



**WEST BENGAL STATE UNIVERSITY**  
B.Sc. Honours 5th Semester Examination, 2022-23

**PHSACOR11T-PHYSICS (CC11)**

**QUANTUM MECHANICS AND APPLICATIONS**

Time Allotted: 2 Hours

Full Marks: 40

*The figures in the margin indicate full marks.  
Candidates should answer in their own words and adhere to the word limit as practicable.  
All symbols are of usual significance.*

**Question No. 1 is compulsory and answer any two from the rest**

1. Answer any *ten* questions from the following: 2×10 = 20
  - (a) Explain the physical significance of energy time uncertainty relation.
  - (b) Give the physical interpretation of the wave function  $\psi(x, t)$ .
  - (c) What are stationary states in quantum mechanics?
  - (d) If the commutation relation between  $x$  and  $p$  is  $[x, p] = i\hbar$ . Find the commutation value of  $[x^2, p]$ .
  - (e) What is the implication of the result:  $[\hat{H}, \hat{L}] = 0$ ?
  - (f) Consider the operator  $\hat{Q} = i \frac{d}{d\phi}$  where  $\phi$  is usual polar coordinates in two dimensions. Write down its eigenvalue equation and find its eigenvalues.
  - (g) What is normal Zeeman effect? Under what conditions it may be observed?
  - (h) What is Larmor precession of electron in an atom?
  - (i) Explain bound and unbound states in quantum mechanics.
  - (j) A wavefunction  $\psi$  is constructed as a linear combination of a set of orthonormal eigenfunctions  $\psi_n$  :
$$\psi = \sum_{n=1}^{\infty} c_n \psi_n$$

where  $c_n$  are constants. Show that if  $\psi$  is normalized then  $\sum_{n=1}^{\infty} |c_n|^2 = 1$
  - (k) If the wavefunction of a particle trapped in space between  $x=0$  and  $x=L$  is given by  $\psi(x) = A \sin \frac{2\pi x}{L}$ , where  $A$  is a constant, for which value(s) of  $x$  will the probability of finding the particle be maximum?
  - (l) Electron configuration of Sodium is given by  $1s^2 2s^2 2p^6 3s^1$ . Find the ground state term symbol of Sodium.
  - (m) What is Stark effect?

1. (n) A beam of spin  $\frac{1}{2}$  particle is prepared in the state  $|\psi\rangle = \frac{3}{\sqrt{34}}|+\rangle + i$

where  $|+\rangle$  and  $|-\rangle$  are eigen states of  $\hat{S}_z$  with eigenvalues  $+\frac{\hbar}{2}$  respectively. Find the average value in  $S_z$  measurement.

2. (a) If  $\psi_1$  and  $\psi_2$  are two eigen states with energy  $E_1$  and  $E_2$  respectively, check whether the state  $(\psi_1 + \psi_2)$  is stationary or not. 2

(b) (i) Prove that the time rate of change of the expectation value of a dynamical variable satisfies the following relation 3+2

$$\frac{d}{dt}\langle\hat{A}\rangle = -\frac{i}{\hbar}\langle[\hat{A}, \hat{H}]\rangle + \left\langle\frac{\partial\hat{A}}{\partial t}\right\rangle \text{ where the symbols have their usual meanings.}$$

(ii) Using the above relation show that the time rate of change of expectation value of momentum is equal to the average value of force.

(c) Prove that the parity of spherical harmonics  $Y_{l,m}(\theta, \phi)$  is  $(-1)^l$ . 2

(d) What do you mean by degenerate wavefunction? 1

3. (a) The potential in a region is given as:

$$V(x) = 0 \text{ for } x < 0$$

$$= V_0 \text{ for } 0 \leq x \leq a$$

$$= 0 \text{ for } x > a$$

A particle of mass  $m$  and energy  $E < V_0$  travelling from left to the right is incident on the potential barrier.

(i) Write down Schrodinger equations in three regions of the potential. 2

(ii) Write down appropriate boundary conditions. 2

(b) The wavefunction of a hydrogen atom is given by the following superposition of energy eigenfunctions  $\psi_{nlm}(\vec{r})$  ( $n, l, m$  are the usual quantum numbers):

$$\psi(\vec{r}) = \frac{\sqrt{2}}{\sqrt{7}}\psi_{100}(\vec{r}) - \frac{3}{\sqrt{7}}\psi_{210}(\vec{r}) + \frac{1}{\sqrt{14}}\psi_{322}(\vec{r})$$

(i) Determine the ratio of expectation value of the energy to the ground state energy. 2

(ii) What are the expectation value of  $\hat{L}^2$  and  $\hat{L}_z$  operators? 2

(iii) What is the probability that the atom is found in a state of even parity? 2

4. (a) Hamiltonian for the linear harmonic oscillator is given by  $\hat{H} = \frac{1}{2}\hat{p}^2 + \frac{1}{2}m\omega^2\hat{x}^2$ , where the symbols have usual meanings. Using the basic commutation relation between  $\hat{x}$  and  $\hat{p}$  show that,

(i)  $[\hat{a}, \hat{a}^*] = 1$  and 2+2

(ii) the Hamiltonian is given by

$$H = (\hat{a}^+ \hat{a} + 1/2) \hbar \omega$$

$$\text{given that } \hat{a} = \left( \frac{m\omega}{2\hbar} \right)^{1/2} \left( \hat{x} + \frac{i}{m\omega} \hat{p} \right) \text{ and } \hat{a}^+ = \left( \frac{m\omega}{2\hbar} \right)^{1/2} \left( \hat{x} - \frac{i}{m\omega} \hat{p} \right)$$

Then find the normalized ground state wavefunction of linear harmonic oscillator.

- (b) A particle constrained to move along x-axis in the domain  $0 \leq x \leq L$  has a wavefunction  $\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$ , where  $n$  is an integer. What is the expectation value of its momentum? 3

5. (a) State Moseley's Law. Derive this law from Bohr's theory. 1+4

(b) Considering the L-S coupling scheme for helium atom, find the spectroscopic terms for (i)  $1s^1 2s^1$  and (ii)  $1s^1 2p^1$  configurations. 3

(c) In a Stern-Gerlach experiment on turning on the magnetic field, the beam splits into seven components. What is the angular momentum of the atoms in the beam? 2

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**WEST BENGAL STATE UNIVERSITY**  
B.Sc. Honours 5th Semester Examination, 2022-23

**PHSACOR12T-PHYSICS (CC12)**

**SOLID STATE PHYSICS**

Time Allotted: 2 Hours

Full Marks: 40

*The figures in the margin indicate full marks.*

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*All symbols are of usual significance.*

**Question No. 1 is compulsory and answer any two from the rest**

1. Answer any **ten** questions from the following: 2×10 = 20
  - (a) A plane makes intercepts of  $1\text{\AA}$ ,  $2\text{\AA}$ ,  $3\text{\AA}$  on the crystallographic axes of an orthorhombic crystal with  $a : b : c = 3 : 2 : 1$ . Determine the Miller indices of the plane.
  - (b) Calculate the Einstein frequency ( $\nu_E$ ) for copper for which Einstein temperature ( $\theta_E$ ) is 230 K. [Given:  $h = 6.6 \times 10^{-34}$  J. s.,  $k = 1.37 \times 10^{-23}$  JK $^{-1}$ , the symbols having their usual meanings].
  - (c) What is "Geometrical Structure Factor"?
  - (d) Explain briefly how the classical free electron theory leads to Ohm's law. 6 =
  - (e) Why diamagnetic materials have negative susceptibility? Give an example of such material.
  - (f) Define polarisation of a dielectric material. Which type of polarisation is most effective in the visible region?
  - (g) Bragg found that for a KCl crystal, strong reflection from the sets of planes (100); (110) and (111) are obtained at the same order for angles  $5^\circ 23'$ ,  $7^\circ 25'$  and  $9^\circ 25'$ ; respectively. Show that the KCl crystal has a simple cubic crystal structure.
  - (h) Explain briefly, why the inert gases do not exhibit paramagnetism.
  - (i) The thermal conductivity of aluminium at  $20^\circ\text{C}$  is  $210 \text{ Wm}^{-1}\text{K}^{-1}$ . Calculate the electrical resistivity of aluminium at this temperature. The Lorentz number for aluminium is  $2.02 \times 10^{-8} \text{ W}\Omega\text{K}^{-2}$ .
  - (j) Discuss briefly the differences between Type I and Type II superconductors.
  - (k) What is Bloch Theorem? Explain the significance of this theorem.
  - (l) KBr crystal has cubic structure. Its density is  $2.75 \times 10^3 \text{ kg/m}^3$  and its molecular weight is 119.01. Calculate its lattice constant.
  - (m) Calculate the reciprocal lattice of FCC lattice.
  - (n) Why semiconductor acts as an insulator at 0 K?

