ससन विश्वविद्यालय

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2022-23

PHSACOR01T-PHYSICS (CC1)

Time Allotted: 2 Hours — Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any ***ten*** questions from the following: $2\times10=20$

(a) Sketch: $f(x)=x^2e^{-x}$ for $x\geq 0$ and indicate the maxima.

(b) Examine the convergence of the series $\sum\limits_{n=1}^{\infty}\frac{1}{n^p}$.

(c) Find the first three terms in the Maclaurin's series expansion of $\tan^{-1}x$.

(d) $\left[y+x\left(\frac{dy}{dx}\right)^2\right]^{4/3}=x\frac{d^2y}{dx^2}$ Find the degree and order of the differential equation.

(e) Show that the function 1, $\sin x$ and $\cos x$ are linearly independent.

(f) Given $u=\frac{1}{r}$ where $r=\sqrt{x^2+y^2+z^2}$; Evaluate $\nabla^2 u$.

(g) Show that $\vec{A}\cdot(\vec{A}\times\vec{C})=0$.

(h) In which direction from the point (1, 3, 2) is the directional derivative of $\mathit{\Phi}=2xz-y^2$ is maximum? What is the magnitude of this maximum?

(i) Prove that $\vec{\nabla}\times(\varphi\vec{A})=\varphi\vec{\nabla}\times\vec{A}+\vec{\nabla}\varphi\times\vec{A}$.

(j) Find $\vec{\nabla}\times(\vec{r}f(r))=\vec{0}$.

(k) State Divergence theorem in vector calculus.

(l) Find the unit vector to the sphere $x^2+y^2+z^2=9$. Hence evaluate $\oiint\hat{r}\cdot d\vec{S}$, where S is the surface of the given sphere.

(m) A problem of physics is given to three students A, B, C whose chance of solving it $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved?

(n) If $f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(x-\mu)^2}{2\sigma^2}}$ is a probability density function find out the mean and draw the distribution.

2. (a) Show that $\vec{A}=(4xy-z^3)\hat{i}+2x^2\hat{j}-3xz^2\hat{k}$ is an irrotational vector. Then find the corresponding potential. (2+3)+3+2

(b) Solve the ordinary differential equation $x\frac{dy}{dx}+3x+y=0$.

(c) A man is known to speak the truth 3/4 times. He draws a card and reports it is king. Find the probability that it is actually a king.

3. (a) Find the particular solution of the following differential equation 3+3+4

$$\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=12xe^{3x}$$

(b) Find the shortest distance from the origin to the plane $x-2y-2z=3$ using Lagrange undetermined multiplier method.

(c) Express Cartesian $\vec{\nabla}$ operator in cylindrical coordinate.

4. (a) Express the integral (1+1+1)+3 +(1+3)

$$I=\int_0^1 dx\int_0^{\sqrt{1-x^2}} e^{-(x^2+y^2)}dy$$

as an integral in polar coordinates (r,θ). Find the new limits by sketching the region of integration and evaluate it.

(b) Show that $\vec{F}=(2xy-z^3)\hat{i}+x^2\hat{j}-(3xz^2+1)\hat{k}$ is conservative, and find a scalar potential ϕ such that $\vec{F}=-\vec{\nabla}\phi$.

(c) Show, by performing convergence tests, that the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}x^n}{n}$ is convergent for the values of x lies in the range $-1<x\le 1$.

5. (a) Given $\vec{V}=4y\hat{i}+x\hat{j}+2z\hat{k}$, find: 4+2+1+3

$$\iint \vec{\nabla}\times\vec{V}\cdot d\vec{S}$$

over the hemisphere $x^2+y^2+z^2=a^2, z\ge 0$.

(b) Show that $\vec{\nabla}\cdot\vec{\nabla}\times\vec{A}=0$ for any vector $\vec{A}$.

(c) Write down the expression for Poission distribution function.

(d) The number of particle emitted each minute by a radioactive source is recorded for a period of 10 Hrs., a total of 1800 counts are registered. During how many 1-minute intervals should we expect to observe on particle. [Consider $e^{-3}\approx 0.05$].

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2022-23

PHSACOR02T-PHYSICS (CC2)

MECHANICS

Time Allotted: 2 Hours | Full Marks: 40

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All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any ***ten*** questions from the following: $2\times 10 = 20$

(a) What is geostationary satellite? What is its time period of revolution?

(b) A solid sphere and a solid cylinder having same mass and same radii roll down an inclined plane without slipping. Show that the sphere will reach the bottom first.

(c) Show that theoretically Poisson's ratio lies between -1 and 0.5.

(d) Show that Lorentz transformation leads to Galilean transformation when the relative velocity (v) is much less than the velocity of light (c).

(e) Prove that the force field,

$$\vec{F} = (yz - y)\hat{i} + (xz - x - 1)\hat{j} + (xy - 2z)\hat{k}$$ is conservative.

(f) Prove that Kepler's first and second laws lead to the conservation of angular momentum.

(g) What is Coriolis force? Write down an expression for this force in terms of the angular velocity of the rotating frame of reference.

(h) What do you understand by the terms "proper length of a body" and "proper time"?

(i) What do you understand by Quality factor (Q)? Give brief explanation.

(j) A bar pendulum oscillates about a point 15 cm away from its C.G., with a period of 1 sec. Calculate the distance between the C.G. and centre of oscillation.

(k) A particle moves along the curve $r = ae^{\theta}$ with constant angular velocity. Show that the radial acceleration is zero and the cross-radial acceleration varies with its distance from the origin.

(l) Using Newton's law of gravity, determine the dimension of the universal Gravitational constant.

