-4}_{Z-2}Y + \ ^{4}_{2}He + Q_\alpha$

Hence disintegration energy-$Q_\alpha = (M_X - M_Y - M_\alpha)c^2$ where M_X, M_Y & M_α are the mass of parent, daughter and α −particle.

From conservation of momentum, $0 = M_\alpha v_\alpha - M_Y v_Y$ OR, $v_Y = \frac{M_\alpha v_\alpha}{M_Y}$ [assuming parent nucleus be at rest during decay].

Then, $Q_\alpha = \frac{1}{2}M_\alpha v_\alpha^2 + \frac{1}{2}M_Y v_Y^2 = \frac{1}{2}M_\alpha v_\alpha^2 + \frac{1}{2}M_Y\left(\frac{M_\alpha v_\alpha}{M_Y}\right)^2 = \frac{1}{2}M_\alpha v_\alpha^2\left(1 + \frac{M_\alpha}{M_Y}\right) = T_\alpha\left(1 + \frac{M_\alpha}{M_Y}\right) \approx T_\alpha\left(1 + \frac{4}{A-4}\right)$

$Q_\alpha = T_\alpha \frac{A}{A-4}$ For A large $Q_\alpha \approx T_\alpha$

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Thus if the daughter nucleus does not carry in the energy of the alpha particle has a discrete value of energy as the all other factors are constant. Even if the daughter nucleus carries some energy the alpha particle energy will be less by the same amount but it will still have a discrete value.

Show that $\ ^{236}_{94}Pu$ will be spontaneously decay by $\alpha -$ emission. Given $M_{Pu} = 236.046$ u , $M_U = 232.037$ u , $M_{He} = 4.0020$ u,

- The decay mode is $\ ^{236}_{94}Pu \rightarrow \ ^{232}_{92}U + \ ^{4}_{2}He + Q_\alpha$

$\frac{Q_\alpha}{c^2} = (M_{Pu} - M_U - M_\alpha) = (236.046 - 232.037 - 4.0020)\text{u} = 0.007 \text{ u} > 0$

Then spontaneous decay is possible.

The K.E of the α −particle. Is nearly equal to $\frac{232}{236}(0.007 \times 931.5)$ MeV= 6.40 MeV

Polonium 212 emits an α −particle of 8.776 MeV energy. Calculate the disintegration energy.

- $Q_\alpha = T_\alpha \frac{A}{A-4} = 8.776\left(1 + \frac{4}{208}\right) = 8.945$

Calculate the K.E of the α −particle emitted by $\ ^{222}_{84}Rn$. Given that $M_{Rn} = 222.017531$ u M_{Po} 218.0089 u $M_{He} = 4.0020$ u,

Here $\frac{Q_\alpha}{c^2} = (M_{Rn} - M_{Po} - M_\alpha) = (222.017531 - 218.0089 - 4.0020)\text{u} = 0.016278 \text{ u} > 0$

$Q_\alpha = 0.016278 \times 931.5 \text{ u} = 15.256$ MeV

$T_\alpha = Q_\alpha \frac{218}{222} = 14.98$ MeV

Explain why proton emission (spontaneous) is not possible from $\ ^{232}_{92}U$

Let consider the imaginary decay process $\ ^{232}_{92}U \rightarrow \ ^{231}_{91}Pa + \ ^{1}_{1}H + Q_p$

$\frac{Q_\alpha}{c^2} = \left(M_U - M_{Pa} - M_p\right) = (232.0371562 - 231.03588 - 1.007276)\text{u} = -5.99 \times 10^{-3} \text{ u} < 0$

$Q_\alpha = -5.99 \times 10^{-3} \times 931.5 \text{ u} = -5.58$ MeV

Here disintegration energy is negative hence emission is not possible spontaneously.

<u>What is meant what is meant by the structure α −ray spectrum? How because it leads to the existence of energy levels in nuclei?</u>

Accurate Energy measurements of α −particle emitted by the radioactive substance indicate that the number of α −particle emitter show a group of discrete α −particle energies rather than a single energy. This is known as fine structure in α −ray energy spectrum.