2. (a) Energy  $E(k)$  of electron of wave vector  $\vec{k}$  in a solid is given by  $E(k) = Ak^2 + Bk^4$ , where  $A$  and  $B$  are positive non-zero constant. Find the effective mass of the electron at  $|\vec{k}| = k_0$ . 4
- (b) Derive the expression for paramagnetic susceptibility on the basis of Langevin's theory. 4
- (c) Explain the Meissner effect from the second London equation, using the Maxwell's relation  $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}_s$ . 4
3. (a) Consider the model of one dimensional monoatomic lattice chain of  $N$  atoms, equally spaced with lattice separation  $a$ , and each with the same mass  $m$ . Find the following: 2+2+2
- (i) Derive an expression for the group velocity  $v_g$  with the wave vector  $k$ .
- (ii) Using the result of (i) Evaluate  $v_g$  at small values of  $k$  ( $k \rightarrow 0$ ) and briefly discuss the physical significance of this low  $k$  group velocity.
- (b) Show that in vector form, the Bragg's Law is given by  $G^2 + 2\vec{k} \cdot \vec{G} = 0$ , where  $\vec{k}$  represents the wave vector and  $\vec{G}$  is the reciprocal lattice vector. 4
4. (a) Distinguish between Pyroelectric and Piezoelectric materials. Give proper examples. 3
- (b) Using Kronig Penney model discuss briefly how this model led to the formation of energy bands inside a solid. 3
- (c) What is Hall effect? Deduce the expression for Hall Coefficient in the case of a semiconductor. 1+3
5. (a) What are Bravais lattices and crystal system? 2
- (b) What is the packing fraction of FCC crystal? 3
- (c) The primitive translation vectors of the space lattice are: 3
- $$\vec{a} = 2\hat{i} + \hat{j}, \vec{b} = 2\hat{j}, \vec{c} = \hat{k}$$
- Find the primitive translation of the reciprocal lattice.
- (d) Mobilities of electrons and holes in a sample of intrinsic Germanium at 300 K are  $0.36 \text{ m}^2\text{V}^{-1}\text{s}^{-1}$  and  $0.17 \text{ m}^2\text{V}^{-1}\text{s}^{-1}$  respectively. If the conductivity of the specimen is  $2.12 \Omega^{-1}\text{m}^{-1}$ , estimate the intrinsic carrier density. 2

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**WEST BENGAL STATE UNIVERSITY**  
B.Sc. Honours 5th Semester Examination, 2022-23

**PHSADSE01T-PHYSICS (DSE1/2)**

**ADVANCED MATHEMATICAL PHYSICS-I**

Time Allotted: 2 Hours

Full Marks: 40

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**Question No. 1 is compulsory and answer any two from the rest**

1. Answer any **ten** questions from the following:

2×10 = 20

✓ (a) Define isomorphism of vector space.

✓ (b) Consider the following two kets

$$|\psi\rangle = \begin{pmatrix} -3i \\ 2+i \\ 4 \end{pmatrix} \text{ and } |\phi\rangle = \begin{pmatrix} 2 \\ -i \\ -3i \end{pmatrix}$$

Evaluate the scalar product  $\langle\phi|\psi\rangle$ .

(c) Write in full the following

$$g_{ij} dx^i dx^j \quad (i, j = 1, 2, 3)$$

✓ (d) Show that  $f_1(x) = Ae^{mx}$  and  $f_2(x) = Be^{nx}$  ( $m \neq n$ ) are linearly independent.

(e) Show that the operator  $|\phi\rangle\langle\phi|$  is a projection operator only when  $|\phi\rangle$  is normalized.

(f) Write down the Minkowski metric in the mixed tensor form showing the components.

(g) Prove that every vector in a finite dimensional vector space  $V$  over the field  $F$  can be uniquely expressed as a linear combination of the basis vectors.

✓ (h) Define Laplace transform of a function. What conditions are to be satisfied for the transform to exist?

✓ (i) Evaluate  $L[e^{-2x}(2\cos 5x - 3\sin 5x)]$

✓ (j) Find the metric for the three-dimensional Euclidean space in terms of spherical polar coordinates.

✓ (k) Find  $L^{-1}\left[\frac{1}{\sqrt{s+2}}\right]$ .

✓(l) State the convolution theorem of Laplace transform.

✓(m) Show that for Laplace inverse transform

$$L^{-1}\{c_1 f_1(p) + c_2 f_2(p)\} = L^{-1}\{c_1 f_1(p)\} + L^{-1}\{c_2 f_2(p)\}$$

✓(n) Any arbitrary second rank tensor  $M_{ij}$  can be written as a sum of a symmetric and an antisymmetric tensor.

2. (a) Find the eigenvalues and eigenvectors of the matrix  $\begin{pmatrix} 3 & i \\ -i & 3 \end{pmatrix}$ . 2+2+2

Check whether the eigenvalues are real and the eigenvectors are orthogonal. Find the normalized eigenvectors of the same.

(b) Prove that  $\nabla^2 \phi = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^k} \left( \sqrt{g} g^{kr} \frac{\partial \phi}{\partial x^r} \right)$  4

✓3. (a) Solve  $\frac{dy}{dx} + 2y = \cos x$ , given  $y(0) = 1$ , using Laplace transform method. 4

(b) Find the Laplace transform of the Heaviside function: 2

$$H(t-a) = \begin{cases} 0 & \text{for } t < a, \\ 1 & \text{for } t \geq 0, \quad a \geq 0 \end{cases}$$

(c) Prove that  $L[f(t)] = \frac{1}{1 - e^{-pt}} \int_0^T e^{-pt} f(t) dt$  where,  $f(t+T) = f(t)$ . 4

✓ 4. (a) Find a unit vector orthogonal to 2

$$\alpha_1 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \text{ and } \alpha_2 = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

(b) Prove that  $\delta_j^i a^{jk} = a^{ik}$ . 2

(c) Show that  $\epsilon_{ijk} \epsilon_{ijk} = 6$ . 3

(d) Consider the states  $|\psi\rangle = 3i|\phi_1\rangle - 7i|\phi_2\rangle$  and  $|\chi\rangle = -|\phi_1\rangle + 2i|\phi_2\rangle$ , where  $|\phi_1\rangle$  and  $|\phi_2\rangle$  are orthonormal. Show that the states  $|\psi\rangle$  and  $|\chi\rangle$  satisfy the Schwarz inequality and triangle inequality. 3

5. (a) Expand  $\phi = a_{ij} u^i v^j$ , where  $i, j = 1, 2, 3$  assuming Einstein summation convention. 2

(b) Show that  $\delta_i^i = n$  where,  $n$  is the dimension. 2

(c) Prove that every basis of a finite dimensional vector space has the same number of vectors. 4

(d) Show that  $\frac{\partial A_p}{\partial x^q}$  is not a tensor even though  $A_p$  is a covariant tensor of rank one. 2

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**WEST BENGAL STATE UNIVERSITY**  
B.Sc. Honours 5th Semester Examination, 2022-23

**PHSADSE02T-PHYSICS (DSE1/2)**

**ADVANCED DYNAMICS**

Time Allotted: 2 Hours

Full Marks: 50

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**Question No. 1 is compulsory and answer any two from the rest**

1. Answer any **fifteen** questions from the following:

2×15=30

- (a) Show that if the Lagrangian is independent of any 'generalised' coordinate then the corresponding 'generalised' momentum is conserved.
- (b) A particle of mass  $m$  is constrained to move along a vertical circle of radius  $a$  under the field of gravity. Determine the force of constraint.
- (c) The Lagrangian of a particle of mass  $m$  moving in a plane is given by  $L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + a(x\dot{y} - y\dot{x})$  where  $\dot{x}$  and  $\dot{y}$  are velocity components and  $a$  is a constant.
- (d) Find the canonical momenta of the particle.
- (e) Show that the kinetic energy of a rigid body can be represented as  $T = T = \frac{1}{2}\vec{\omega} \cdot \vec{J}$ .
- (f) What is meant by 'principal axes of inertia'? What is the property of a rigid body associated with them?
- (g) A particle of mass  $m$  moves in one dimension with the following potential energy  $V(x) = \frac{k}{2}x^2 + \frac{k^2}{x}$ . Find the frequency for small oscillation about position of stable equilibrium.
- (h) Find the fixed points for the map  $x_{n+1} = x_n^2$  and determine their stability.
- (i) Show that fluid velocity  $\vec{v} = \frac{-\hat{i}y + \hat{j}x}{x^2 + y^2}$  is a possible motion of an incompressible ideal fluid. Is this motion irrotational?
- (j) A particle of unit mass moves in a potential  $V(x) = \frac{a}{x^2} + bx^2$  where  $a$  and  $b$  are positive constants. Find the angular frequency of small oscillations about the minimum of the potential.
- (k) What is laminar and turbulent flow of fluid?



- (h) What do you understand by bifurcation?
- (m) Show that the phase trajectory for a linear harmonic oscillator is an ellipse.
- (n) Derive the equation of continuity for an incompressible fluid.
- (o) What is streamline motion? What is turbulent motion?
- (p) Define Reynold number. How estimation of Reynold number helps to determine whether a motion is turbulent or streamline?
- (q) Draw the 2D phase space diagram of a point particle of mass  $m$  falling freely under the action of earth's gravity.
- (r) Classify all the fixed points of the first order differential equations.
- (s) Write the dimension of the co-efficient of viscosity and the surface tension.
- (t) Define Euler's angles for the orientation of a rigid body.
2. (a) A particle of mass  $m$  is constrained to move on the plane curve  $xy = c$  ( $c > 0$ ) under gravity ( $y$ -axis vertical). Obtain the Lagrangian of the particle. 3
- (b) Show that the transformation 3
- $$Q = \log(1 + \sqrt{q} \cos p)$$
- $$P = 2\sqrt{q}(1 + \sqrt{q} \cos p) \sin p$$
- is canonical.
- (c) Find the moment and product of inertia of a uniform square plate of side  $a$  about X, Y and Z axes, X any axes being taken as the adjacent sides of the plate and Z axis perpendicular to its plane. 4
3. (a) What do you mean by normal modes of vibration? Explain the meaning of normal coordinates and normal frequencies. 1+1+1
- (b) A massless spring of force constant  $k$  has masses  $m_1$  and  $m_2$  attached to its two ends. The system rests on a horizontal table. Obtain the normal frequencies of the system. 4
- (c) The potential energy of a particle is given by  $V = 3x^4 - 8x^3 - 6x^2 + 24x$ . Find the points of stable and unstable equilibrium. 3
4. (a) Show that  $Q = -p$ ,  $P = q + Ap^2$  (where  $A$  is a constant) is a canonical transformation. The Hamiltonian for a particle moving vertically in a gravitational field  $g$  is  $H = \frac{p^2}{2m} + mgq$ . Find the new Hamiltonian for new canonical variables  $Q, P$  given above. 4
- (b) Obtain the normal modes of vibration of a double pendulum, assuming equal lengths but unequal masses. Show that if the lower mass is small compared to the upper one, the two resonant frequencies are almost equal. 2+4
5. (a) Show that Poisson bracket remains invariant under canonical transformation. 5
- (b) A circular disc of mass  $M$  and radius  $R$  rolls down an inclined plane. The angle of inclination is  $\phi$ . Write the Lagrangian for the rolling disc. Write the equation of motion using Lagrange's multipliers and then find the force of constraint. 5