(m) Calculate the angular momentum $\vec{L}$ of a rigid body spinning steadily about a fixed axis with angular velocity $\vec{\omega}$. Are the directions of $\vec{\omega}$ and $\vec{L}$ necessarily the same? Explain.

(n) A flat plate of area $2\times 10^{-2}\,\text{m}^2$ is separated from a large flat surface by a film of oil of uniform thickness $1.5\times 10^{-3}\,\text{m}$ and viscosity $2\,\text{N-s/m}^2$. Determine the force required to slide the plate over the surface at a velocity of 4.5×10^{-2} m/s.

2. (a) Derive the continuity equation for fluid motion, clearly describing the notations used.

(b) Establish the expression for the internal bending moment of a beam.

(c) A metal rod of length L and cross-section α suffers a small longitudinal strain and is streched by l in length. Show that the potential energy stored in the rod due to this strain is $\frac{Y\alpha l^2}{2L}$, where Y is the Young's modulus of the material.

3. (a) Show that the differential equation of motion of a particle of mass 'm' under the influence of a central force $F(r)$ can be written as 4

$$\frac{d^2u}{d\theta^2}+u=-\frac{m}{L^2u^2}F\left(\frac{1}{u}\right),$$

where L is the angular momentum and $u=\frac{1}{r}$, r and θ being the polar coordinates.

(b) Show that under a central force, the total energy of the system remains constant. 3

(c) Two balls of masses m and $3m$ moving with velocities of equal magnitude but opposite direction undergo linear elastic collision. Find the final velocities of the two balls. 3

4. (a) Prove that the total angular momentum of a system of particles about any point is the sum of the angular momentum of the total mass assumed to be located at the centre of mass and the total angular momentum of the system about the centre of mass. 4

(b) Solve the equation of motion of a simple harmonic oscillator subject to the following forces: 3

(i) a damping force which is proportional to velocity and

(ii) an external sinusoidal force.

(c) Show that the self gravitational potential energy of an uniform sphere of mass M and radius R is $-\frac{3}{5}\frac{GM^2}{R}$. 3

5. (a) State the postulates of Galilean relativity and special relativity. 3

(b) Two reference frames S_1 and S_2 are moving with a uniform velocity 'v' relative to each other. If 'u_1' and 'u_2' are velocities of a body in the two frames respectively, then using Lorentz transformation equations, show that, 4

$$u_1=\frac{u_2+v}{1+\frac{u_2v}{c^2}},$$

where 'c' is the velocity of light.

(c) Quality Factor for a damped oscillator is 10^3. Its frequency is 512 Hz. Find the time taken for its energy to get reduced to $1/e$ of its energy in the absence of damping. 3

How many oscillations are executed during this time?

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2021-22

PHSACOR01T-PHYSICS (CC1)

MATHEMATICAL PHYSICS-I

Time Allotted: 2 Hours | Full Marks: 40

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Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any ***ten*** questions from the following: $2\times10=20$

(a) Sketch: $f(\theta)=\sin\theta\cos\theta$ for $0\leq\theta\leq4\pi$.

(b) If $\vec{r}$ be the position vector of a point on a closed contour, prove that the line integral $\oint\vec{r}\cdot d\vec{r}=0$.

(c) If $\vec{A}$ and $\vec{B}$ are irrotational then prove that $\vec{A}\times\vec{B}$ is solenoidal.

(d) Find the Taylor series of the function $f(x)=\dfrac{1}{x^2+4}$ about the point $x=0$.

(e) State the Uniqueness theorem of the solution of a differential equation for initial value problems.

(f) Find the volume of the parallelepiped whose edges are represented by $\vec{A}=2\hat{i}-3\hat{j}+4\hat{k}$, $\vec{B}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{C}=3\hat{i}-\hat{j}+2\hat{k}$.

(g) The position vector of a particle is $\vec{r}(t)=\cos(\omega t)\hat{i}+\sin(\omega t)\hat{j}$, where ω is a constant. Prove that, at every instant, its velocity and acceleration are perpendicular to each other.

(h) Find the directional derivative of the scalar field $\phi(x,y,z)=2x^2+yz$ in the direction of the vector $\hat{i}+\hat{j}$ at the point (0, 1, –1).

(i) Find the value of k so that the average value of the function

$$f(t)=\frac{3t^2}{8}+kt \text{ in the range } 0\leq t\leq2 \text{ vanishes.}$$

(j) A particle moves along a curve, $x=2t^2$, $y=t^2-4t$ and $z=3t-5$ where "t" is time. Find its component velocity at time $t=1$ in the direction of vector $(\hat{i}-2\hat{j}+2\hat{k})$.

(k) Solve the following differential equation.

$$ye^y dx=(y^3+2xe^y)\,dy.$$

(l) Show that, $\vec{\nabla}\times\left(\dfrac{\vec{r}}{r^2}\right)=0$.

(m) In a normal distribution, 31% of items are under 45 and 8% are over 64. Find the mean and the standard deviation of the distribution.

(n) An integer is chosen at random from the first 200 positive integers. What is the probability that the integer chosen is divisible by 6 or 8?

2. (a) Evaluate $\iint_R x(y-1)\,dxdy$, where R is the region bounded by the parabola $y=1-x^2$ and $y=0$. 3+3+4

(b) Prove that for a scalar field ϕ,

$$\oint_S \phi\, d\vec{S} = \int_V (\vec{\nabla}\phi)\, dV\,,$$

where V is the volume bounded by the closed surface S.

(c) Solve: $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} + 6y = 2\sin(4x)$.

