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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2019

PHSACOR01T-PHYSICS (CC1)

Time Allotted: 2 Hours — Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No.1 and any *two* questions from the rest

1. Answer any ***ten*** questions from the following: $2\times 10 = 20$

(a) Prove that the series $x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots$ is convergent for $0 < x < 1$.

(b) Find the general solution of the equation

$$2x\frac{dy}{dx} + y = 2xe^{5/2}$$

(c) A function $f(x)$ is defined by

$$\begin{aligned} f(x) &= \cos x \text{ for } x \geq 0 \\ &= -\cos x \text{ for } x < 0 \end{aligned}$$

Is $f(x)$ continuous at $x = 0$? Give reasons.

(d) If $f(x) = |x|$, show that $f(0)$ is a minimum although $f'(0)$ does not exist.

(e) Prove $\nabla^2 f(r) = \frac{d^2 f}{dr^2} + \frac{2}{r}\frac{df}{dr}$

(f) Calculate the Laplacian of the scalar field $\ln(x^2 + y^2)$.

(g) Expand $f(x) = \frac{1}{x-2}$ in a Taylor series about the point $x = 1$.

(h) Wronskian of two functions is $\omega(t) = t\sin^2 t$. Are the functions linearly independent or linearly dependent? Explain.

(i) If $\vec{A} = 2\hat{i} + \hat{j} - 3\hat{k}$ and $\vec{B} = \hat{i} - 2\hat{j} + \hat{k}$, find a vector of magnitude 5 perpendicular to both $\vec{A}$ and $\vec{B}$.

(j) Show that $\vec{\nabla}\phi$ is a vector perpendicular to the surface $\phi(x, y, z) = c$ where c is a constant.

(k) Find an expression for ds^2 in curvilinear co-ordinates u, v, w. Then determine ds^2 for the special case of an orthogonal system.

(l) An integer is chosen at random from the first 200 positive integers. What is the probability that the integer chosen is divisible by 6 or 8?

(m) A distribution function is given by $f(x)=\frac{1}{\pi\sqrt{A^2-x^2}}$ where x is the random variable. Find the value of $<|x|>$. (Assume 'A' is a constant).

(n) Show that, for large number of trials, Binomial distribution yields Poisson distribution.

2. (a) Check if the following differential equation $\frac{dy}{dx}-y\tan x=e^x\sec x$ is exact. Hence, solve the equation. 1+3

(b) Show that $\vec{A}\cdot(\vec{B}\times\vec{C})$ is an absolute value equal to the volume of a parallelepiped with sides $\vec{A}, \vec{B}$ and $\vec{C}$. Hence, find the condition for these vectors to be coplanar. 2+1

(c) Find the extrema of $f(x, y)=5x-3y$ subject to the constraint $x^2+y^2=136$. 3

3. (a) Solve: $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=e^{2x}\sin x$. 6

(b) Show that $r^n\vec{r}$ is an irrotational vector for any value of n, but is a solenoidal only if $n=-3$. 4

4. (a) Suppose that X is exponentially distributed with $\lambda=3$. $2\frac{1}{2}+2\frac{1}{2}+1$

(i) What is $P\{X>2\}$?

(ii) What is $P\{X>5 \mid X>3\}$?

(iii) What did you notice about these two answers? Is it a coincidence?

(b) A person makes steps of length 'l' is just likely to step forwards as backwards. Prove that after n steps in this random walk, the person will have gone forward a distance rl with a probability $\left(\frac{1}{2}\right)^n\ {}^nC_{\frac{n+r}{2}}$. 4

5. (a) Find the expression of Laplacian in cylindrical coordinate system. 4

(b) Prove that $\oint u\nabla v\cdot d\lambda=-\oint v\nabla u\cdot d\lambda$ 3

(c) Evaluate $\oint_C y^3dx-x^3dy$ where C is the positively oriented circle of radius 2 centered at the origin. 3

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2019

PHSACOR02T-PHYSICS (CC2)

MECHANICS

Time Allotted: 2 Hours

Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *two* from the rest

1. Answer any ***ten*** questions from the following: $2\times 10=20$

(a) Find the equation of motion for center of mass vector of an isolated system consisting of two particles interacting under a central force.