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**WEST BENGAL STATE UNIVERSITY**  
B.Sc. Honours 5th Semester Examination, 2022-23

**PHSADSE03T-PHYSICS (DSE1/2)**

**NUCLEAR AND PARTICLE PHYSICS**

Time Allotted: 2 Hours

Full Marks: 50

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**Question No. 1 is compulsory and answer any two from the rest**

1. Answer any **fifteen** questions from the following: 2×15 = 30
  - (a) What is the significance of non-zero electric quadrupole moment of a nucleus?
  - (b) What is the minimum kinetic energy required to probe a nucleus of diameter 10 fm?
  - (c) What is straggling of range of an  $\alpha$ -particle?
  - (d) Mention two differences between direct nuclear reaction and compound nuclear reaction.
  - (e) Explain why an isolated photon cannot produce an electron-positron pair.
  - (f) Discuss the working principle of a scintillation detector.
  - (g) How close can a proton with kinetic energy 2 MeV get to a gold nucleus ( $Z = 79$ ) at rest?
  - (h) Why is nuclear fusion not possible beyond the iron group of elements?
    - (i) Consider the reaction among nuclei:
 
$$A + B \rightarrow C + D^*,$$
 where the nucleus  $D$  is created in an excited state with excitation energy  $E_D$ . If the masses of the nuclei are given, write down an expression for  $Q$ -value of this reaction.
    - (j) What are fertile and fissile nuclei?
    - (k) Can you accelerate an electron by a cyclotron? Discuss.
    - (l) Give names and symbols for the antiparticles of  $e$ ,  $p$ ,  $\nu_e$  and  $k^+$ .
    - (m) Calculate the mass of U-238 with 1 Curie activity.
    - (n) What are baryons and mesons? Give one example for each of them.
    - (o) What are anomalous about the magnetic dipole moment of a neutron?
    - (p) What is the difference between beta decay and internal conversion process?
    - (q) Give an example each for a LINAC and a cyclotron situated in India.
    - (r) What are the quark contents of a proton and an electron?
    - (s) Give the spin and parity of two stable isotopes of Li.
    - (t) Give an example of a hyperon. What is a hyper nucleus?

2. (a) Define range of an  $\alpha$ -particle in a medium. Why is it expressed in  $\text{kg/m}^2$  unit? 1+1
- (b) Mention two inadequacies in the nuclear liquid drop model. 1
- (c) What is mass parabola? What is its utility? 3
- (d) A nucleus of mass number 240 decays by  $\alpha$ -emission to the ground state of the daughter nucleus. The Q-value of this process is 5.26 MeV. Find out the energy of the  $\alpha$ -particle. 2
3. (a) What is a Geiger-Müller counter? How does it work? 1+3
- (b) In Compton scattering between a photon and a stationary electron, what is the maximum wavelength of the scattered photon if the incident photon has wavelength  $\lambda$ ? 1
- (c) Explain three processes by which  $\gamma$ -rays lose energy by interaction with matter. Give your answer in brief. 3
- (d) Write down and complete the nuclear reaction  $^{15}\text{N}_7(p,d)$ , indicating the compound nucleus. 2
4. (a) An experimentalist found a radioactive source that emits both  $\alpha$  and  $\beta$  particles with half-lives 1600 years and 400 years respectively. After what time would one-fourth of the material remain undecayed? 3
- (b) Write down the CPT conservation law. 2
- (c) What is the definition of binding energy of a nucleus? How much is it for a valence neutron of a nucleus lying on the neutron drip line? 1+1
- (d) What is bremsstrahlung radiation? Why is it important in the context of electrons interacting with matter? 1+2
5. (a) Show, using weight diagram, the octet symmetry of mesons and baryons. 2+2
- (b) Check whether the following reactions are allowed or forbidden:  $1\frac{1}{2} + 1\frac{1}{2}$
- (i)  $p + \bar{p} \rightarrow 2\pi^+ + 2\pi^- + 2\pi^0$
- (ii)  $\pi^+ + p^- \rightarrow \Sigma^- + k^-$
- (c) Show that an electron is a clean probe for probing a nucleus at high beam energies. 3

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# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2021-22

## PHSACOR11T-PHYSICS (CC11)

### QUANTUM MECHANICS AND APPLICATIONS

Time Allotted: 2 Hours

Full Marks: 40

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Candidates should answer in their own words and adhere to the word limit as practicable.  
All symbols are of usual significance.*

#### Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any **ten** questions from the following: 2×10 = 20
  - (a) What is meant by expectation value?
  - (b) Explain what is meant by spin-orbit coupling.
  - (c) If  $L_{\pm} |1, m\rangle = C_{\pm} |1, m \pm 1\rangle$ , find  $C_{\pm}$ . Here  $L_{\pm} = L_x \pm iL_y$ .
  - (d) Find the value of the commutator  $[\sin(x), p_x]$ , where symbols have their usual meanings.
  - (e) The eigenvalue equations corresponding to two operators  $A$  and  $B$  are respectively given by  $Af(x) = af(x)$  and  $Bf(x) = bf(x)$ , where  $a$  and  $b$  are the corresponding eigenvalues of the operators  $A$  and  $B$ . Prove that the operators  $A$  and  $B$  commute.
  - (f) The one-dimensional wave function is given by  $\psi(x) = \sqrt{a}e^{-ax}$ . Find the probability of finding the particle between  $x = \frac{1}{a}$  and  $x = \frac{2}{a}$ .
  - (g) Consider a particle whose Hamiltonian matrix is  $H = \begin{pmatrix} 2 & i & 0 \\ -i & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ . Is  $|\lambda\rangle = \begin{pmatrix} i \\ 7i \\ -2 \end{pmatrix}$  an eigenstate of  $H$ ?
  - (h) What is Larmor precession of electron in an atom?
  - (i) Two operators  $A$  and  $B$  have simultaneous eigen-functions. Show that  $[A \cdot B] = 0$ .
  - (j) Can the principal quantum number for an electron in a hydrogen atom be zero? Explain your answer.
  - (k) Justify the statement that the probability current density cannot be directly measured.
  - (l) What is Lande  $g$ -factor? Obtain an expression for it in terms of  $l$ ,  $s$  and  $j$ .
  - (m) Show that if a quantum particle has the wave function  $\psi = e^{ikz}$ , the  $z$ -component of its angular momentum is zero.
  - (n) Can Lithium ( $Z = 3$ ) give rise to normal Zeeman effect? Justify your answer.

2. (a) What is stationary state? If  $\psi_1$  and  $\psi_2$  are two eigen states with energy  $E_1$  and  $E_2$  respectively, check whether the state  $(\psi_1 + \psi_2)$  is stationary or not. 1+2
- (b) Does a stationary state evolve with time? Explain your answer. 2
- (c) If  $\psi_l^m(\vec{r}, t)$  be the simultaneous eigenfunctions of the angular momentum operator  $L$  and  $L_z$ , what are the eigenvalue equations corresponding to the operators  $L^2$  and  $L_z$ ? 3+2
3. (a) The initial state of a two level system is a superposition of the ground state,  $|E_1\rangle$ , and first excited state,  $|E_2\rangle$  (symbols bearing usual meaning), as follows:  $|\Psi\rangle = 3|E_1\rangle + 2|E_2\rangle$ . 2+2
- (i) Find the possible results of energy measurement with their corresponding probabilities.
- (ii) Find the average value of energy. Will it be time dependent?
- (b) Explain the origin of spin-orbit interaction. 3
- (c) What is meant by space quantization? What role does magnetic quantum number play in space quantization? Explain in the light of vector atom model. 3
4. (a) A particle of mass  $m$  and momentum  $p$  is incident from left on the potential step of height  $V_0$ . Calculate the probability that the particle is scattered backward by the potential if  $\frac{p^2}{2m} < V_0$ . 4
- (b) Calculate the expectation value of the potential energy of the electron in the 1s state of H-atom. The wave function of the 1s-electron of H-atom is given by  $\psi_{100} = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$  where  $a_0$  is the Bohr radius  $= 4\pi\epsilon_0 \hbar^2 / me^2$ , with usual meanings of symbols. 3
- (c) Determine stating reasons whether each of the following functions is acceptable or not as a state function over the indicated intervals. 3
- (i)  $\sqrt{\frac{2}{l}} \sin \frac{n\pi x}{l}$  in the range  $-0$  to  $+l$
- (ii)  $\sin^{-1} x$  in the range  $+1$  to  $-1$ .
5. (a) Find the energy of  $n^{\text{th}}$  state of a linear harmonic oscillator with mass  $m$  and frequency  $\omega$ . Show that the average potential energy of the  $n^{\text{th}}$  state of a linear harmonic oscillator is half of the energy of the oscillator in this state. 3+3
- (b) Discuss the goal of Stern-Gerlach experiment. Why is it necessary to apply an inhomogeneous magnetic field in this experiment? 3+1

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**WEST BENGAL STATE UNIVERSITY**  
B.Sc. Honours 5th Semester Examination, 2021-22