3. (a) State Green's theorem in a plane. 1+5+(1+3)

(b) Verify Gauss' divergence theorem for the vector field $\vec{F} = 4y\hat{i} - 2x\hat{j} + z^2\hat{k}$, where V is the volume bounded by the upper half surface of the sphere $x^2+y^2+z^2=1$ together with the plane $z=0$.

(c) State the condition of convergence of a Taylor series expansion. Find interval of x for which the Taylor series of $\ln(1+x)$ about $x=0$, converges.

4. (a) If $r=\sqrt{x^2+y^2}$ and $z=\phi(r)$, show that $\dfrac{\partial^2 z}{\partial x^2}+\dfrac{\partial^2 z}{\partial y^2}=\dfrac{1}{r}\phi'(r)+\phi''(r)$. (2+2)+3+3

(b) Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube.

(c) If $\vec{A}=(y-2x)\hat{i}+(3x+2y)\hat{j}$, compute the circulation of $\vec{A}$ about a circle C in the XY plane with center at the origin and radius 2, if C is traversed in the positive direction.

5. (a) Find the area of the ellipse described by $x=a\cos\theta$, $y=b\sin\theta$. 3+3+1+3

(b) Prove $\vec{\nabla}\cdot\left(\dfrac{\vec{r}}{r^3}\right)=0$, where $\vec{r}(\neq 0)$ is the position vector.

(c) What is meant by probability distribution function?

(d) A box ***A*** contains 2 white and 4 black balls. Another box ***B*** contains 5 white and 7 black balls. A ball is transferred from the box ***A*** to the box ***B***. Then a ball is drawn from the box ***B***. Find the probability that the drawn ball is white.

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2021-22

PHSACOR02T-PHYSICS (CC2)

MECHANICS

Time Allotted: 2 Hours | Full Marks: 40

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Candidates should answer in their own words and adhere to the word limit as practicable.
Answers must be precise and to the point to earn credit.
All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any ***ten*** questions from the following: $2\times10 = 20$

(a) Show that Newton's 2nd law is invariant under Galilean transformation.

(b) An object of mass *m* slides on the surface of a block, whose coefficient of friction is μ. Initially, the speed of the block is v_0 and it comes to rest after travelling a distance x_0. Find the value of μ.

(c) Determine whether the force field,

$$\vec{F} = (y^2z^3 - 6xz^2)\hat{i} + (2xyz^3)\hat{j} + (3xy^2z^2 - 6x^2z)\hat{k}$$

is "conservative" or not.

(d) Calculate the angular momentum of a rigid body spinning steadily about a fixed axis with angular velocity $\vec{\omega}$. Are the direction of $\vec{\omega}$ and $\vec{L}$ necessarily the same? Explain.

(e) Find the moment of inertia of a solid circular cylinder of radius *a*, height *h* and mass *M*, about an axis passing through the center of the cylinder and parallel to its height.

(f) Show that the areal velocity of a particle moving under central force is constant.

(g) State Kepler's law of planetary motion.

(h) What is Reynolds number? What is the significance of it?

(i) A plate of area 200 cm^2 rests on a layer of castor oil 1 mm thick. The coefficient of viscosity of castor oil is 15.5 poise. Calculate the force required to move the plate horizontally with a speed of 4 cm/sec.

(j) A rod of circular cross-section of length *l* and radius *r* is stretched such that the volume of the rod is not changed. Show that Poisson's ratio is $\frac{1}{2}$.

(k) The potential energy of a particle is $V = 3x^4 - 8x^3 - 6x^2 + 24x$. Find the points of stable and unstable equilibrium.

(l) What do you understand by Quality factor? What is its importance?

(m) What is the 'apparent mass' of an object with 1 kg rest mass when it has been accelerated to 92.9% of light's speed. Speed of light, $c = 3\times10^8\,\text{m/sec}$. Give your answer in kg.

(n) A particle of mass 100 gm is placed in a field of potential $U = 5x^2 + 10$ ergs/gm. Find the frequency of small oscillations.

2. (a) A body is thrown at an angle with the horizontal. Prove that the mechanical energy is conserved at every point of its motion. 3

(b) Write the general conditions for equilibrium of a system. What do you mean by stable and unstable equilibrium? 3

(c) An upward force $F = 196$ Newton is applied on a body of mass 10 kg till it is raised vertically upwards by a distance of 10 meters. Here the work done by F is much greater than the gain in gravitational potential energy. Show that the law of conservation of energy is satisfied here $(g = 9.8\,\text{m/s}^2)$. 3

(d) Define the centre of mass of a body. 1

3. (a) Show that the torsional rigidity of a hollow cylinder of outer and inner radii r_1 and r_2 respectively is given by $\dfrac{n\pi(r_1^4 - r_2^4)}{2l}$, where l is the length of the cylinder and n is the modulus of rigidity of the material. 4

(b) Show that for a body undergoing combined rotational and translational motion, the total angular momentum can be written as, $\vec{L} = \vec{L}_{CM} + M(\vec{r}_0 \times \vec{v}_0)$,where, $\vec{L}_{CM}$ is the angular momentum in the center of mass frame, $\vec{r}_0$ is position vector of the center of mass and $\vec{v}_0$ is the velocity of center of mass. 3

(c) Find the diameter of a gold wire which elongates by 1 mm when stretched by a force of 330 gm-wt and twists through 1 radian when equal and opposite torques of 145 dyne - cm are applied at its ends. Poisson's ratio for gold is 0.435. 3

4. (a) Derive Poiseuilles formula for the steady flow of an incompressible viscous liquid through a horizontal capillary tube of uniform cross section. 4