As an example consider the decay of $^{212}_{83}Bi$ by α −emission to Thallium $^{208}_{81}Tl$. The most energetic α −ray corresponds to a disintegration energy Q_{α} = 6.201 MeV. In addition to this, there are α-rays corresponding to disintegration energies of 6.161, 5.873, 5.728, 5.708 and 5.583 MeV giving rise to a fine structure.

The fine structure in the α −ray spectrum can easily be explained in terms of nuclear energy levels. The nuclear-energy level diagram for $^{212}_{83}Bi$ decaying to $^{208}_{81}Tl$ is shown in Fig1.

According to the explanation, when a parent element decays by α-emission, the daughter element may go over to any one of the number of allowed energy states. The state corresponding to the lowest energy is known as the ground state and those corresponding to higher energies are called excited states. The most energetic α-particle corresponds to decay to the ground state of the daughter nucleus and other α-particles correspond to decay to the excited state of the daughter nucleus. The difference Fig. 24.10. between pairs of disintegration energies gives directly the energy difference between various levels of the daughter nucleus. Thus, the first excited level of $^{208}_{81}Tl$ has an energy $(6.201 - 6.161) = 0.040$ MeV and the second excited level has an energy (6.201 - 5.873)= 0.328 MeV and so on. Thus, the fine structure in the α-ray spectrum not only gives an indication of the existence of energy states in the nucleus but also provides a method of measuring their energies. A daughter nucleus formed in an excited state sooner or later passes to its ground state

successive emission of one or more γ— rays. The energies of these coincide γ— rays within experimental error with the energy level difference found from α-particle fine structure.

<u>Long range a-particles.</u> Decay of $^{212}_{83}Bi$ by α −emission to Thallium $^{208}_{81}Tl$ happens in 34% cases. In the remaining 66% of the cases, $^{212}_{83}Bi$ decays to $^{212}_{84}Po$ by the emission of a ß-particle. In has been observed that $^{212}_{84}Po$ emits α-particles a few of which have energies much greater than that of the main group. These are known as long range a-particles.
For example, the main group of α-particles from $^{212}_{84}Po$ − corresponds to a disintegration energy of 8.945 MeV whereas the long range α-particles energies of 9.67 MeV, 10.167 MeV and 10.738 MeV.
The explanation is again based on energy levels. When $^{212}_{83}Bi$decays by the of a ß-particle, $^{212}_{84}Po$ which is formed, may exist in the ground state or in one of the excited states. As $^{212}_{84}Po$ is very short lived in the ground state, the excited states are even more short lived. Normally these excited nuclei pass to the lower level within 10^{-12}sec. or so by γ −emission. But this γ-emission is a relatively slow process on a nuclear time scale and sometimes the excited nucleus emits an α-particle in less than 10^{-12}sec. i.e., before the nucleus can de-excite itself by γ −emission. This α-particle carries more energy than the normal i. e., the energy from the ground state of the parent to the ground state of the daughter plus the energy from the excited state of the parent to the ground state of the parent. Thus, the study of the long range α-particles gives us information about the energy levels of the parent α-emitter itself, whereas the study of fine structure α-ray spectrum gives us information about the energy levels of daughter nucleus.

<u>Discuss the origin of fine structure of α-ray spectra in short.</u>
α −emission by the radioactive substance are usually mono-energetic. This shows that they are emitted from a definite energy state of parent nucleus to a definite energy state of a daughter nucleus (product nucleus).
In case of some nuclei more than one group of α −particles, each of a definite energy are found to be emitted by some radioactive substance. This can happen due to transmission from different discrete states (ground or excited) of the parent nucleus to the different discrete states (ground or excited) of the daughter nucleus.
Transition may be grouped into two classes-

3. Transition from different excited state of parent nucleus to the ground state of the product nucleus (long-range)
4. Transition from ground state of parent nucleus to the different excited states of product or daughter nucleus (fine structure).

Note that: if the product nucleus is formed in the excited state, then it subsequently emits γ − rays to transit to the ground state.