**PHSACOR12T-PHYSICS (CC12)**

**SOLID STATE PHYSICS**

Time Allotted: 2 Hours

Full Marks: 40

*The figures in the margin indicate full marks.  
Candidates should answer in their own words and adhere to the word limit as practicable.  
Answers must be precise and to the point to earn credit.  
All symbols are of usual significance.*

**Question No. 1 is compulsory and answer any *two* from the rest**

1. Answer any **ten** questions from the following: 2×10 = 20
- (a) “Crystals are common in nature” — True or False. Explain briefly.
  - (b) Calculate the coordination number for SC, and BCC crystal.
  - (c) Discuss the construction of the first two Brillouin zones for a square lattice.
  - (d) Derive Bragg’s relation from Laue’s equations.
  - (e) The forbidden energy band is 0.75 eV in germanium. To what wavelength of light is this substance transparent? [ $h = 6.6 \times 10^{-34}$  Jsec]
  - (f) Calculate the glancing angle on the plane (110) of a cubic rock salt crystal ( $a = 2.81 \text{ \AA}$ ) corresponding to second order diffraction maxima of wavelength  $0.71 \text{ \AA}$ .
  - (g) What is Einstein temperature? Calculate the Einstein temperature given Einstein frequency as  $9 \times 10^{11}$  Hz.
  - (h) A single electron in an energy band may have positive or negative effective mass: True or False. Give proper logic.
  - (i) The resistivity of aluminium at room temperature is  $2.62 \times 10^{-8}$  ohm. Calculate the drift velocity and their mobility.
  - (j) Draw the susceptibility vs. temperature graph for dia, para and ferromagnetic materials.
  - (k) A paramagnetic substance has  $10^{28}$  atoms/m<sup>3</sup>. The magnetic moment of each atom is  $1.8 \times 10^{-23}$  Am<sup>2</sup>. Calculate the paramagnetic susceptibility at 300 K.
  - (l) What is the physical significance of the hysteresis loop in magnetic or dielectric materials?
  - (m) Define Hall coefficient. Why it is positive in metals?
  - (n) Why does dielectric loss occur?

2. (a) Explain orientational polarization giving proper examples. 2
- (b) Derive an expression for the electronic polarizability of an atom on the basis of classical theory. 4
- (c) Show that the diamagnetic susceptibility of an element is independent of temperature. 4
3. (a) The conductivity of a metal decreases with rise of temperature, whereas the conductivity of a semiconductor increases with increase of temperature. Explain both the cases clearly giving appropriate examples. 2
- (b) Consider the local field at an atomic site in a cubic structure in terms of the polarization  $\vec{P}$  produced by the applied electric field  $\vec{E}$ . Hence, arrive at the Clausius-Mossotti relation for non-polar dielectric medium. 1+3
- (c) Derive Curie-Weiss law from Weiss's Molecular theory of magnetism. Sketch the variation of the magnetic susceptibility with temperature above the Curie point. 3+1
4. (a) Show that the Einstein's relation for the heat capacity per k.mol of a solid reduces to the classical value of  $3R_u$ , for the condition when  $k_B T \geq h\nu$ . 3
- (b) Obtain the dispersion relation for one-dimensional diatomic lattice. Hence, explain the concept of optical branches. 3+2
- (c) Calculate the effective mass as a function of  $k$  for a one-dimensional crystal of lattice constant ' $a$ ' having dispersion the relation  $E(k) = E_0 - \alpha - 4\beta \cos ka$ , where  $\cos ka$ ,  $E_0$ ,  $\alpha$ ,  $\beta$  are constants. 2
5. (a) The intrinsic carrier density of Ge at  $27^\circ\text{C}$  is  $2.4 \times 10^{17} \text{ m}^{-3}$ . Calculate its intrinsic resistivity, if the electron and hole mobilities are  $0.35 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$  and  $0.18 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$ ; respectively. 3
- (b) State Wiedemann Franz law and give its physical significance. 2
- (c) The Hall voltage for the metal sodium is  $0.001 \text{ mV}$ , measured at  $I = 100 \text{ mA}$ ,  $B = 2.0 \text{ Weber/m}^2$  and the width of the specimen is  $0.05 \text{ mm}$ . Calculate the number of carriers per cubic meter in sodium. 2
- (d) Write the domain hypothesis of Weiss and explain the physical origin of domain formation from the general thermodynamic principle. 1+2

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**WEST BENGAL STATE UNIVERSITY**  
B.Sc. Honours 5th Semester Examination, 2021-22

**PHSADSE01T-PHYSICS (DSE1/2)**  
**ADVANCED MATHEMATICAL PHYSICS-I**

Time Allotted: 2 Hours

Full Marks: 40

*The figures in the margin indicate full marks.*  
*All symbols are of usual significance.*

**Question No. 1 is compulsory and answer any two from the rest**

1. Answer any **ten** questions from the following:

2×10 = 20

- (a) Prove that the contraction of tensors  $A_q^p$  is an invariant.
- (b) If  $\varphi = a_{jk} A^j A^k$  then show that one can write  $\varphi = b_{jk} A^j A^k$ , where  $b_{jk}$  is symmetric.
- (c) Show that the Kronecker delta is a mixed tensor of order two.
- (d) For an orthogonal basis prove that norm of a nonzero vector is positive definite.
- (e) Why is Laplace transformation a linear operation?
- (f) Show that the Laplace transform of the integral of  $f(x)$ , i.e.

$$L\left[\int_0^x f(x) dx\right] = \frac{1}{p} \bar{f}(p), \text{ where } L[f(x)] = \bar{f}(p).$$

- (g) Prove that every vector in a finite-dimensional vector space  $V$  over the field  $F$  can be uniquely expressed as a linear combination of the vectors of its basis.
- (h) Show that the vectors  $(2, -5, 3)$  cannot be expressed as a linear combination of the vectors  $\alpha_1 = (1, -3, 2)$ ,  $\alpha_2 = (2, -4, -1)$  and  $\alpha_3 = (1, -5, 7)$ .

- (i) Show that the four matrices  $E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ,  $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ ,  $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$  and  $C = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$  form a group under matrix multiplication.

- (j) Find Laplace transform of  $f(x)$ , where  $f(x) = \begin{cases} 0 & , x < a \\ 1 & , x > a \end{cases}$ .

- (k) Find Laplace transform of  $e^{4x} \sin(2x) \cos(x)$ .

- (l) Examine if the following operator is linear:

$$Af(x) = x^2 f(x)$$

- (m) Evaluate  $L^{-1}\left[\frac{1}{s-2} + \frac{2}{s+5} + \frac{6}{s^4}\right]$ .

2. (a) Show that the velocity at any point of a fluid  $\frac{dx^k}{dt} = v^k$  is a tensor but acceleration  $\frac{dv^k}{dt}$  is not a tensor. 2+1
- (b) Using Levi-Civita symbol establish the relation  $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$ . 4
- (c) What are stress and strain tensors? Write down the tensorial form of Hooke's law in Elasticity. 3
3. (a) Distinguish between isomorphism and homomorphism in connection with two groups. 2
- (b) Define basis and dimension of a vector space. 2
- (c) Define orthogonal and orthonormal set of vectors. 2
- (d) Show that the sets  $S\{(1, 0, 0), (1, 1, 1), (0, 1, 0)\}$  spans a vector space  $R^3$  but is not a basis set. 4
4. (a) For the set of basis vectors,  $(0, 2, 0, 0)$ ,  $(3, -4, 0, 0)$  and  $(1, 2, 3, 4)$  use Gram-Schmidt process to construct an orthonormal set. 4
- (b) A linear transformation  $T$  is defined as  $T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 - x_3 \end{pmatrix}$  that transforms a vector a 3-D real space to 2-D real space. Show that the transformation matrix is  $T = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$ . 2
- (c) Use the convolution theorem to find the inverse Laplace transform of  $\frac{1}{(s^2 + 4)^2}$ . 4
5. (a) If  $LT[f(x)] = \bar{f}(s)$ , then prove that  $LT[f''(x)] = s^2 \bar{f}(s) - sf(0) - f'(0)$ . 3
- (b) Find inverse Laplace transform of  $\frac{s+2}{s^2(s+1)(s-2)}$ . 4
- (c) Using Laplace transform solve the differential equation  $2 \frac{d^2 y}{dx^2} + 5 \frac{dy}{dx} + 2y = e^{-2x}$  with  $y(0) = 1$ ,  $\frac{dy}{dx}(0) = 1$  3

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**WEST BENGAL STATE UNIVERSITY**  
B.Sc. Honours 5th Semester Examination, 2021-22

**PHSADSE02T-PHYSICS (DSE1/2)**

**ADVANCED DYNAMICS**

Time Allotted: 2 Hours

Full Marks: 50

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Candidates should answer in their own words and adhere to the word limit as practicable.  
All symbols are of usual significance.*

**Question No. 1 is compulsory and answer any *two* from the rest**

1. Answer any *fifteen* questions from the following: 2×15 =30
- (a) Show that for a cyclic co-ordinate, the conjugate momentum is conserved.
  - (b) What do you mean by generalized co-ordinates and what is the advantage of using them?
  - (c) State whether the constraints given by  $x \frac{dy}{dt} - y \frac{dx}{dt} = c$  (constant) is holonomic one.
  - (d) Define Poisson Bracket (PB) and write down the Hamilton's equation of motion using PB.
  - (e) How many degrees of freedom does a rigid body have when the body is rotating about an axis that is fixed in space?
  - (f) Show that if the Lagrangian of a system does not depend on time explicitly, its Hamiltonian is a constant of motion.
  - (g) State the parallel axes theorem of moment of inertia.
  - (h) What are principal moments of inertia and principal axes?
  - (i) Show that the directions of angular velocity and angular momentum, though usually differ, coincide only along principal axes.
  - (j) State the properties of principal moments of inertia of rigid body.
  - (k) What do you understand by stable and unstable equilibria?
  - (l) Find the eigen-frequencies of a vibrating system characterized by a Lagrangian
$$L = \frac{1}{2}(\eta_1^2 + \eta_2^2 + \eta_3^2) - \alpha^2(\eta_1^2 + \eta_2^2 + \eta_3^2 + \eta_1\eta_2).$$
  - (m) A particle of mass  $m$  in one dimensional motion along  $x$ -axis ( $x > 0$ ) with its potential energy given by  $V(x) = x + \frac{1}{x}$ , is executing small oscillation near its stable equilibrium. Find the frequency of the oscillation.
  - (n) Potential energy of a particle is given by  $V = x^4 - 4x^3 - 8x^2 + 48x$ . Find the points of stable and unstable equilibria.
  - (o) What are 'Autonomous' and 'Nonautonomous' systems? Give example for each.