(b) An artificial satellite is moving in a circular orbit 600 km above the surface of a planet of radius $5.85\times10^3\,\text{km}$. The period of revolution of the satellite around the planet is 3.5 hours. Determine the average mass density of the planet. 3

(c) Water flows through a horizontal tube of length 20 cm and internal radius of 0.081 cm under a constant head of the liquid 20 cm high. In 12 minutes 864 c.c. of liquid issue from the tube. Calculate the viscosity of water and verify that the conditions of streamline flow exist. Given $r = 1000$, ρ of water is 1 gm/cc and $g = 981\,\text{cm/s}^2$. 3

5. (a) What is length contraction? Obtain the expression for the relativistic length contraction from Lorentz transformation equation. 3

(b) Draw the potential energy curve for a simple pendulum with respect to angular position 'θ', clearly indicating the positions of stable and unstable equilibrium. 2

(c) What are non-inertial frames and fictitious forces? Is the centrifugal force a fictitious one? Explain. 3

(d) State the postulates of special theory of relativity. 2

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2020, held in 2021

PHSACOR01T-PHYSICS (CC1)

MATHEMATICAL PHYSICS I

Time Allotted: 2 Hours | Full Marks: 40

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All symbols are of usual significance.

Question No. 1 is compulsory and answer any *two* from the rest.

1. Answer any ***ten*** questions from the following: 2×10=20

(a) Sketch: $f(\theta)=1+\frac{1}{2}\sin^2\theta$ for $0\le\theta\le 2\pi$.

(b) Show that $f(x)=\frac{|x|}{x}$ is discontinuous at $x=0$, where $f(0)=0$.

(c) If $d\varphi(x,\, y)=M(x,\, y)\,dx+N(x,\, y)\,dy$, where φ is a well-behaved function of its arguments, then show that

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$$

(d) Solve: $\frac{dy}{dx}+2xy=4x$

(e) Determine a unit vector perpendicular to the plane of $\vec{A}=2\hat{i}-6\hat{j}-3\hat{k}$ and $\vec{B}=4\hat{i}+3\hat{j}-\hat{k}$.

(f) The position vector $\vec{r}$ of any arbitrary point on the surface satisfies the equation $|\vec{r}\,|=k$, a constant. Identify the geometry of the surface and justify your answer.

(g) $\vec{r}$ is the position vector of an arbitrary point in a three-dimensional space. Using Cartesian coordinate system, find gradient of $1/|\vec{r}\,|$ at any point away from the origin.

(h) For a vector field $\vec{F}(x,\, y,\, z,\, t)$ show that $dF=(d\vec{r}\cdot\vec{\nabla})\,\vec{F}+\frac{\partial\vec{F}}{\partial t}\,dt$.

(i) Show that $\vec{\nabla}\times\vec{r}f(r)=0$ where r is the magnitude of position vector of any arbitrary point in three-dimensional space ($f(r)$ being differentiable everywhere).

(j) ϕ is a scalar function satisfying the equation $\nabla^2\phi=0$. Show that $\vec{\nabla}\phi$ is both solenoidal and irrotational.

(k) Show that $\oint\!\!\oint_S d\vec{S}=0$.

(l) A dice is thrown. What is the probability that the number obtained is a prime number?

(m) There are five green and seven red balls. Two balls are selected one by one without replacement. Find the probability that the first is green and the second is red.

(n) What is meant by a probability distribution function? Cite an example.

2. (a) Prove the identity $\vec{\nabla}.(\vec{A}\times\vec{B}) = \vec{B}.(\vec{\nabla}\times\vec{A}) - \vec{A}.(\vec{\nabla}\times\vec{B})$. Hence show that $(\vec{A}\times\vec{r})$ is solenoidal if $\vec{A}$ is irrotational. 3+1

(b) If $f(x,\ y,\ z) = 0$, then show that $\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$. 4

(c) A multiple-choice test consists of 100 questions. Answer to each question has four possible options among which only one is correct. If a student answers all the questions by guessing at random, then what is the expected number of correct answers given by him? 2

3. (a) Solve: $\frac{y}{x^2} + 1 + \frac{1}{x}\frac{dy}{dx} = 0$ 2

(b) Determine the constant a so that the following vector is solenoidal: 4

$$\vec{V} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+az)\hat{k}$$

(c) Calculate the mean and the variance of a binomial distribution. 2+2

4. (a) The relativistic sum w of two velocities u and v in the same direction is given by 4

$$\frac{w}{c} = \frac{\frac{u}{c} + \frac{v}{c}}{1 + \frac{uv}{c^2}}$$

If $u/c = v/c = 1-\alpha$, where $0 \le \alpha \le 1$, find w/c in powers of α. Show terms only up to α^3.

(b) Find the directional derivative of $\varphi(x,\ y,\ z) = x^2 y + xz$ at (1, 2, –1) along the direction of $\vec{A} = 2\hat{i} - 2\hat{j} + \hat{k}$. 3

(c) Use Green's theorem on a plane to show that the area bounded by a simple closed curve C is $\frac{1}{2}\oint_C (x\,dy - y\,dx)$. 3

5. (a) An integer N is chosen at random with $1 \le N \le 100$. What is the probability that N is a perfect square? 2

(b) Obtain the complementary function of the differential equation 3

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4y = \cos 2x$$

(c) Evaluate the line integral of $\vec{A}(x,\ y,\ z) = x^2\hat{i} + y^2\hat{j} - z^2\hat{k}$, from the origin to $(a,\ b,\ c)$, along the path given parametrically by $x = at^2,\ y = bt,\ z = c\sin(\pi t/2)$. Does the result depend on the path? Justify your answer. 3+2

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2020, held in 2021

PHSACOR02T-PHYSICS (CC2)

MECHANICS

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Question No. 1 is compulsory and answer any *two* from the rest

1. Answer any ***ten*** questions from the following: $2\times10 = 20$

(a) A solid spherical ball rolls on a table. What fraction of its total kinetic energy is rotational?