(b) A solid cylinder of mass 1 kg, length 20 cm and diameter 4 cm rotates about its own axis. Find the kinetic energy necessary to rotate the cylinder at 100 r.p.m.

(c) Power supplied to a particle of mass 2 kg varies with time as $P=\frac{3t^2}{2}$ Watt (t in second). If velocity of the particle at $t=0$ is zero. Find the velocity of the particle at $t=2$ s.

(d) A rocket is set for vertical projection. If the exhaust speed is $10^3\,\text{ms}^{-1}$, how much gas must be ejected per second to supply the thrust needed (i) to overcome the weight of the rocket (ii) to give the rocket an initial vertical upward acceleration of $19.6\ \text{ms}^{-2}$. Given mass of the rocket $=6\times 10^3\,\text{kg}$.

(e) Suppose the radius of gyration and the distance of the centre of mass of a body from the axis of rotation are 'k' and 'R' respectively. Explain which of them is bigger.

(f) Differentiate clearly between inertial and non-inertial frame of reference.

(g) A metal wire does not change its volume when stretched, find the value of the Poisson's ratio of the metal.

(h) How much energy would have to be supplied to break up the earth completely (i.e. remove all masses to ∞).

(i) The radius of the earth is 6400 km, its mean density is $5.62\times 10^3\,\text{kg}\cdot\text{m}^{-3}$ and the Gravitational constant is 6.658×10^{-11} S.I. units. Calculate the potential at the earth's surface, assuming earth to be a solid sphere.

(j) Show that areal velocity of a particle moving under central force is constant.

(k) What do you understand by Quality factor (Q)? Give brief explanation.

(l) A rod of length 1 m moves past an observer standing on the ground with velocity 3×10^7 m/s, along the direction of its length. What is the apparent length of the rod with respect to the observer?

(m) What do you understand by the terms "proper length of a body" and "proper time"?

(n) Describe briefly about trilateration in the case of GPS system.

2. (a) Write down the expression of Newton's 2nd law of motion in an accelerated frame of reference. Give brief explanation of the terms involved. 2

(b) Starting from Newton's law show that, 2

$$\int_{x_a}^{x_b} F(x)dx = \frac{1}{2}mv_b^2 - \frac{1}{2}mv_a^2$$

(c) Two balls of masses m and $3m$ moving with velocities of equal magnitude but opposite direction undergo linear elastic collision. Find the final velocities of the two balls. 3

(d) Define the centre of mass of a body. Show that it is unique for a body. 1+2

3. (a) Determine the moment of inertia of a solid cylinder of mass M, length L and radius R about the axis perpendicular to its length and passing through its centre of mass. 4

(b) Two horizontal capillary tubes are connected together in series so that a steady stream of water flows through them. The first tube is 0.4 mm in internal radius and 256 cm long while the other is 0.3 mm in internal radius and 40 cm long. The pressure of water at the entrance is 8 cm of mercury above the atmosphere and that at the exit is atmospheric (76 cm of Hg). Calculate the pressure at the junction of the tube. 3

(c) A horizontal wire of length $2l$ and cross-section α is tightly fixed to rigid supports at both ends. A weight W is suspended from the mid-point of the wire. Show that depression 'd' of the mid-point is given by 3

$$d^3 = \frac{Wl^3}{Y\alpha}$$

4. (a) A particle of mass m moves under the action of a central force whose potential is 4

$$V(r) = kr^3 (k > 0)$$

For what energy and angular momentum will the orbit be a circular of radius a about the origin.

(b) An artificial satellite circles around the earth in the plane of the equator concentric with earth in such a way that the direction of rotation of the satellite is same as the direction of rotation of the earth about its own axis. Find the expression for time-period of the revolution of the satellite. Sketch the appropriate figure. 4

(c) Write down the expression for coriolis force. Describe the effect of coriolis force on moving bodies on earth. 2

5. (a) Find out the energy of a particle executing simple harmonic motion. 3

(b) Write down the equation of motion of a damped harmonic oscillator and identify the damping factor. Graphically illustrate that how the energy of a damped oscillator varies with time. 2

(c) Two reference frames S and S' are moving with a uniform velocity 'v' relative to each other. If 'u' and 'u'' are velocities of a body in the two frames respectively, then using Lorentz transformation equations, show that, 5

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

'c' is the velocity of light.