- (p) Find and classify all the fixed points of the following first order differential equation:

$$\frac{dy}{dt} = X(y) = -y(y^2 - 4)$$

- (q) Draw the 2D phase space diagram of a point particle of mass  $m$  falling freely under the action of earth's gravity.

2. (a) A pendulum bob of mass  $m$  is suspended by a string of length  $l$  from a point of support. The point of support moves along a horizontal  $x$ -axis according to the equation  $x = a \cos \omega t$ . Assuming the pendulum swings only in the vertical plane containing the  $x$ -axis. 3+1

(i) Set up the Lagrangian and write out the Lagrange equation.

(ii) Show that small values of the angle which the string makes with a line vertically downward, the equation reduces to that of a forced harmonic oscillator.

- (b) Show that the gauge transformation  $\mathbf{A}' = \mathbf{A} + \nabla f(\mathbf{r}, t)$ ,  $\phi' = \phi - \frac{\partial f}{\partial t}$  effected by a generating function  $F_2(\mathbf{r}, \mathbf{p}) = \mathbf{r} \cdot \mathbf{p} - ef(\mathbf{r}, t)$  can be regarded as a canonical transformation. 3

- (c) Sketch the phase portrait corresponding to  $\dot{x} = x - \cos x$ , and determine the stability of all the fixed points. 3

3. (a) The equation of a damped harmonic oscillator is  $\frac{d^2x}{dt^2} + 2b \frac{dx}{dt} + \omega_0^2 x = 0$ . Discuss the phase trajectory for  $b^2 < \omega_0^2$  and  $b^2 > \omega_0^2$ . 4

- (b) The Van der Pol's equation is given by  $\frac{d^2x}{dt^2} - \varepsilon(1 - x^2) \frac{dx}{dt} + x = 0$ . Write parametric equations of the system. 3

- (c) What do you understand by a limit cycle? What is an attractor?  $1\frac{1}{2} + 1\frac{1}{2}$

4. (a) For rotational motion of rigid bodies, derive an expression for kinetic energy in terms of moment of inertia and angular velocity. 3

- (b) Write down Euler's equation for free symmetrical top and solve for angular velocity  $\omega$ . Show that the angular velocity vector  $\omega$  rotates about the body symmetry axis describing a cone with the vertex at the origin. 3+1

- (c) Show that the transformation  $P = \frac{1}{2}(p^2 + q^2)$  and  $Q = \tan^{-1} \frac{q}{p}$  is canonical. 3

5. (a) Find the canonical transformation generated by 2+1+1+2

$$F_1(Q, q) = \lambda q^2 \cot Q,$$

$\lambda$  being a constant. If the Hamiltonian in  $(q, p)$  representation is

$$H(q, p) = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 q^2,$$

find the Hamiltonian in  $(Q, P)$  representation. Choose  $\lambda$  to make this Hamiltonian independent of  $Q$  and hence find the equation of motion in each representation.

- (b) Draw the potential for the system  $\frac{dx}{dt} = x - x^3$  and identify all the equilibrium points. 2
- (c) Show that for a first order dynamical system (with,  $\frac{dx}{dt} = f(x)$ ), there are no periodic solutions. 2

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**WEST BENGAL STATE UNIVERSITY**  
B.Sc. Honours 5th Semester Examination, 2021-22

**PHSADSE03T-PHYSICS (DSE1/2)**

**NUCLEAR AND PARTICLE PHYSICS**

Time Allotted: 2 Hours

Full Marks: 50

*The figures in the margin indicate full marks.  
Candidates should answer in their own words and adhere to the word limit as practicable.  
All symbols are of usual significance.*

**Question No. 1 is compulsory and answer any two from the rest**

1. Answer any **fifteen** questions from the following: 2×15 = 30
- (a) What is mirror nuclei? Give example.
  - (b)  ${}^2\text{He}^4$  nucleus has no magnetic moment. Explain.
  - (c) Determine the radii of a  ${}^{16}\text{O}$  nucleus, given  $r_0 = 1.2 \text{ fm}$
  - (d) State Geiger-Nuttall law.
  - (e) Find the Q-value of the reaction  ${}^{14}_7\text{N}(\alpha, p){}^{17}_8\text{O}$  Take mass of  ${}^4\text{He} = 4.0026 \text{ u}$ ,  ${}^{14}\text{N} = 14.0031 \text{ u}$ ,  ${}^1\text{H} = 1.0078 \text{ u}$ ,  ${}^{17}\text{O} = 16.9994$  and  $1 \text{ u} = 931 \text{ Mev}$ .
  - (f) A muon is not a meson. Explain.
  - (g) Calculate the minimum speed of a charged particle for emission of Cerenkov radiation in a medium of refractive index 1.5.
  - (h) What are the differences between Compound nuclear reaction and direct reaction?
  - (i) What is strangeness of an elementary particle?
  - (j) How does interaction of  $\gamma$ -ray in matter differ from interaction of charged particles?
  - (k) What are packing fraction and binding energy fraction?
  - (l) What are the quark contents of proton and neutron?
  - (m) What is straggling of range of  $\alpha$ -particle?
  - (n) An ultra-relativistic proton moves in a magnetic field. Can it radiate  $\pi^+$ ,  $\pi^-$  and  $\pi^0$  mesons, electrons and positrons?
  - (o) Why the following reactions are not found in nature?
    - (i)  $e^- \rightarrow e^- + \gamma$  in vacuum
    - (ii)  $K^+ \rightarrow \pi^+ + \gamma$  in vacuum
  - (p) Can photoelectric emission take place with free electron? Explain.
  - (q) Give example of two hyperons. What is hyper nucleus?
  - (r) Define parity of a nucleus.
  - (s) What is resonant reaction?
  - (t) Why neutron show magnetic moment though it lacks charge?

2. (a) 1 g of  $^{226}\text{Ra}$  has an activity of 1 curie. Calculate the mean life and half life of Radium. 3
- (b) What are the problems to explain continuous  $\beta$ -decay spectrum? How they were solved? 3
- (c) Why  $\text{U}^{235}$  is fissile with slow neutrons but  $\text{U}^{238}$  requires fast neutrons for fission process? 4
3. (a) Why spin-orbit coupling is necessary in Shell model to reproduce magic numbers? 3
- (b) What are the spin-parity ( $J^\pi$ ) of  ${}_4\text{Be}^{11}$  in its ground state? 1
- (c) What are the basic assumptions behind Fermi gas model of the nucleus? 2
- (d) Calculate the threshold energy for the nuclear reaction  $^{14}\text{N}(n,\alpha)^{11}\text{B}$  in MeV. Use the following data: 4
- Mass of  $^{14}\text{N}$  (14.007550 u), mass of n (1.008987 u), mass of  $\alpha$  (4.003879 u), mass of  $^{11}\text{B}$  (11.012811 u).
4. (a) Explain how the Wave theory failed to explain photoelectric effect and Compton effect. 2
- (b) What are the major interaction processes by which energized electron loses energy within matter? Explain them. 4
- (c) How fast neutrons interact with matter? 2
- (d) Why  $e^- - e^+$  pair production cannot occur in vacuum? 2
5. (a) What is meant by SU(3) symmetry of strong interaction? How is this broken? 4
- (b) Write CPT conservation law. 2
- (c) The isospin, baryon number and strangeness of a particle are given by  $I = 0$ ,  $B = +1$  and  $S = -3$ , respectively. Find the electric charge of the particle. 2
- (d) Identify the type of the reaction 2
- (i)  $\mu^- \rightarrow e^- + \nu + \bar{\nu}$
- (ii)  $\Sigma^0 \rightarrow \Lambda^0 + \Lambda$

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**WEST BENGAL STATE UNIVERSITY**  
B.Sc. Honours 5th Semester Examination, 2020, held in 2021

**PHSACOR11T-PHYSICS (CC11)**

**QUANTUM MECHANICS AND APPLICATIONS**

Time Allotted: 2 Hours

Full Marks: 40

*The figures in the margin indicate full marks.  
Candidates should answer in their own words and adhere to the word limit as practicable.  
All symbols are of usual significance.*

**Question No. 1 is compulsory and answer any two from the rest**

1. Answer any **ten** questions from the following: 2×10 = 20

(a) Consider a system whose Hamiltonian is given by  $\hat{H} = \alpha(|\phi_1\rangle\langle\phi_2| + |\phi_2\rangle\langle\phi_1|)$ , where  $\alpha$  is a real number having appropriate dimension and  $|\phi_1\rangle, |\phi_2\rangle$  are normalized eigenstates of an Hermitian operator  $\hat{A}$  that has no degenerate eigenvalue. Determine if  $|\phi_1\rangle, |\phi_2\rangle$  are eigenstates of  $\hat{H}$ .

(b) State Heisenberg uncertainty principle.

(c) Find the lowest energy of an electron confined to move in a one dimensional box of length 1 Å.