(b) What is the significance of "quality factor"?

(c) A 5 kg stone is dropped on a nail and drives the nail 0.025 m into a piece of wood. If the stone is moving at 10 m/s, when it hits the nail, calculate the average force exerted by the stone on the nail.

(d) A particle of mass m is moving in a circular path of radius r such that its centripetal acceleration a varies with time t as $a = k^2rt^2$, where k is a constant. Calculate the power delivered to the particle by the forces acting on it.

(e) A ball is dropped from a height h. When it bounces off the floor, its speed is 80% of what it was just before it hit the floor. Determine the height up to which the ball will now rise.

(f) Find the ratio of the radii of gyration of two objects, one of which is a solid sphere and the other is a thin spherical shell, both having same mass and same radius.

(g) A particle of unit mass undergoes one-dimensional motion such that its velocity varies according to $v(x) = \beta x^{-n}$, where β and n are constants, and x is the position of the particle. Obtain the acceleration of the particle as a function of x.

(h) What is the difference between angle of twist and angle of shear?

(i) What is a cantilever? What is its difference with an ordinary lever?

(j) A satellite of mass m orbits a planet of mass M in a circular orbit of radius R. Find out the time required for one revolution of the satellite around the planet.

(k) A wire of length L and radius r is elongated by applying a force. If the volume of the wire remains unchanged during the process, then determine the Poisson's ratio of the material of the wire.

(l) What are the differences between "streamline" flow and "turbulent" flow of fluid?

(m) Using Lorentz transformation relations express $(x^2 - c^2t^2)$ in terms of transformed spacetime coordinates (x', t').

(n) A particle moves on the x-axis according to the equation $x = A + B\sin\omega t$. Show that this is a simple harmonic motion.

2. (a) In a nonrelativistic, one dimensional collision, a particle of mass $2m$ collides with a particle of mass m initially at rest. If the particles stick together after the collision, what fraction of the initial kinetic energy is lost in the collision? 3

(b) The smallest and the largest speeds of a satellite are given by $v_{\min}$ and $v_{\max}$ respectively. The time period is T. Show that the semi major axis of the elliptic orbit is given by $\frac{T}{2\pi}\sqrt{v_{\min} v_{\max}}$. 3

(c) A cylinder is released from rest from the top of an inclined plane of angle θ. If the cylinder rolls without slipping through a distance l, what will be its final speed? 4

3. (a) Derive the continuity equation for fluid motion, clearly describing the notations used. 4

(b) The potential of an object is given by $U(x) = 5x^2 - 4x^3$. Determine the positions where the object is in equilibrium and find the nature of the equilibria. 2+2

(c) If the speed of water in pipe with a diameter of 12 cm is 10 cm/s, what is the speed of water in a pipe with a diameter of 8 cm? 2

4. (a) State the postulates of Galilean relativity. State the postulates of special theory of relativity. 2+2

(b) Two inertial frames S and S' move with respect to each other with a constant speed $0.8c$ along the common y-axis. Write down the Lorentz transformation equations between the S and S' frames. 3

(c) Determine the theoretically admissible range of values of the Poisson's ratio. 3

5. (a) Determine the gravitational potential at a distance r from the centre, inside a homogeneous solid sphere of radius R and mass M. 4

(b) For a damped harmonic oscillator the equation of motion is given by 2+2+2

$$m\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + kx = 0$$

Using the solution of the above equation, calculate

(i) The period of oscillation.

(ii) The number of oscillations in which its mechanical energy will drop to one-half of its initial value.

(iii) The quality factor.

Given that $m = 0.25$ kg, $\gamma = 0.7$ kg/s and $k = 85$ N/m.

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2019

PHSACOR01T-PHYSICS (CC1)

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No.1 and any *two* questions from the rest

1. Answer any ***ten*** questions from the following: $2\times 10 = 20$

(a) Prove that the series $x+\frac{x^2}{2}+\frac{x^3}{3}+\cdots$ is convergent for $0<x<1$.

(b) Find the general solution of the equation

$$2x\frac{dy}{dx}+y=2xe^{5/2}$$

(c) A function $f(x)$ is defined by

$$\begin{aligned} f(x) &= \cos x \ \text{ for } x\geq 0 \\ &= -\cos x \ \text{ for } x<0 \end{aligned}$$

Is $f(x)$ continuous at $x=0$? Give reasons.

(d) If $f(x)=|x|$, show that $f(0)$ is a minimum although $f'(0)$ does not exist.

(e) Prove $\nabla^2 f(r)=\frac{d^2f}{dr^2}+\frac{2}{r}\frac{df}{dr}$

(f) Calculate the Laplacian of the scalar field $\ln(x^2+y^2)$.

(g) Expand $f(x)=\frac{1}{x-2}$ in a Taylor series about the point $x=1$.

(h) Wronskian of two functions is $\omega(t)=t\sin^2 t$. Are the functions linearly independent or linearly dependent? Explain.

(i) If $\vec{A}=2\hat{i}+\hat{j}-3\hat{k}$ and $\vec{B}=\hat{i}-2\hat{j}+\hat{k}$, find a vector of magnitude 5 perpendicular to both $\vec{A}$ and $\vec{B}$.

(j) Show that $\vec{\nabla}\phi$ is a vector perpendicular to the surface $\phi(x,y,z)=c$ where c is a constant.