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 1st Semester Examination, 2019

MTMHGEC01T/MTMGCOR01T-MATHEMATICS (GE1/DSC1)

DIFFERENTIAL CALCULUS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Evaluate the right hand and left hand limits of the function $f(x)=\frac{|x|}{x}$ at the point $x=0$. Examine whether the function has a limit at 0.

(b) Find the points of discontinuity of the function $f(x)=\frac{x-1}{(x^2-1)(x-1)x}$.

(c) Verify Rolle's theorem for the function $f(x)=x^2-5x+10$ on $[2, 3]$.

(d) Investigate the extremum for the function $f(x)=2x^3-15x^2+42x+10$.

(e) Show that $\frac{\sin\alpha-\sin\beta}{\cos\alpha-\cos\beta}=\cot\theta$, where $0<\alpha<\theta<\beta<\frac{\pi}{2}$.

(f) Show that the function $f(x)=\sqrt{\frac{x^{1/3}+y^{1/3}}{x^{1/4}+y^{1/4}}}$ is homogeneous and find its degree.

(g) Find the points on the curve $y=x^2+3x+4$, where the tangents pass through the origin.

(h) If $y=\sin(m\sin^{-1}x)$, show that $(1-x^2)y_2-xy_1+m^2y=0$.

2. (a) Show that the limit that $\lim_{x\to 0}\sin\frac{1}{x}$ does not exist. 2

(b) If two functions f and g are continuous at a point c, then show that $f+g$ is also continuous at c. 3

(c) Discuss the continuity of the function $f(x)=|x-3|$ at $x=3$ and find $f'(3)$, if exists. 2+1

3. (a) If a function f is differentiable at some point c in its domain, then prove that it is also continuous at c. Give a suitable example to show that the converse of the above result is not true. 3

(b) If $u = \tan^{-1}\dfrac{x^3+y^3}{x-y}$, then prove that $x\dfrac{\delta u}{\delta x}+y\dfrac{\delta u}{\delta y}=\sin 2u$. 3

(c) Find the equation of the normal to the curve $x^2-y^2=a^2$ at the point $(a\sqrt{2}, a)$. 2

4. (a) If $y^{1/m}+y^{-1/m}=2x$, prove that $(x^2-1)y_{n+2}+(2n+1)xy_{n+1}+(n^2-m^2)y_n=0$. 4

(b) State and prove Euler's theorem on homogeneous functions. 4

5. (a) Find the radius of curvature at $(\frac{1}{4}, \frac{1}{4})$ of the curve $\sqrt{x}+\sqrt{y}=1$. 3

(b) State and prove Lagrange's mean value theorem. Write the geometrical interpretation of this theorem. 4+1

6. (a) Find the Taylor's series expansion of the function $f(x)=\sin x,\ x\in\mathbb{R}$. 5

(b) Determine the asymptotes of the curve $x=\dfrac{2t}{t^2-1},\ y=\dfrac{(1+t)^2}{t^2}$. 3

7. (a) Verify Rolle's theorem for the function $f(x)=x(x+3)e^{-x/2}$ in $[-3, 0]$. 4

(b) If $f:\mathbb{R}\longrightarrow\mathbb{R}$ is a differentiable function such that $f'(x)=0$ everywhere then show that $f(x)$ is a constant function on $\mathbb{R}$. 4

8. (a) If $f(h)=f(0)+hf'(0)+\dfrac{h^2}{2!}f''(\theta h),\ 0<\theta<1$, find θ when $h=1$ and $f(x)=(1-x)^{3/2}$. 3

(b) Discuss maxima and minima of the function $f(x)=(\frac{1}{x})^x,\ x>0$, if there be any. 5

9. (a) If $u=f(x, y),\ x=r\cos\theta,\ y=r\sin\theta$, then prove that 6

$$\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2=\left(\frac{\partial u}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$$

(b) If $f(x, y)=\dfrac{xy(x^2-y^2)}{x^2+y^2}$ for $(x, y)\neq(0, 0)$ and $f(0, 0)=0$ then find $f_x(0, 0)$ and $f_y(0, 0)$. 1+1

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