[Given,  $m = 9.11 \times 10^{-31}$  kg,  $\hbar = 1.05 \times 10^{-34}$  Js,  $1 \text{ eV} = 1.6 \times 10^{-19}$  J]

(d) Consider the operator  $\hat{Q} = i \frac{d}{d\phi}$ , where  $\phi$  is plane polar azimuthal angle in two dimensions. Write down its eigenvalue equation and find its eigenvalues.

(e) What is Larmor precession of an electron in an atom?

(f) For one dimensional bound state motion of a particle of mass  $m$ , prove that the expectation value of its kinetic energy is given by  $\langle K \rangle = \frac{\hbar^2}{2m} \int_{-\infty}^{\infty} \frac{\partial \psi}{\partial x} \frac{\partial \psi^*}{\partial x} dx$ .

(g) If  $\psi_1(x, t)$  and  $\psi_2(x, t)$  are both solutions of the time dependent Schrödinger equation for the motion of a particle with potential energy  $V(x)$ , prove that the linear combination  $\psi(x, t) = A_1 \psi_1(x, t) + A_2 \psi_2(x, t)$  is also a solution, where  $A_1$  and  $A_2$  are constants.

(h) Calculate the probability current density of a quantum mechanical system of mass  $m$  and described by the state function  $\psi(r) = \frac{1}{r} e^{ikr}$ .

- (i) A particle constrained to move along  $x$ -axis in the domain  $0 \leq x \leq L$  has a wavefunction  $\psi(x) = \sqrt{\frac{2}{L}} \sin(n \frac{\pi x}{L})$ , where  $n$  is an integer. What is the expectation value of its momentum?
- (j) Plot the wavefunctions of the ground state and the 1<sup>st</sup> excited state for a particle within an infinite square well potential of width  $a$ . Also plot the probability density for those states.
- (k) Find the probability of finding an electron within Bohr radius for the ground state of hydrogen atom. Given that the ground state wavefunction is  $\psi_{1s} = \sqrt{\frac{1}{\pi a^3}} \exp(-\frac{r}{a})$ , where  $a$  is the Bohr radius.
- (l) Find the degeneracy of the  $n$ -th energy eigenstate for the electron in a hydrogen atom neglecting its spin.
- (m) In a Stern-Gerlach experiment, on turning on the magnetic field, the beam splits into seven components. What is the angular momentum of the atoms in the beam?
- (n) Find the uncertainty in the measurement of  $S_z$  on a system prepared in a state  $|\psi\rangle = \frac{(|\uparrow\rangle + |\downarrow\rangle)}{\sqrt{2}}$  where  $|\uparrow\rangle$  and  $|\downarrow\rangle$  are the eigenvectors of the operator  $\hat{S}_z$  with eigenvalues  $+\frac{\hbar}{2}$  and  $-\frac{\hbar}{2}$  respectively.
2. A particle of mass  $m$  is in a normalized state  $\psi(x) = Ae^{-a\left[\frac{mx^2}{\hbar} + it\right]}$ , where  $A$  and  $a$  are constants.
- (a) Find  $A$ . 2
- (b) For what potential energy function  $V(x)$  does  $\psi(x)$  satisfy Schrödinger equation. 2
- (c) Calculate the expectation values of  $x$  and  $p_x$ . 3+3
3. (a) Starting from the time-dependent Schrödinger equation in one dimension, derive the equation of continuity of probability. 4
- (b) If a dynamical variable is represented by the quantum mechanical operator  $\hat{A}$  that does not depend on time explicitly, prove  $\frac{d}{dt}\langle \hat{A} \rangle = \frac{i}{\hbar}[\hat{H}\hat{A} - \hat{A}\hat{H}]$ , where  $\hat{H}$  is the Hamiltonian operator. 3
- (c) If  $\{|\vec{r}\rangle\}$  and  $\{|\vec{p}\rangle\}$  represent the bases in position and momentum representations respectively, then prove that  $\langle \vec{p} | \vec{r} \rangle = \langle \vec{r} | \vec{p} \rangle^* = \frac{1}{(2\pi\hbar)^{3/2}} e^{-i(\vec{p} \cdot \vec{r})/\hbar}$  3
4. (a) Ground state eigenfunction of a one dimensional quantum oscillator is 3

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{\left\{-\frac{m\omega}{2\hbar}x^2\right\}}.$$

Show that its ground state energy is  $E_0 = \frac{1}{2}\hbar\omega$

- (b) Write down the Schrödinger equation in spherical polar co-ordinates corresponding to the hydrogen atom problem. Write down the radial part of the wave function. 2
- (c) What is space quantization of orbital angular momentum? 2
- (d) Find the normalized ground state wavefunction of linear harmonic oscillator using the operator  $\hat{a} = \frac{m\omega\hat{x} + ip}{\sqrt{2m\omega\hbar}}$ . 3
5. (a) Explain the term 'spin-orbit coupling'. 2
- (b) In a many electron atom, the orbital, spin and total angular momenta are denoted by  $L=2$ ,  $S=1$  and  $J=2$ . Find the angle between  $\vec{L}$  and  $\vec{S}$  using the vector atom model. 2
- (c) Derive an expression for the magnetic moment for an electron moving in a circular orbit. Hence show that the ratio of the orbital magnetic moment to its angular momentum is  $\frac{e}{2m}$ . 3+3

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**WEST BENGAL STATE UNIVERSITY**  
B.Sc. Honours 5th Semester Examination, 2020, held in 2021

**PHSACOR12T-PHYSICS (CC12)**

**SOLID STATE PHYSICS**

Time Allotted: 2 Hours

Full Marks: 40

*The figures in the margin indicate full marks.  
Candidates should answer in their own words and adhere to the word limit as practicable.  
All symbols are of usual significance.*

**Question No. 1 is compulsory and answer any two from the rest**

1. Answer any **ten** questions from the following: 2×10 = 20
- (a) Show that the lattice constant for a cubic crystal with  $n$  number of molecules per unit cell, molar mass  $M$  and density  $\rho$  is given by  $\left(\frac{nM}{\rho N_A}\right)^{1/3}$ , where  $N_A$  is the Avogadro number.
  - (b) The radius of an argon atom is  $10^{-10}$  m. Calculate the electronic polarizability of an argon atom. Given that  $\epsilon_0 = 8.854 \times 10^{-12} \text{ Fm}^{-1}$ .
  - (c) Calculate the wavelength of the X-ray if the glancing angle for the 1<sup>st</sup> order is  $30^\circ$  for a crystal with  $2.8 \times 10^{-10}$  m separation between the atomic planes.
  - (d) The Hall voltage for the metal sodium is found to be 0.001 mV, for a current (through the sample)  $I = 100$  mA and a magnetic field  $B = 2.0 \text{ Wb m}^{-2}$ . The width of the specimen is 0.05 mm. Calculate the number of carriers per cubic meter in sodium.
  - (e) All primitive cells are unit cells but the reverse is not true. Illustrate with an example.
  - (f) Estimate the specific heat  $C_V$  for a material at 30 K where the Einstein temperature for it is 157 K. Find your answer in terms of the universal gas constant  $R$ .
  - (g) Could you explain the existence of band gap in solids using the Drude model? Explain.
  - (h) Show on the same graph the schematic variations of frequency  $\omega$  as a function of the wave number  $q$  (considering a one-dimensional solid) for (i) optical phonons and (ii) acoustic phonons near the point  $q \rightarrow 0$ .
  - (i) How does the magnetic susceptibility, according to Weiss' theory, depend on absolute temperature  $T$  for a ferromagnetic material above its Curie temperature? Plot the susceptibility as a function of  $T$ .

- (j) Band gaps between the highest occupied band and the lowest empty band for five materials,  $A$ ,  $B$ ,  $C$ ,  $D$  and  $E$ , are given below

$$A \rightarrow 0.8 \text{ eV}; B \rightarrow 0.69 \text{ eV}; C \rightarrow 5.3 \text{ eV}; D \rightarrow 10 \text{ eV}; E \rightarrow 1.09 \text{ eV}.$$

Identify with justification the prospective semiconductors among these.

- (k) For a metal kept in a magnetic field  $\vec{H}$  at a very low temperature, it is found that the sample develops a magnetic induction  $\vec{B} = 0$  inside it. Calculate its magnetic susceptibility. How do you classify the material in terms of its magnetic property?
- (l) What is a Wigner-Seitz cell? Show with a diagram how it is constructed for a two dimensional square lattice.
- (m) The two plates of a parallel plate capacitor are identical and carry equal amount of opposite charges. The separation between the plates is 5 mm and the space between the plates is filled with a solid slab of dielectric constant 3. The electric field within the dielectric is  $10^6 \text{ V/m}$ . Calculate the magnitude of the polarization vector ( $\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$ ).
- (n) Why is the Dulong-Petit law not useful for calculation of specific heat of a solid at low temperatures?

2. (a) Show that the reciprocal lattice to a bcc lattice is an fcc lattice. 3
- (b) A copper wire has length 0.5 m, diameter 0.3 mm and its resistance at  $20^\circ\text{C}$  is  $0.12 \Omega$ . The thermal conductivity of copper at  $20^\circ\text{C}$  is  $390 \text{ Wm}^{-1}\text{K}^{-1}$ . Estimate the Lorentz number. 2
- (c) The frequency of an elastic wave passing through a one dimensional monatomic lattice is given by  $\omega(q) = \omega_0 \sin\left(\frac{qa}{2}\right)$ , where  $a$  is the lattice spacing and  $q$  is the wave number and  $\omega_0$  is a material-specific constant. How does  $\omega_0$  depend on atomic mass? Calculate the velocity of the wave when the wavelength becomes much greater than the lattice spacing. Explain how a lattice could be used as a mechanical frequency filter. 1+2+2
3. (a) Starting from Laue's equations of X-ray diffraction, arrive at the condition for Bragg reflection. 3
- (b) Show that the dc electrical conductivity of a metal is given by  $\sigma = \frac{ne^2\tau}{m}$ , where the symbols carry their usual meanings. State clearly the assumptions, if any, involved in the derivation. 2+2
- (c) Using the Clausius-Mossotti relation, make an estimate of the Avogadro number from the data set given below. 3

Dielectric constant of Ne gas at normal pressure and temperature:  $\epsilon = 1.000148$ .