(k) Find an expression for ds^2 in curvilinear co-ordinates u, v, w. Then determine ds^2 for the special case of an orthogonal system.

(l) An integer is chosen at random from the first 200 positive integers. What is the probability that the integer chosen is divisible by 6 or 8?

(m) A distribution function is given by $f(x) = \dfrac{1}{\pi\sqrt{A^2 - x^2}}$ where x is the random variable. Find the value of $\langle |x| \rangle$. (Assume 'A' is a constant).

(n) Show that, for large number of trials, Binomial distribution yields Poisson distribution.

2. (a) Check if the following differential equation $\dfrac{dy}{dx} - y\tan x = e^x \sec x$ is exact. Hence, solve the equation. 1+3

(b) Show that $\vec{A}\cdot(\vec{B}\times\vec{C})$ is an absolute value equal to the volume of a parallelepiped with sides $\vec{A}, \vec{B}$ and $\vec{C}$. Hence, find the condition for these vectors to be coplanar. 2+1

(c) Find the extrema of $f(x, y) = 5x - 3y$ subject to the constraint $x^2 + y^2 = 136$. 3

3. (a) Solve: $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} + 2y = e^{2x}\sin x$. 6

(b) Show that $r^n\vec{r}$ is an irrotational vector for any value of n, but is a solenoidal only if $n = -3$. 4

4. (a) Suppose that X is exponentially distributed with $\lambda = 3$. $2\frac{1}{2}+2\frac{1}{2}+1$

(i) What is $P\{X > 2\}$?

(ii) What is $P\{X > 5 \mid X > 3\}$?

(iii) What did you notice about these two answers? Is it a coincidence?

(b) A person makes steps of length 'l' is just likely to step forwards as backwards. Prove that after n steps in this random walk, the person will have gone forward a distance rl with a probability $\left(\frac{1}{2}\right)^n \, {}^nC_{\frac{n+r}{2}}$. 4

5. (a) Find the expression of Laplacian in cylindrical coordinate system. 4

(b) Prove that $\oint u\nabla v\cdot d\lambda = -\oint v\nabla u\cdot d\lambda$ 3

(c) Evaluate $\oint_C y^3dx - x^3dy$ where C is the positively oriented circle of radius 2 centered at the origin. 3

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D^2+3D+2

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2019

PHSACOR02T-PHYSICS (CC2)

MECHANICS

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *two* from the rest

1. Answer any ***ten*** questions from the following: $2\times10=20$

(a) Find the equation of motion for center of mass vector of an isolated system consisting of two particles interacting under a central force.

(b) A solid cylinder of mass 1 kg, length 20 cm and diameter 4 cm rotates about its own axis. Find the kinetic energy necessary to rotate the cylinder at 100 r.p.m.

(c) Power supplied to a particle of mass 2 kg varies with time as $P=\frac{3t^2}{2}$ Watt (t in second). If velocity of the particle at $t=0$ is zero. Find the velocity of the particle at $t=2$ s.

(d) A rocket is set for vertical projection. If the exhaust speed is $10^3\,\text{ms}^{-1}$, how much gas must be ejected per second to supply the thrust needed (i) to overcome the weight of the rocket (ii) to give the rocket an initial vertical upward acceleration of $19.6\ \text{ms}^{-2}$. Given mass of the rocket $=6\times10^3\,\text{kg}$.

(e) Suppose the radius of gyration and the distance of the centre of mass of a body from the axis of rotation are 'k' and 'R' respectively. Explain which of them is bigger.

(f) Differentiate clearly between inertial and non-inertial frame of reference.

(g) A metal wire does not change its volume when stretched, find the value of the Poisson's ratio of the metal.

(h) How much energy would have to be supplied to break up the earth completely (i.e. remove all masses to ∞).

(i) The radius of the earth is 6400 km, its mean density is $5.62\times10^3\,\text{kg}\cdot\text{m}^{-3}$ and the Gravitational constant is 6.658×10^{-11} S.I. units. Calculate the potential at the earth's surface, assuming earth to be a solid sphere.

(j) Show that areal velocity of a particle moving under central force is constant.

(k) What do you understand by Quality factor (Q)? Give brief explanation.

(l) A rod of length 1 m moves past an observer standing on the ground with velocity $3\times10^7\,\text{m/s}$, along the direction of its length. What is the apparent length of the rod with respect to the observer?

(m) What do you understand by the terms "proper length of a body" and "proper time"?

(n) Describe briefly about trilateration in the case of GPS system.

2. (a) Write down the expression of Newton's 2^{nd} law of motion in an accelerated frame of reference. Give brief explanation of the terms involved. 2

(b) Starting from Newton's law show that,

$$\int_{x_a}^{x_b} F(x)dx = \frac{1}{2}mv_b^2 - \frac{1}{2}mv_a^2$$

3

(c) Two balls of masses m and $3m$ moving with velocities of equal magnitude but opposite direction undergo linear elastic collision. Find the final velocities of the two balls. 3

(d) Define the centre of mass of a body. Show that it is unique for a body. 1+2

3. (a) Determine the moment of inertia of a solid cylinder of mass M, length L and radius R about the axis perpendicular to its length and passing through its centre of mass. 4

(b) Two horizontal capillary tubes are connected together in series so that a steady stream of water flows through them. The first tube is 0.4 mm in internal radius and 256 cm long while the other is 0.3 mm in internal radius and 40 cm long. The pressure of water at the entrance is 8 cm of mercury above the atmosphere and that at the exit is atmospheric (76 cm of Hg). Calculate the pressure at the junction of the tube. 3

(c) A horizontal wire of length $2l$ and cross-section α is tightly fixed to rigid supports at both ends. A weight W is suspended from the mid-point of the wire. Show that depression 'd' of the mid-point is given by 3

$$d^3 = \frac{Wl^3}{Y\alpha}$$

4. (a) A particle of mass m moves under the action of a central force whose potential is 4

$$V(r) = kr^3 (k > 0)$$

For what energy and angular momentum will the orbit be a circular of radius a about the origin.

(b) An artificial satellite circles around the earth in the plane of the equator concentric with earth in such a way that the direction of rotation of the satellite is same as the direction of rotation of the earth about its own axis. Find the expression for time-period of the revolution of the satellite. Sketch the appropriate figure. 4

(c) Write down the expression for coriolis force. Describe the effect of coriolis force on moving bodies on earth. 2

5. (a) Find out the energy of a particle executing simple harmonic motion. 3

(b) Write down the equation of motion of a damped harmonic oscillator and identify the damping factor. Graphically illustrate that how the energy of a damped oscillator varies with time. 2

(c) Two reference frames S and S' are moving with a uniform velocity 'v' relative to each other. If 'u' and 'u'' are velocities of a body in the two frames respectively, then using Lorentz transformation equations, show that, 5

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

'c' is the velocity of light.

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2019

PHSACOR02T-PHYSICS (CC2)

MECHANICS

Time Allotted: 2 Hours | Full Marks: 40

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$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

'c' is the velocity of light.

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2018

PHSACOR01T-PHYSICS (CC1)

MATHEMATICAL PHYSICS-I

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks
Candidates should answer in their own words and adhere to the word limit as practicable
All symbols are of usual significance

Answer Question No. 1 and any *two* questions from the rest

1. Answer any *ten* questions from the following: $2\times 10 = 20$

(a) Plot $f(x)=xe^{-x}$ for $x\geq 0$. Indicate any extremum within this range.

(b) Test whether $\cos(x)$ and $\sin(x)$ are linearly independent within the interval $-\infty < x < \infty$

(c) Expand $f(x,y)=xy$ in a Taylor series around $(1,1)$.

(d) Verify that $y=e^{t^2}\int_0^t e^{-s^2}ds+e^{t^2}$ is a solution of the differential equation $\frac{dy}{dt}-2ty=1$

(e) What do you mean by a regular singular point of a second order linear differential equation?

(f) Prove that the volume of a sphere with radius 'a' is $\frac{4}{3}\pi a^3$

(g) Find the value of $\varepsilon_{ijk}\varepsilon_{ijk}$, where ε_{ijk} = Levi-Civita symbol and assume Einstein's summation convention.

(h) Given a vector $\vec{A}=2x\hat{i}-3y\hat{j}+5z\hat{k}$. Verify that $l^2+m^2+n^2=1$, where the (l,m,n) are the directional cosines along three mutually perpendicular directions $(\hat{i},\hat{j},\hat{k})$.

(i) Find the directional derivative of the function $\Phi(x,y,z)=2xy+z^2$ in the direction of the vector $\vec{i}+2\vec{j}+2\vec{k}$ at the point $(1,-1,3)$.

(j) Find $\nabla^2 \gamma^4$ using cartesian coordinate where $\vec{\gamma}$ is the position vector of an arbitrary point.

(k) Using Gauss's theorem prove that

$$\oiint_s d\vec{s}=0$$

(l) Show that $\vec{\nabla}\times(\vec{r}f(r))=0$, where $\vec{r}$ is a position vector and $f(r)$ is differentiable.

(m) A dice is rolled and a coin is tossed simultaneously. Find the probability that the dice shows an odd number and the coin shows a head.

(n) If x is a random variable obeying Poisson distribution of probability, find the mean of x^2.

2. (a) Calculate the extreme values of the function $f(x, y, z) = xyz$, subject to the constraint $x^2 + y^2 + z^2 = 3$, using Lagrange's multipliers. 4+(3+2)+1

(b) Show that,

(i) $\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A}$ and

(ii) $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0$

(c) What is meant by a random variable?

3. (a) Show that the following differential equation is exact, 2+(1+3+1)+3

$$2xy\frac{dy}{dx} + y^2 - 2x = 0$$

(b) What is meant by linear independence of three vectors? Find the necessary and sufficient condition for three vectors $\vec{A}, \vec{B}$ & $\vec{C}$ (in three dimension) being linearly independent in terms of their scalar triple product. State the geometrical significance of the result.

(c) For an exhaustive set of events $\{A_1, A_2, \ldots, A_n\}$ which are mutually exclusive and B is another event defined on the same space, show that

$$P(A_j \mid B) = \frac{P(B \mid A_j)P(A_j)}{\sum_{i=1}^{n} P(B \mid A_i)P(A_i)}$$

4. (a) Solve the differential equation, 3+(2+3)+2

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4y = \cos(2x)$$

(b) State Gauss' divergence theorem.

Prove the vector integral theorem, $\int \vec{\nabla}\phi\, dV = \oint \phi\, d\vec{S}$, where $\phi(\vec{r})$ denotes a well behaved scalar field and $\vec{S}$ is the closed surface enclosing a volume V.

(c) Calculate the mean of a normal distribution, $f(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$ using the fact that the mean and variance of standard normal variate z is 0 and 1 respectively, given that the probability density function of z is, $g(z) = \frac{1}{\sqrt{2\pi}}e^{\frac{-z^2}{2}}$

5. (a) Find the expression of Laplacian in Spherical polar co-ordinate system. 5+(1+3)+1

(b) State the condition of convergence of a Taylar series expansion. Find interval of x for which the Taylor Series of $\ln(1+x)$ about $x = 0$, converges.