Electronic polarizability of Ne:  $\alpha = 0.4 \times 10^{-24} \text{ cm}^3$ .

Assume an ideal gas behaviour for Ne.

4. (a) Consider the following one dimensional periodic potential  $V(x)$  in which an electron is constrained to move. 1+3

$$V(x) = 0 \text{ for } 0 < x < a$$

$$= V_0 \text{ for } a < x < a + b$$

Suggest a form of the wave function that is expected to satisfy the corresponding Schrödinger equation. In the limit  $b \rightarrow 0$  and  $V_0 \rightarrow \infty$ , the quantization condition of the wave-vector  $k$  in the above problem (subject to suitable boundary conditions) turns out to be  $\frac{P}{Qa} \sin Qa + \cos Qa = \cos ka$ , where  $P \propto ba$  is a finite quantity, and  $Q \propto \sqrt{E}$  ( $E$  is the energy eigenvalue). Hence show that this model explains formation of band gaps of disallowed energy values.

- (b) Using Langevin's theory, obtain the temperature dependence of magnetic susceptibility of a paramagnetic gas (mention the inherent assumption in the derivation). 5
- (c) Mention an application of Hall effect. 1

5. (a) Consider a lattice with lattice constants  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$ . Define the reciprocal lattice vectors and find a relation between the volumes of primitive cells in the direct and the reciprocal lattices. 2+2
- (b) Iron is a ferromagnetic material. However, an iron nail usually does not show ferromagnetic properties even below the Curie temperature. Why? What happens to its microscopic structure above the Curie temperature? 2+1
- (c) "The dispersion (frequency  $\omega$  vs. wave-vector  $k$ ) relation of an elastic wave in a fluid is linear in  $k$ . But it is not so in a solid in general" – why? Why does the group velocity of an elastic wave propagating in a solid vanish at the Brillouin zone boundaries? 1+2

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**WEST BENGAL STATE UNIVERSITY**  
B.Sc. Honours 5th Semester Examination, 2020, held in 2021

**PHSADSE01T-PHYSICS (DSE1/2)**  
**ADVANCED MATHEMATICAL PHYSICS I**

Time Allotted: 2 Hours

Full Marks: 40

*The figures in the margin indicate full marks.  
Candidates should answer in their own words and adhere to the word limit as practicable.  
Answer must be precise and to the point to earn credit  
All symbols are of usual significance.*

**Question No. 1 is compulsory and answer any two from the rest**

1. Answer any **ten** questions from the following: 2×10 = 20
- Show that Laplace transformation is a linear transformation.
  - Find the Laplace transform  $\mathcal{L}\{f(t)\}$  of the function  $f(t) = \begin{cases} 1 & \text{for } t \geq t_0 \\ 0 & \text{for } t \leq t_0 \end{cases}$ ,  $t_0$  being a constant.
  - If  $\tilde{f}(s)$  is the Laplace transform of  $f(t)$ , show that multiplying  $f(t)$  by  $e^{at}$  moves the origin of  $s$  by and amount  $a$ .
  - Find the Laplace transform of  $f'(t)$  in terms of that of  $f(t)$ .
  - From the definition of Laplace transform, show that  $\mathcal{L}\{f(\omega t)\} = \frac{1}{\omega} \tilde{f}\left(\frac{s}{\omega}\right)$ , for  $\omega > 0$ , where  $\tilde{f}(s)$  is the Laplace transform of  $f(t)$ .
  - Show that the vectors  $|a\rangle = (2, 1, 1)$ ;  $|b\rangle = (-2, 1, 2)$ ;  $|c\rangle = (0, 0, 1)$  are linearly independent.
  - What is the significance of a non-singular transformation in a vector space?
  - If a vector  $|a\rangle \in \mathbb{R}^3$  in standard basis is  $|a\rangle = (3, 2, -1)$ , find its components in the new basis  $(1, 1, 1), (1, 1, 0), (1, 0, 0)$ .
  - When are two finite dimensional vector spaces  $V$  and  $V'$  isomorphic?
  - Show that members of an orthogonal set of vectors are linearly independent of each other.
  - In the relation  $L = I\omega$ , show using quotient law that  $I$  is a tensor of rank two if  $L$  and  $\omega$  are tensors of rank one.
  - Prove the vector identity  $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$  using properties of Levi-Civita symbol  $\varepsilon_{ijk}$ .
  - Show that in three dimensions any anti-symmetric second rank tensor  $A_{ij}$  can be expressed in terms of a dual vector.
  - If  $v_i$  are the components of a first rank tensor, show that  $\frac{\partial v_i}{\partial x_i}$  is a tensor of rank zero.



2. (a) Find the Laplace transform of the periodic saw-tooth function with period  $T$  defined by 4+3+3

$$V(t) = V_0 \frac{t}{T}, \text{ for } 0 \leq t \leq T.$$

- (b) In a two-dimensional complex linear vector space,  $\{|a\rangle, |b\rangle\}$  form a non-orthogonal normalized basis where inner product of  $|a\rangle$  and  $|b\rangle$  is given by  $k$ . Form another orthonormal basis where  $|c\rangle = \alpha|a\rangle + \beta|b\rangle$  is a basis element.
- (c) From the defining properties of Kronecker delta, show that it is an invariant second-rank Cartesian tensor.

3. (a) Using tensorial symbols, establish the following vector identity: 4+4+2

$$\vec{\nabla} \times (\vec{A} \times \vec{B}) = (\vec{B} \cdot \nabla) \vec{A} - (\vec{A} \cdot \nabla) \vec{B} + A(\vec{\nabla} \cdot \vec{B}) - B(\vec{\nabla} \cdot \vec{A}).$$

- (b) For a 3-dimensional vector space with an orthonormal basis formed by  $|1\rangle, |2\rangle$  and  $|3\rangle$ , An operator  $\hat{A}$  is defined by the relations:  $\hat{A}|1\rangle = |2\rangle$ ,  $\hat{A}|2\rangle = |1\rangle$  and  $\hat{A}|3\rangle = |3\rangle$ .

Find the matrix representation of  $A$ , using this basis. Is this operator unitary?

- (c) Prove that the negative gradient of a scalar field is a first rank tensor.

4. (a) Show that  $\mathcal{L}\left\{\frac{d^2 f}{dt^2}\right\} = s^2 \tilde{f}(s) - sf(0+) - \frac{df}{dt}(0+)$ , where  $\tilde{f}(s) = \mathcal{L}\{f(t)\}$ . 3+3+(3+1)

- (b) Using techniques of linear transformation, decouple the following coupled first order differential equations:  $\frac{dx}{dt} = -ay$ ,  $\frac{dy}{dt} = ax$ .
- (c) Consider a collection of rigidly connected  $N$  particles, rotating about an axis through the origin with angular velocity  $\vec{\omega}$ . If the particles, positioned at  $\vec{r}_\alpha$ , have mass  $m_\alpha$  (where  $\alpha = 0, 1, 2, \dots, N$ ), find the component  $I_{ij}$  of the inertia tensor. Find the number of independent components of this tensor.

5. (a) From Newton's law for impulsive force acting on a particle of mass  $m$ , initially resting at the origin, the governing equation of its displacement  $x(t)$  is written as 4+4+2

$$m \frac{d^2 x}{dt^2} = P \delta(t), \text{ where } P \text{ is a constant denoting the strength of the impulsive}$$

force. Show that the Laplace transform of  $x(t)$  may be written as  $\tilde{x}(s) = \frac{P}{ms^2}$ .

- (b) Apply Gram-Schmidt process to obtain an orthonormal set for the given vectors

$$\vec{A} = (-1, 0, 1); \vec{B} = (1, -1, 0); \vec{C} = (0, 0, 1) \text{ in } \mathbb{R}^3.$$

- (c) Find the value of  $\varepsilon_{ijk} \varepsilon_{ijk}$ , where  $\varepsilon_{ijk}$  is the antisymmetric Cartesian tensor in three dimensions.

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**WEST BENGAL STATE UNIVERSITY**  
B.Sc. Honours 5th Semester Examination, 2020, held in 2021

**PHSADSE02T-PHYSICS (DSE1/2)**

**ADVANCED DYNAMICS**

Time Allotted: 2 Hours

Full Marks: 50

*The figures in the margin indicate full marks.  
Candidates should answer in their own words and adhere to the word limit as practicable.  
Answer must be precise and to the point to earn credit.  
All symbols are of usual significance.*

**Question No. 1 is compulsory and answer any two from the rest**

1. Answer any **fifteen** questions from the following: 2×15 =30
- (a) What is a semi-holonomic constraint? Give an example.
  - (b) What do you mean by forces of constraint in a system?
  - (c) A bead is sliding along a smooth massless circular ring that is rotating about its diameter. State the nature of constraints present in the system.
  - (d) What is the significance of fundamental Poisson bracket in connection with the canonical transformations?
  - (e) If all the generalised coordinates  $(q_1, q_2, \dots, q_n)$  of a system become cyclic what will be the time dependence of those coordinates?
  - (f) What is the value of  $x$  if the following transformation is canonical?  
$$Q = \sqrt{2q}e^{-1-2x} \cos(p), \quad P = \sqrt{2q}e^{-1-x} \sin(p).$$
  - (g) State, with reasons, the number of degrees of freedom of a rigid body.
  - (h) Show that the angular velocity and the angular momentum of a rotating rigid body may not be parallel in general.
  - (i) A semi-circular arc and a quadrant of a circle, each of mass  $M$  and radius  $R$ , are rotating separately about an axis passing through the centres of their respective circles and perpendicular to their planes. Find the ratio of their moments of inertia.
  - (j) What is meant by a spherical top? Write down Euler's dynamical equations for it.
  - (k) Explain the difference between "local" and "global" minima in case of a generic potential for one dimensional motion.
  - (l) A particle of mass  $m$  in one dimensional motion along  $x$ -axis ( $x > 0$ ) with its potential energy given by  $V(x) = x + 1/x$ , is executing small oscillation near its stable equilibrium. Find the frequency of the oscillation.
  - (m) Two identical point masses (each of mass  $m$ ), connected by a massless spring (with spring constant  $k$ ), are placed on a smooth horizontal table. Find the frequency of oscillation when the spring is stretched and then released.