(c) What is meant by probability distribution function?

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2018

PHSACOR02T-PHYSICS (CC2)

MECHANICS

Time Allotted: 2 Hours

Full Marks: 40

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *two* from the rest

1. Answer any *ten* questions from the following: $2\times10=20$

(a) Show that the total external force applied for a system of particles equals the rate of change of the system's momentum.

(b) A particle of mass m moves along the x-axis under the influence of a conservative force field having potential $V(x)$. If the particle is located at position x_1 and x_2 at respective times t_1 and t_2, prove that if E is the total energy,
$$t_2-t_1=\sqrt{\frac{m}{2}}\int_{x_1}^{x_2}\frac{dx}{\sqrt{E-V(x)}}.$$

(c) Consider a particle of mass m moving in a potential $V(\theta)=2mg\cos\theta(L-l\cos\alpha)$, where $L, g, l, \alpha>0$ and $\theta<\frac{\pi}{2}$.

Find the condition of stable equilibrium.

(d) The moment of inertia of a circular disc about an axis perpendicular to its plane and passing through its center is $\frac{1}{2}Mr^2$, where M is mass of the disc and r is its radius. Show that the moment of inertia of the disc about an axis along any tangent to the circle is $\frac{5}{4}Mr^2$.

(e) Two bodies of masses M_1 and M_2 are placed with distance d apart. Show that at a point, where the gravitational field is zero, the potential is given by $V=-\frac{G}{d}\left(M_1+M_2+2\sqrt{M_1M_2}\right)$.

(f) Establish the relation $Y=K(1-2\sigma)$, where the symbols have their usual meaning.

(g) How does the coefficient of viscosity (η) of a liquid vary with temperature? Write down the dimension of η.

8

(h) Equation of a particle vibrating simple harmonically is $x = 10\sin\left(\omega t + \frac{\pi}{3}\right)m$.

What is the significance of $\frac{\pi}{3}$ occurring in the argument of sine?

(i) What is Reynold's number? Discuss its significance.

(j) If E_k, p and m_0 are the kinetic energy, momentum and rest mass respectively, of a relativistic particle, then show that $p = \frac{1}{C}(E_k^2 + 2m_0C^2E_k)^{1/2}$.

(k) Explain 'power-factor' using the expression for average power in case of a forced vibration. What is its value at resonance?

(l) Prove that when $v \ll c$, Lorentz transformations reduce to Galilean transformation.

(m) Can a reference frame attached to the earth be considered to be an inertial frame?

(n) The effective length of a simple pendulum is 'l' and its angular amplitude is θ_0. Find the velocity of the pendulum at its equilibrium position.

2. (a) What is the neutral layer? Establish the expression for the internal bending moment of a beam. 1+4

(b) At time t, the mass of a rocket moving under an external force $\vec{F}$ is M and its velocity is $\vec{v}$ with respect to certain reference frame. The fuel expulsion takes place at a constant velocity $\vec{u}$ relative to the rocket. Calculate the impulse $\vec{F}\Delta t$ and use it to show that $M\frac{d\vec{v}}{dt} + \vec{u}\frac{dM}{dt} = \vec{F}$. 3+2

Assuming a constant rate of change of mass, obtain the expression for $\vec{v}$ as a function of M from the above equation.

3. (a) Establish the equation of continuity for the flow of a fluid. 2

(b) A capillary tube of radius 'a' and length 'l' is fitted horizontally at the bottom of a cylindrical vessel of cross section 'A'. If initially the height is 'h_1' and liquid is allowed to flow, find the time required for the height to reduce to 'h_2'. (Assume the flow to be streamline). Mention any formula you have used. 3+1

(c) A particle follows an orbit given by $r = c\theta^2$ under an unknown force. Prove whether such an orbit is possible in a central field. If so, find the form of the force law. 4

4. (a) Prove that the total angular momentum of a system of particles about any point is the sum of the angular momentum of the total mass assumed to be located at the centre of mass and the total angular momentum of the system about the centre of mass. 4

(b) Consider two uniform spherical concentric shells having masses M_1, M_2 and radii R_1, R_2 $(R_2 > R_1)$ respectively. Find the gravitational force on a particle of 3 mass m placed at a distance d from the centre when (i) $d > R_2$, (ii) $R_1 < d < R_2$ and (iii) $d < R_1$.

(c) The half-life of a radioactive particle is 4×10^{-8} sec when its speed is 0.8c. If the speed of the particle is reduced by 25%, then what will be its half-life? 3

5. (a) The displacement of a particle of mass m subjected to an external periodic force $F_0e^{i\omega t}$, a restoring force proportional to displacement and resisting force proportional to its velocity is given by $y = y_1 + y_2$ 1+1+1+1

with $y_1 = Re^{-\frac{kt}{2}}\cos\left(\frac{\sqrt{4\omega_0^2 - k^2}}{2}t - \phi\right)$

and $y_2 = \dfrac{F_0}{m\sqrt{(\omega_0^2 - \omega^2)^2 + k^2\omega^2}}e^{i(\omega t - \phi)}$

(i) Explain physical meanings of y_1 and y_2.

(ii) What will be the expression for y after long time?

(iii) Name the physical phenomenon which will occur when $\omega = \omega_0$.

(b) Define coefficient of restitution. Two masses m_1 and m_2 travelling in the same straight line collide. Find the velocity of the particles after collision in terms of the velocities before collision. Hence show that kinetic energy is conserved for a perfectly elastic collision. 1+

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