- (n) What is meant by a nonlinear dynamical system?
- (o) Draw the 2D phase space diagram of a point particle of mass  $m$  falling freely under the action of earth's gravity.
- (p) Show that an oscillatory motion of a system far away from its stable equilibrium configuration under an arbitrary potential, is nonlinear in general.
- (q) Show that for a first order dynamical system (with,  $\dot{x} = f(x)$ ), there are no periodic solutions.
- (r) What is meant by a limit cycle in a dynamical system?
- (s) Draw a typical phase space trajectory of damped harmonic oscillator under over-damped condition. What is the nature of its fixed point?
- (t) Explain the idea of 'negative damping' in connection with van-der-Pol oscillator.
2. (a) Consider the planar motion of a point particle of mass  $m$  suspended from a point with the help of a massless rigid rod of length  $l$ . Find the expression of the force of constraint. 4+4+2
- (b) Identify the fixed points on the phase space of the system and comment on their nature with justification.
- (c) Find the frequency of small amplitude oscillation of a system. Will the frequency differ if the amplitude increases?
3. (a) Show that the Poisson bracket is invariant under canonical transformation. 4+4+2
- (b) Four particles each of mass  $m$  are placed at the corners of a square of side  $a$  in the  $x-y$  coordinate system. One corner of the square is coincident with the origin and its two sides lie along  $x$ -axis and  $y$ -axis respectively.  $x'-y'$  coordinate system is obtained by rotating the  $x, y$  axes through an angle  $\theta$  about an axis passing through the origin and perpendicular to the plane of the square. What is the value of  $\theta$  if the  $x', y'$  axes be the principal axes for the system of masses?
- (c) Determine the points of stable and unstable equilibrium for a potential
- $$V(x) = -\frac{1}{2}x^2 + \frac{1}{3}x^3,$$
- and sketch the typical nature of phase trajectories near those points.
4. (a) A light string is stretched with tension  $F$  between two fixed points A and D on a frictionless horizontal table. Two point particles B and C, each of mass  $m$ , are attached to the string at the points of trisection. Consider the small oscillations of the particles in the plane of the table and at right angle to the string. (1+2+2+1)  
+ 2+2
- (i) Draw a neat, labeled diagram of the arrangement.
- (ii) Derive the equations of motion for the above system.
- (iii) From the eigenvalue equation find the positive roots of the system.
- (iv) Draw two diagrams to illustrate the normal modes of oscillation of the system. Clearly label the phase and the anti-phase oscillations.
- (b) Find all the fixed points for  $\dot{x} = x^2 - 1$ , and classify them according to their nature of stability.
- (c) A bead (of mass  $m$ ) slides on a uniformly rotating (with angular velocity  $\omega$ ) straight wire in a force-free space. Find its Lagrangian.

5. (a) Find the canonical transformation generated by (2+1+1+2)

$$F_1(Q, q) = \lambda q^2 \cot Q,$$

+4

$\lambda$  being a constant. If the Hamiltonian in  $(q, p)$  representation is

$$H(q, p) = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 q^2,$$

find the Hamiltonian in  $(Q, P)$  representation. Choose  $\lambda$  to make this Hamiltonian independent of  $Q$  and hence find the equation of motion in each representation.

- (b) Hamiltonian of an anharmonic oscillator is given by,

$$H(q, p) = \frac{p^2}{2m} + x^4 - x^2 + \alpha^2 x^2,$$

$\alpha$  being a positive parameter. Find the value of  $\alpha$  across which a bifurcation occurs in the system, clearly showing the change of the number and nature of the fixed points in the system across that value of  $\alpha$ .

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**WEST BENGAL STATE UNIVERSITY**  
B.Sc. Honours 5th Semester Examination, 2020, held in 2021

**PHSADSE03T-PHYSICS (DSE1/2)**

**NUCLEAR AND PARTICLE PHYSICS**

Time Allotted: 2 Hours

Full Marks: 50

*The figures in the margin indicate full marks.  
Candidates should answer in their own words and adhere to the word limit as practicable.  
All symbols are of usual significance.*

**Question No. 1 is compulsory and answer any two from the rest**

1. Answer any **fifteen** questions from the following: 2×15 = 30
- (a) What are isotones? Give an example.
  - (b) What is meant by 'saturation' of nuclear force?
  - (c) What does non-zero quadrupole moment of a nucleus signify? Additionally, what does its sign indicate?
  - (d) Which nuclei would you expect to be more stable –  ${}^7\text{Li}_3$  or  ${}^8\text{Li}_3$ ? Justify your answer.
  - (e) Using the extreme single particle shell model determine the ground state spin-parity of  ${}^{25}\text{Mg}_{12}$ .
  - (f) What is meant by 'internal conversion'?
  - (g) Complete the nuclear reaction  ${}^{15}\text{N}_7 (p, d)$ , including the compound nucleus in the intermediate stage.
  - (h) Explain very briefly how Coulomb interaction affects the probabilities of  $\beta^+$  and  $\beta^-$  decays.
  - (i) What is the most common way of absorption of low energy  $\gamma$ -rays? What are the other methods of absorption of  $\gamma$ -rays in matter?
  - (j) What are the reasons for straggling of range of  $\alpha$ -particles?
  - (k) Explain why electrons cannot be a part of a nucleus.
  - (l) In connection with strong force, what are charge independence and charge symmetry?
  - (m) Define range of an  $\alpha$ -particle in a medium. Why is it expressed in  $\text{kg/m}^2$  unit?
  - (n) What is pair production? Why cannot it take place in vacuum?
  - (o) Calculate the energy released by fission of 1 kg of  ${}^{235}\text{U}$  in kWh. Given: Energy released per fission is 200 MeV and Avogadro's number is  $6.03 \times 10^{23}$ .
  - (p) What is nuclear magneton? Write down its expression.
  - (q) The isospin, baryon number, and strangeness of a particle are given by  $I = 0$ ,  $B = +1$  and  $S = -3$  respectively. Find the electric charge of the particle.
  - (r) What are the quark contents of a proton and an electron?

- (s) Define Q-value of a nuclear reaction.
- (t) In case of intrinsic spin of a neutron, what is the gyromagnetic ratio? Comment on the value.
2. (a) What do you mean by binding energy of a nucleus? Write down its formal expression. Show graphically the variation of average binding energy per nucleon with mass number and hence discuss the stability of a nucleus. 2+3
- (b) Assuming that  $1\text{ u} = 1.66 \times 10^{-27}\text{ kg}$  and the radius of a nucleus to be given by  $R = r_0 \times A^{1/3}$  ( $r_0 = 1.2\text{ fm}$ ,  $A = \text{mass number}$ ), calculate the density of nuclear matter. 3
- (c) What is the parity of a  $p$ -electron? Is it same as that of  $^{16}\text{O}$  nucleus in its ground state? 2
3. (a) What is Bohr's 'independence hypothesis' for a compound nuclear reaction? 3
- (b) What is a resonant reaction? Is it as fast as a direct nuclear reaction? 1+1
- (c) Calculate the threshold energy required to initiate the reaction  $^{31}\text{P} (n, p) ^{31}\text{Si}$ . 3  
(Given  $m_p = 1.00814\text{ u}$ ,  $m_n = 1.00898\text{ u}$ ,  $M_P = 30.98356\text{ u}$  and  $M_{\text{Si}} = 30.98515\text{ u}$ .)
- (d)  $^{238}\text{U}$  is 'fertile' whereas  $^{235}\text{U}$  is 'fissile'— very briefly give the meanings. 2
4. (a) Write down the Bethe-Weizsäcker semi-empirical mass formula of a nucleus, clearly mentioning the meaning of each term. 5
- (b) Find the Cerenkov radiation angle for an electron moving with velocity  $0.577c$  inside a material of refractive index 2.0. 2
- (c) Why is the energy spectrum of  $\alpha$ -particles usually mono-energetic whereas  $\beta$  shows a continuous energy spectrum? 3
5. (a) What are "isotopic spin" and "strangeness" of an 'elementary particle'? Are the following reactions possible? Give reasons. 2+2
- (i)  $n \rightarrow p + e^- + \bar{\nu}_e$ , (ii)  $\pi^+ + n \rightarrow p + \pi^-$
- (b) A hadron has quark content  $u\bar{s}$ . Find the baryon number, charge, spin and strangeness of this hadron. 2
- (c) The decay  $\Sigma^0 \rightarrow \Lambda^0 + \gamma$  occurs in nature; whereas the apparently similar decay  $\Sigma^+ \rightarrow p + \gamma$  never occurs. What is the reason? 2
- (d) Identify the type of interaction (strong, weak or electromagnetic) which is responsible for each of the following decays: 2
- (i)  $\pi^0 \rightarrow \gamma + \gamma$  (ii)  $K^+ \rightarrow \mu^+ + \pi^0 + \nu_\mu$

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